

# Security Engineering (6)

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Slides: KEATS (also homework is there)

# Hashes for History

## Q: What is the hash for?

### Dismantling Megamos Crypto: Wirelessly Lockpicking a Vehicle Immobilizer

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#### 1 Disclaimer

Due to a interim injunction, ordered by the High Court of London on Tuesday 25th June 2013, the authors are restrained from publishing the technical contents of the scientific article *Dismantling Megamos Crypto: Wirelessly Lockpicking a Vehicle Immobilizer* [1] until further notice.

#### 2 Historical claim

Figure 1 contains the cryptographic hash (SHA-512) of the original final paper which was scheduled to appear in the proceedings of the 22nd USENIX Security Symposium, Washington DC, August 2013.

```
9d05ba88740499eecea3d8609174b444
43683da139f78b783666954ccc605da8
4601888134bf0c23ba46fb4a88c056bf
bbb629e1ddffcf60fa91880b4d5b4aca
```

Figure 1: SHA-512 hash of the final paper

#### References

# Checking Solutions

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*“an **individual** leaf of paper or parchment, either loose as one of a series or forming part of a bound volume, which is numbered on the recto or front side only.”*

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- folio  $\xrightarrow{I}$  individual  
*“a single **human** being as distinct from a group”*

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- folio  $\xrightarrow{1}$  individual  $\xrightarrow{2}$  human

*“relating to **or** characteristic of humankind”*

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this is essentially a hash function...but Bob can only check once he has also found the solution

# Zero-Knowledge Proofs

Two remarkable properties of **Zero-Knowledge Proofs**:

- Alice only reveals the fact that she knows a secret, not the secret itself (meaning she can convince Bob that she knows the secret, but does not give it to him).
- Having been convinced, Bob cannot use the evidence in order to convince Carol that Alice knows the secret.

# Interactive Protocols

Q: How to cut a cake into two equal slices?



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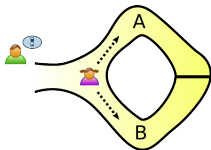


Solves the problem of communication when both parties distrust each other.

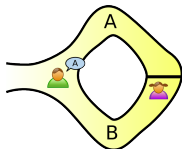
# The Idea

The Alibaba cave  
protocol:

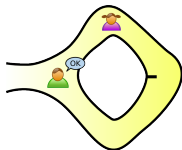
1.



2.

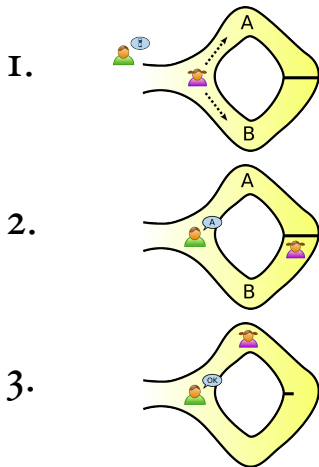


3.



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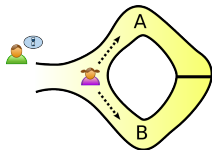


Even if Bob has a hidden camera, a recording will not be convincing to anyone else (Alice and Bob could have made it all up).

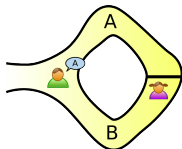
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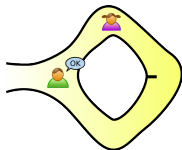
1.



2.



3.



Even worse, an observer present at the experiment would not be convinced.

# Applications of ZKPs

- authentication, where one party wants to prove its identity to a second party via some secret information, but doesn't want the second party to learn anything about this secret
- to enforce honest behaviour while maintaining privacy: the idea is to force users to prove, using a zero-knowledge proof, that their behaviour is correct according to the protocol

digital currencies, smart cards, id cards

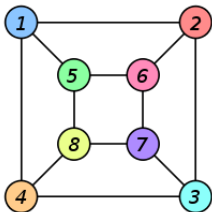
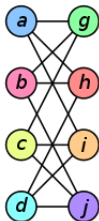


# Central Properties

Zero-knowledge proof protocols should satisfy:

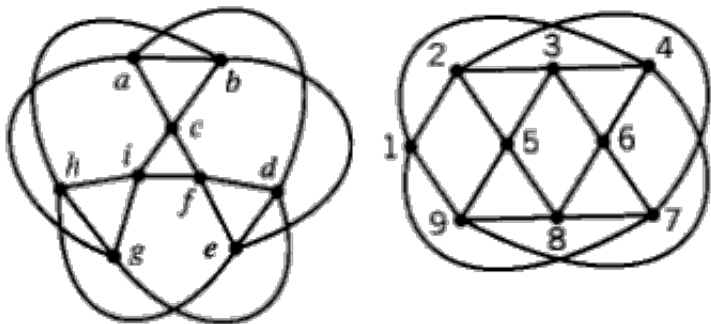
- **Completeness** If Alice knows the secret, Bob accepts Alice's "proof" for sure.
- **Soundness** If Alice does not know the secret, Bob accepts her "proof" with a very small probability.
- **Zero-Knowledge** Even if Bob accepts the proof by Alice, he cannot convince anybody else.

# Graph Isomorphism



Graph A	Graph B
Graph G <sub>1</sub>	Graph G <sub>2</sub>
a	1
b	6
c	8
d	3
g	5
h	2
i	4
j	7

Finding an isomorphism between two graphs is an NP problem.



Creating a new isomorphic graph is easy; finding an isomorphism is hard; checking an isomorphism is easy again

# Graph Isomorphism Protocol

Alice starts with knowing an isomorphism  $\sigma$  between graphs  $G_1$  and  $G_2$

- 1 Alice generates an isomorphic graph  $H$  which she sends to Bob
- 2 Bob asks either for an isomorphism between  $G_1$  and  $H$ , or  $G_2$  and  $H$
- 3 Alice and Bob repeat this procedure  $n$  times

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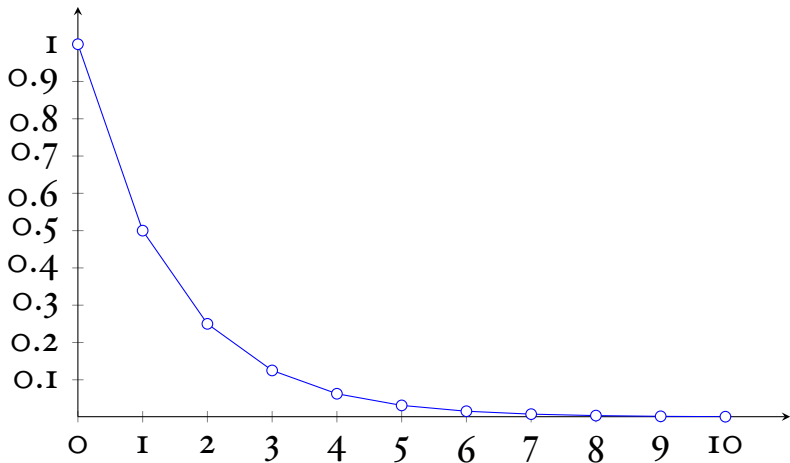
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- these are called commitment algorithms

# Graph Isomorphism Protocol (2)

If Alice knows the isomorphism, she can always calculate  $\sigma$ .

If she doesn't, she can only correctly respond if Bob's choice of index is the same as the one she used to form  $H$ . The probability of this happening is  $\frac{1}{2}$ , so after  $n$  rounds the probability of her always responding correctly is only  $\frac{1}{2}^n$ .

# Plot of $\frac{1}{2}^n$



# Graph Isomorphism Protocol (3)

Why is the GI-protocol zero-knowledge?



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Why is the GI-protocol zero-knowledge?

A: We can generate a fake transcript of a conversation, which cannot be distinguished from a “real” conversation.

Anything Bob can compute using the information obtained from the transcript can be computed using only a forged transcript and therefore participation in such a communication does not increase Bob’s capability to perform any computation.

# Non-Interactive ZKPs

This is amazing: This can all be done “offline”:

Alice can publish some data that contains no data about her secret, but this data can be used to convince anyone of the secret’s existence (whether Alice knows it, must be established by other means).

# Non-Interactive ZKPs (2)

Alice starts with knowing an isomorphism  $\sigma$  between graphs  $G_1$  and  $G_2$

- 1 Alice generates  $n$  isomorphic graphs  $H_{1..n}$  which she makes public
- 2 she feeds the  $H_{1..n}$  into a hashing function (she has no control over what the output will be)
- 3 Alice takes the first  $n$  bits of the output:  
whenever output is 0, she shows an isomorphism with  $G_1$ ; for 1 she shows an isomorphism with  $G_2$

# Problems of ZKPs

- “grand chess master problem”  
(person in the middle again)
- Alice can have multiple identities; once she committed a fraud with one, she stops using one

# Other Methods for ZKPs

Essentially every NP-problem can be used for ZKPs

- modular logarithms: Alice chooses public  $A$ ,  $B$ ,  $p$ ; and private  $x$

$$A^x \equiv B \pmod{p}$$

# Commitment Stage

- 1 Alice generates  $z$  random numbers  $r_1, \dots, r_z$ , all less than  $p - 1$ .

- 2 Alice sends Bob for all  $i..z$

$$b_i = A^{r_i} \text{ mod } p$$

- 3 Bob generates random bits  $b_1, \dots, b_z$  by flipping a coin

- 4 For each bit  $b_i$ , Alice sends Bob an  $s_i$  where

$$b_i = 0: s_i = r_i$$

$$b_i = 1: s_i = (r_i - r_j) \text{ mod } (p - 1)$$

where  $r_j$  is the lowest  $j$  where  $b_j = 1$

# Confirmation Stage

- 1 For each  $b_i$  Bob checks whether  $s_i$  conforms to the protocol

$$b_i = 0: A^{s_i} \equiv b_i \pmod{p}$$

$$b_i = 1: A^{s_i} \equiv b_i * b_j^{-1} \pmod{p}$$

Bob was sent

$$r_j - r_j, r_m - r_j, \dots, r_p - r_j \pmod{p - 1}$$

where the corresponding bits were 1; Bob does not know  $r_j$ , he does not know any  $r_i$  where the bit was 1

# Proving Stage

- 1 Alice proves she knows  $x$ , the discrete log of  $B$  she sends

$$s_{z+1} = (x - r_j)$$

- 2 Bob confirms

$$A^{s_{z+1}} \equiv B * b_j^{-1} \text{ mod } p$$



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In order to cheat, Alice has to guess all bits in advance. She has only  $\frac{1}{2}^z$  chance of doing so.

(explanation  $\rightarrow$  <http://goo.gl/irL9GK>)

# Take Home Points

- this is pretty old work (in theory); seems little used in practice (surprising)
- for use in privacy, the incentives are not yet right
- most likely applied with digital cash (Bitcoins are not yet good enough)