#### Access Control and Privacy Policies (6)

#### Email: christian.urban at kcl.ac.uk Office: S1.27 (1st floor Strand Building) Slides: KEATS (also homework is there)

### **Problem with Key Systems**



How can you check somebody's solution without revealing the solution?

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Alice and Bob solve crosswords. Alice knows the answer for 21D (folio) but doesn't want to tell Bob.

You use an English dictionary:

folio

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• folio

"an individual leaf of paper or parchment, either loose as one of a series or forming part of a bound volume, which is numbered on the recto or front side only."

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You use an English dictionary:

• folio  $\stackrel{I}{\rightarrow}$  individual

"a single human being as distinct from a group"

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folio → individual → human
 *"relating to or characteristic of humankind"*

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• folio  $\xrightarrow{I}$  individual  $\xrightarrow{2}$  human  $\xrightarrow{3}$  or ...

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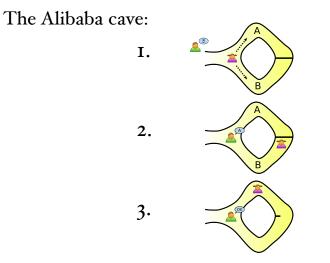
this is essentially a hash function...but Bob can only check once he has also found the solution

#### **Zero-Knowledge Proofs**

Two remarkable properties of Zero-Knowledge Proofs:

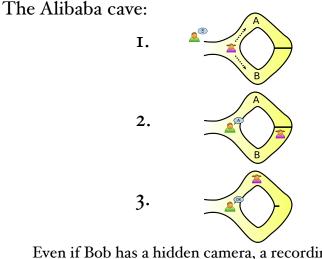
- Alice only reveals the fact that she knows a secret, not the secret itself (meaning she can convince Bob that she knows the secret).
- Having been convinced, Bob cannot use the evidence in order to convince Carol that Alice knows the secret.

#### **The Idea**



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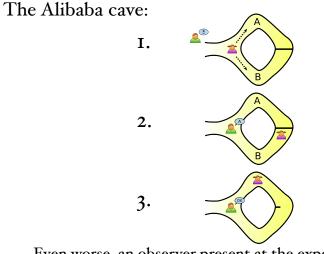
#### **The Idea**



Even if Bob has a hidden camera, a recording will not be convincing to anyone else (Alice and Bob could have made it all up).

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#### **The Idea**



Even worse, an observer present at the experiment would not be convinced.

### **Applications of ZKPs**

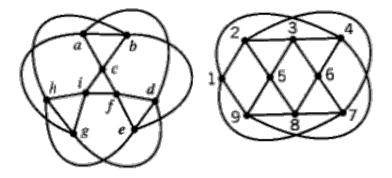
- authentication, where one party wants to prove its identity to a second party via some secret information, but doesn't want the second party to learn anything about this secret
- to enforce honest behaviour while maintaining privacy: the idea is to force a user to prove, using a zero-knowledge proof, that its behaviour is correct according to the protocol

#### **Central Properties**

Zero-knowledge proof protocols should satisfy:

- **Completeness** If Alice knows the secret, Bob accepts Alice "proof" for sure.
- **Soundness** If Alice does not know the secret, Bob accepts her "proof" with a very small probability.

### **Graph Isomorphism**



Finding an isomorphism between two graphs is an NP complete problem.

Alice starts with knowing an isomorphism  $\sigma$  between graphs  $G_1$  and  $G_2$ 

- Alice generates an isomorphic graph *H* which she sends to Bob
- <sup>(a)</sup> Bob asks either for an isomorphism between  $G_{I}$  and H, or  $G_{2}$  and H
- Alice and Bob repeat this procedure n times

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these are called commitment algorithms

If Alice knows the isomorphism, she can always calculate  $\sigma$ .

If she doesn't, she can only correctly respond if Bob's choice of index is the same as the one she used to form *H*. The probability of this happening is  $\frac{1}{2}$ , so after *n* rounds the probability of her always responding correctly is only  $\frac{1}{2}^{n}$ .

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A: We can generate a fake transcript of a conversation, which cannot be distinguished from a "real" conversation.

Anything Bob can compute using the information obtained from the transcript can be computed using only a forged transcript and therefore participation in such a communication does not increase Bob's capability to perform any computation.

#### **Non-Interactive ZKPs**

This is amazing: Alison can publish some data that contains no data about her secret, but this data can be used to convince anyone of the secret's existence.

#### Non-Interactive ZKPs (2)

Alice starts with knowing an isomorphism  $\sigma$  between graphs  $G_1$  and  $G_2$ 

- Alice generates *n* isomorphic graphs  $H_{I...n}$  which she makes public
- (a) she feeds the  $H_{I.n}$  into a hashing function (she has no control over what the output will be)
- Alice takes the first *n* bits of the output: whenever output is 0, she shows an isomorphism with  $G_1$ ; for 1 she shows an isomorphism with  $G_2$

#### **Problems of ZKPs**

- "grand chess master problem" (person in the middle)
- Alice can have multiple identities; once she committed a fraud she stops using one

#### **Other Methods for ZKPs**

- Essentially every NP-problem can be used for ZKPs
- modular logarithms: Alice chooses public A, B, p; and private x

 $A^x \equiv B \mod p$ 

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#### **Commitment Stage**

- Alice generates z random numbers  $r_1, ..., r_z$ , all less than p 1.
- Alice sends Bob for all I..z

 $b_i = A^{r_i} \mod p$ 

- Sob generates random bits  $b_1, ..., b_z$  by flipping a coin
- If For each bit  $b_i$ , Alice sends Bob an  $s_i$  where

$$b_i = 0$$
:  $s_i = r_i$   
 $b_i = 1$ :  $s_i = (r_i - r_j) \mod (p - 1)$ 

where  $r_j$  is the lowest j where  $b_j = 1$ 

#### **Confirmation Stage**

For each b<sub>i</sub> Bob checks whether s<sub>i</sub> conforms to the protocol

$$b_i = 0$$
:  $A^{s_i} \equiv B \mod p$   
 $b_i = 1$ :  $A^{s_i} \equiv b_i * b_j^{-1} \mod p$ 

Bob was send

 $r_j - r_j, r_m - r_j, ..., r_p - r_j \mod p$ 

where the corresponding bits were I; Bob does not know  $r_j$ , he does not know any  $r_i$  where the bit was I

### **Proving Stage**

Alice proves she knows x, the discrete log of B she sends

$$s_{z+1} = (x - r_j)$$

Bob confirms

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In order to cheat, Alice has to guess all bits in advance. She has only I to  $2^z$  chance. (explanation  $\rightarrow http://goo.gl/irL9GK$ )

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  - $\begin{array}{l} a & \text{multiplier} \\ c & \text{increment} \\ X_{\circ} & \text{start value} \end{array}$

and calculate

$$X_{n+1} = (a * X_n + c) \mod m$$

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• Computers are deterministic. H generate random numbers?

$$\begin{array}{rcrcrcrcr}
m = & 16 & 16 \\
X_{\circ} = & 1 & 1 \\
a = & 5 & 5 \\
c = & 1 & 0
\end{array}$$

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