

## Inference Rules

1. Frequently used inference rules:

$$\begin{array}{c}
 \overline{F, \Gamma \vdash F} \\
 \\
 \frac{\Gamma \vdash F_1 \Rightarrow F_2 \quad \Gamma \vdash F_1}{\Gamma \vdash F_2} \quad \frac{F_1, \Gamma \vdash F_2}{\Gamma \vdash F_1 \Rightarrow F_2} \\
 \\
 \frac{\Gamma \vdash F}{\Gamma \vdash P \text{ says } F} \quad \frac{\Gamma \vdash P \text{ controls } F \quad \Gamma \vdash P \text{ says } F}{\Gamma \vdash F} \star \\
 \\
 \frac{\Gamma \vdash P \text{ says } (F_1 \Rightarrow F_2) \quad \Gamma \vdash P \text{ says } F_1}{\Gamma \vdash P \text{ says } F_2} \\
 \\
 \frac{\Gamma \vdash \forall x.F}{\Gamma \vdash F[x := t]} \\
 \\
 \frac{\Gamma \vdash F_1 \quad \Gamma \vdash F_2}{\Gamma \vdash F_1 \wedge F_2} \quad \frac{\Gamma \vdash F_1 \wedge F_2}{\Gamma \vdash F_1} \quad \frac{\Gamma \vdash F_1 \wedge F_2}{\Gamma \vdash F_2} \\
 \\
 \frac{\Gamma \vdash P \mapsto Q \quad \Gamma \vdash P \text{ says } F}{\Gamma \vdash Q \text{ says } F} \star
 \end{array}$$

2. Less frequently used inference rules:

$$\begin{array}{c}
 \frac{\Gamma \vdash \text{slev}(P) = l_1 \quad \Gamma \vdash \text{slev}(Q) = l_2 \quad \Gamma \vdash l_1 < l_2}{\Gamma \vdash \text{slev}(P) < \text{slev}(Q)} \\
 \\
 \frac{\Gamma \vdash \text{slev}(P) = l \quad \Gamma \vdash \text{slev}(Q) = l}{\Gamma \vdash \text{slev}(P) = \text{slev}(Q)} \\
 \\
 \frac{\Gamma \vdash l_1 < l_2 \quad \Gamma \vdash l_2 < l_3}{\Gamma \vdash l_1 < l_3} \\
 \\
 \frac{\Gamma \vdash P \mapsto Q \quad \Gamma \vdash Q \mapsto R}{\Gamma \vdash P \mapsto R} \star \\
 \\
 \frac{\Gamma \vdash F_1}{\Gamma \vdash F_1 \vee F_2} \quad \frac{\Gamma \vdash F_2}{\Gamma \vdash F_1 \vee F_2} \quad \frac{\Gamma \vdash F_1 \vee F_2 \quad F_1, \Gamma \vdash F_3 \quad F_2, \Gamma \vdash F_3}{\Gamma \vdash F_3} \\
 \\
 \frac{\Gamma \vdash F[x := c]}{\Gamma \vdash \forall x.F} \quad c \text{ must be a fresh variable} \\
 \\
 \overline{\Gamma \vdash \text{true}} \\
 \\
 \frac{\Gamma \vdash P \mapsto Q \quad \Gamma \vdash Q \text{ controls } F}{\Gamma \vdash P \text{ controls } F} \star
 \end{array}$$

★ derived rules