Access Control and Privacy Policies (6)

Email: christian.urban at kcl.ac.uk
Office: S1.27 (1st floor Strand Building)
Slides: KEATS (also homework is there)

Access Control Logic

Formulas

$$
\begin{array}{lcll}
F & ::= & \mathrm{true} & \\
  & | & \mathrm{false} & \\
  & | & F \wedge F & \\
  & | & F \vee F & \\
  & | & F \Rightarrow F & \\
  & | & p\,(t_1,\ldots,t_n) & \\
  & | & P\ \mathrm{says}\ F & \text{“saying predicate”}
\end{array}
$$

Judgements

$$\Gamma \vdash F$$

Judgements

$$\Gamma \vdash \boldsymbol{F}$$

Judgements

entails sign

$$\Gamma \vdash \boldsymbol{F}$$

a single formula

Gamma
stands for a collection of formulas
("assumptions")

Inference Rules

premisses

$$\frac{\Gamma \vdash \boldsymbol{F}_1 \quad \Gamma \vdash \boldsymbol{F}_2}{\Gamma \vdash \boldsymbol{F}_1 \wedge \boldsymbol{F}_2}$$

conclusion

Inference Rules

premisses

$$\frac{\Gamma \vdash \boldsymbol{F}_1 \qquad \Gamma \vdash \boldsymbol{F}_2}{\Gamma \vdash \boldsymbol{F}_1 \wedge \boldsymbol{F}_2}$$

conclusion

$\boldsymbol{P}$ says $\boldsymbol{F} \vdash \boldsymbol{Q}$ says $\boldsymbol{F} \wedge \boldsymbol{P}$ says $\boldsymbol{G}$

Inference Rules

premisses

$$\frac{\Gamma \vdash \boldsymbol{F}_1 \quad \Gamma \vdash \boldsymbol{F}_2}{\Gamma \vdash \boldsymbol{F}_1 \wedge \boldsymbol{F}_2}$$

conclusion

$$\underbrace{\boldsymbol{P} \text{ says } \boldsymbol{F}}_{\Gamma} \vdash \underbrace{\boldsymbol{Q} \text{ says } \boldsymbol{F}}_{\boldsymbol{F}_1} \wedge \underbrace{\boldsymbol{P} \text{ says } \boldsymbol{G}}_{\boldsymbol{F}_2}$$

Sending Messages

- Alice sends a message m

  Alice says m

Sending Messages

- Alice sends a message m

$$\text{Alice says } m$$

- Alice sends an encrypted message m (with key K)

$$\text{Alice says } \{m\}_K$$

Sending Messages

- Alice sends a message m

$$\text{Alice says } m$$

- Alice sends an encrypted message m (with key K)

$$\text{Alice says } \{m\}_K$$

- Decryption of Alice's message

$$\frac{\Gamma \vdash \text{Alice says } \{m\}_K \quad \Gamma \vdash \text{Alice says } K}{\Gamma \vdash \text{Alice says } m}$$

Inference Rules

$$\frac{}{\Gamma, \boldsymbol{F} \vdash \boldsymbol{F}}$$

$$\frac{\Gamma \vdash \boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2 \quad \Gamma \vdash \boldsymbol{F}_1}{\Gamma \vdash \boldsymbol{F}_2} \qquad \frac{\boldsymbol{F}_1, \Gamma \vdash \boldsymbol{F}_2}{\Gamma \vdash \boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2}$$

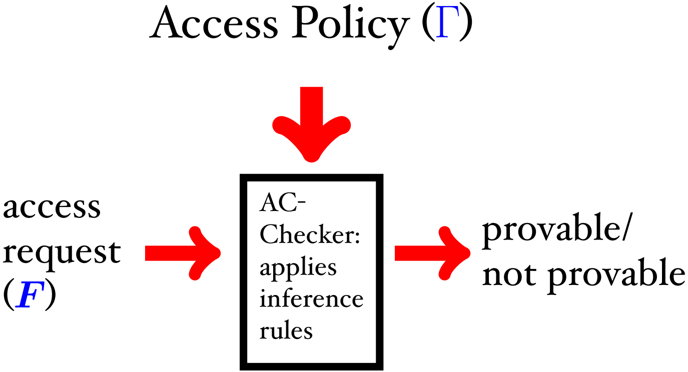
$$\frac{\Gamma \vdash \boldsymbol{F}}{\Gamma \vdash \boldsymbol{P} \text{ says } \boldsymbol{F}}$$

$$\frac{\Gamma \vdash \boldsymbol{P} \text{ says } (\boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2) \quad \Gamma \vdash \boldsymbol{P} \text{ says } \boldsymbol{F}_1}{\Gamma \vdash \boldsymbol{P} \text{ says } \boldsymbol{F}_2}$$

Proofs

$$\dfrac{\dfrac{\dfrac{\vdots}{\vdots} \qquad \dfrac{\vdots \quad \vdots}{\vdots}}{\vdots}}{\Gamma \vdash \boldsymbol{F}}$$

The Access Control Problem



Proofs

$$\frac{}{\vdash}\ \text{axiom}$$

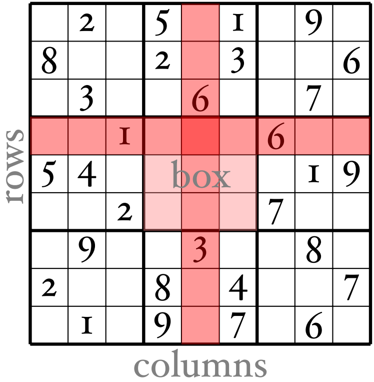
$$\frac{\vdash}{\vdash}$$

$$\frac{\vdash \qquad \vdash}{\vdash}$$

goal

start

Sudoku



1. **Row-Column:** each cell, must contain exactly one number
2. **Row-Number:** each row must contain each number exactly once
3. **Column-Number:** each column must contain each number exactly once
4. **Box-Number:** each box must contain each number exactly once

Solving Sudokus

			7				5	8
	5	6	2	1	8	7	9	3
						1		
							8	1
			3	7	6			
9	6							
		5		3				
		4		2	1	8	3	
8	7				3			

single position rules

$$\frac{\{1..9\} - \{4\} \text{ in one row}}{4 \text{ in empty position}}$$

Solving Sudokus

			7				5	8
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		4		2	1	8	3	
8	7				3			

single position rules

$$\frac{\{1..9\} - \{4\} \text{ in one row}}{4 \text{ in empty position}}$$

$$\frac{\{1..9\} - \{x\} \text{ in one column}}{x \text{ in empty position}}$$

$$\frac{\{1..9\} - \{x\} \text{ in one box}}{x \text{ in empty position}}$$

Solving Sudokus

			7			2	5	8
	5	6	2	1	8	7	9	3
						1	2	2
							8	1
			3	7	6			
9	6							
		5		3				
		4		2	1	8	3	
8	7				3			

candidate rules

$$\frac{\boldsymbol{X} - \{x\} \text{ in one box} \quad \boldsymbol{X} \subseteq \{1..9\}}{x \text{ candidate in empty positions}}$$

Solving Sudokus

			7			2	5	8
4	5	6	2	1	8	7	9	3
						1	2	2
							8	1
			3	7	6			
9	6							
		5		3				
		4		2	1	8	3	
8	7				3			

$$\frac{\{1..9\} - \{4\} \text{ in one row}}{4 \text{ in empty position}}$$

$$\frac{X - \{2\} \text{ in one box} \qquad X \subseteq \{1..9\}}{2 \text{ candidate in empty positions}}$$

Solving Sudokus

			7			2	5	8
4	5	6	2	1	8	7	9	3
						1	2	2
							8	1
			3	7	6			
9	6							
		5		3				
		4		2	1	8	3	
8	7				3			

$$\frac{\{1..9\} - \{4\} \text{ in one row}}{4 \text{ in empty position}}$$

$$\frac{X - \{2\} \text{ in one box} \quad X \subseteq \{1..9\}}{2 \text{ candidate in empty positions}}$$

Solving Sudokus

			7				5	8
	5	6	2	1	8	7	9	3
						1		
							8	1
			3	7	6			
9	6							2
		5		3				
		4		2	1	8	3	
8	7				3			

$$\frac{X - \{2\} \text{ in one box} \qquad X \subseteq \{1..9\}}{2 \text{ candidate}}$$

Sudoku

Are there sudokus that cannot be solved?

Sudoku

Are there sudokus that cannot be solved?

1	2	3	4	5	6	7	8	
								2
								3
								4
								5
								6
								7
								8
								9

Sometimes no rules apply at all....unsolvable sudoku.

Example Proof

$$\frac{?}{P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash Q \text{ says } F_2 \wedge P \text{ says } F_1}$$

Example Proof

We have (by axiom)

$$(1) \quad P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash P \text{ says } F_1 \wedge Q \text{ says } F_2$$

From (1) we get

$$(2) \quad P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash P \text{ says } F_1$$
$$(3) \quad P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash Q \text{ says } F_2$$

From (3) and (2) we get

$$P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash Q \text{ says } F_2 \wedge P \text{ says } F_1$$

Done.

Other Direction

We want to prove

$$P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash Q \text{ says } F_2 \wedge P \text{ says } F_1$$

We are better be able to prove:

$$\begin{array}{ll} (1) & P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash P \text{ says } F_1 \\ (2) & P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash Q \text{ says } F_2 \end{array}$$

For (1): If we can prove

$$P \text{ says } F_1 \wedge Q \text{ says } F_2 \vdash Q \text{ says } F_2 \wedge P \text{ says } F_1$$

then (1) is fine. Similarly for (2).

Recall the following scenario:

- If Admin says that file should be deleted, then this file must be deleted.
- Admin trusts Bob to decide whether file should be deleted.
- Bob wants to delete file.

$$\Gamma = \begin{array}{l} (\text{Admin says del\_file}) \Rightarrow \text{del\_file}, \\ (\text{Admin says } ((\text{Bob says del\_file}) \Rightarrow \text{del\_file})), \\ \text{Bob says del\_file} \end{array}$$

$$\Gamma \vdash \text{del\_file}$$

How to prove $\Gamma \vdash \boldsymbol{F}$?

$$\frac{}{\Gamma, \boldsymbol{F} \vdash \boldsymbol{F}}$$

$$\frac{\boldsymbol{F}_1, \Gamma \vdash \boldsymbol{F}_2}{\Gamma \vdash \boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2}$$

$$\frac{\Gamma \vdash F}{\Gamma \vdash P \text{ says } F}$$

$$\frac{\Gamma \vdash \boldsymbol{F}_1}{\Gamma \vdash \boldsymbol{F}_1 \vee \boldsymbol{F}_2} \qquad \frac{\Gamma \vdash \boldsymbol{F}_2}{\Gamma \vdash \boldsymbol{F}_1 \vee \boldsymbol{F}_2}$$

$$\frac{\Gamma \vdash \boldsymbol{F}_1 \quad \Gamma \vdash \boldsymbol{F}_2}{\Gamma \vdash \boldsymbol{F}_1 \wedge \boldsymbol{F}_2}$$

I want to prove $\Gamma \vdash \mathrm{Pred}$

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1. I found that Γ contains the assumption $\boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2$

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I want to prove $\Gamma \vdash \text{Pred}$

1. I found that Γ contains the assumption $\boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2$
2. If I can prove $\Gamma \vdash \boldsymbol{F}_1$, then I can prove

$$\Gamma \vdash \boldsymbol{F}_2$$

$$\frac{\Gamma \vdash \boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2 \quad \Gamma \vdash \boldsymbol{F}_1}{\Gamma \vdash \boldsymbol{F}_2}$$

I want to prove $\Gamma \vdash \mathrm{Pred}$

1. I found that Γ contains the assumption $\boldsymbol{F}_1 \Rightarrow \boldsymbol{F}_2$
2. If I can prove $\Gamma \vdash \boldsymbol{F}_1$, then I can prove
$$\Gamma \vdash \boldsymbol{F}_2$$
3. So better I try to prove $\Gamma \vdash \mathrm{Pred}$ with the additional assumption $\boldsymbol{F}_2$.
$$\boldsymbol{F}_2, \Gamma \vdash \mathrm{Pred}$$

- P is entitled to do F
  $P \text{ controls } F \overset{\text{def}}{=} (P \text{ says } F) \Rightarrow F$

$$\frac{\Gamma \vdash P \text{ controls } F \quad \Gamma \vdash P \text{ says } F}{\Gamma \vdash F}$$

- P speaks for Q
  $P \mapsto Q \overset{\text{def}}{=} \forall F.(P \text{ says } F) \Rightarrow (Q \text{ says } F)$

$$\frac{\Gamma \vdash P \mapsto Q \quad \Gamma \vdash P \text{ says } F}{\Gamma \vdash Q \text{ says } F}$$

$$\frac{\Gamma \vdash P \mapsto Q \quad \Gamma \vdash Q \text{ controls } F}{\Gamma \vdash P \text{ controls } F}$$

Protocol Specifications

The Needham-Schroeder Protocol:

Message 1 $A \to S : A, B, N_A$
Message 2 $S \to A : \{N_A, B, K_{AB}, \{K_{AB}, A\}_{K_{BS}}\}_{K_{AS}}$
Message 3 $A \to B : \{K_{AB}, A\}_{K_{BS}}$
Message 4 $B \to A : \{N_B\}_{K_{AB}}$
Message 5 $A \to B : \{N_B - 1\}_{K_{AB}}$

Trusted Third Party

Simple protocol for establishing a secure connection via a mutually trusted 3rd party (server):

Message 1 $A \rightarrow S : A, B$
Message 2 $S \rightarrow A : \{K_{AB}\}_{K_{AS}}$ and $\{\{K_{AB}\}_{K_{BS}}\}_{K_{AS}}$
Message 3 $A \rightarrow B : \{K_{AB}\}_{K_{BS}}$
Message 4 $A \rightarrow B : \{m\}_{K_{AB}}$

Sending Messages

- Alice sends a message m

  Alice says m

Sending Messages

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- Alice sends an encrypted message m (with key K)

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Sending Messages

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$$\text{Alice says } \{m\}_K$$

- Decryption of Alice's message

$$\frac{\Gamma \vdash \text{Alice says } \{m\}_K \quad \Gamma \vdash \text{Alice says } K}{\Gamma \vdash \text{Alice says } m}$$

Encryption

- Encryption of a message

$$\frac{\Gamma \vdash \text{Alice says } m \quad \Gamma \vdash \text{Alice says } K}{\Gamma \vdash \text{Alice says } \{m\}_K}$$

Public/Private Keys

- Bob has a private and public key: $K_{Bob}^{pub}, K_{Bob}^{priv}$

$$\frac{\Gamma \vdash \text{Alice says } \{m\}_{K_{Bob}^{pub}} \quad \Gamma \vdash K_{Bob}^{priv}}{\Gamma \vdash \text{Alice says } m}$$

Public/Private Keys

- Bob has a private and public key: K_{Bob}^{pub}, K_{Bob}^{priv}

$$\frac{\Gamma \vdash \text{Alice says } \{m\}_{K_{Bob}^{pub}} \quad \Gamma \vdash K_{Bob}^{priv}}{\Gamma \vdash \text{Alice says } m}$$

- this is **not** a derived rule!

Trusted Third Party

- Alice calls Sam for a key to communicate with Bob
- Sam responds with a key that Alice can read and a key Bob can read (pre-shared)
- Alice sends the message encrypted with the key and the second key it recieved

$$\begin{array}{lcl}
A \text{ sends } S & : & \mathit{Connect}(A, B) \\
S \text{ sends } A & : & \{K_{AB}\}_{K_{AS}} \text{ and } \{\{K_{AB}\}_{K_{BS}}\}_{K_{AS}} \\
A \text{ sends } B & : & \{K_{AB}\}_{K_{BS}} \\
A \text{ sends } B & : & \{m\}_{K_{AB}}
\end{array}$$

Controls

- P controls F $\equiv$ (P says F) $\Rightarrow$ F
- its meaning "P is entitled to do F"
- if P controls F and P says F then F

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Controls

- P controls F $\equiv$ (P says F) $\Rightarrow$ F
- its meaning "P is entitled to do F"
- if P controls F and P says F then F

$$\frac{\Gamma \vdash \text{P controls F} \quad \Gamma \vdash \text{P says F}}{\Gamma \vdash \text{F}}$$

$$\frac{\Gamma \vdash (\text{P says F}) \Rightarrow \text{F} \quad \Gamma \vdash \text{P says F}}{\Gamma \vdash \text{F}}$$

Security Levels

- Top secret ($\boldsymbol{TS}$)
- Secret ($\boldsymbol{S}$)
- Public ($\boldsymbol{P}$)

$$\boldsymbol{slev(P) < slev(S) < slev(TS)}$$

Security Levels

- Top secret (TS)
- Secret (S)
- Public (P)

$$slev(P) < slev(S) < slev(TS)$$

- Bob has a clearance for “secret”
- Bob can read documents that are public or sectret, but not top secret

Reading a File

$$\frac{\text{Bob controls Permitted (File, read)} \qquad \text{Bob says Permitted (File, read)}}{\text{Permitted (File, read)}}$$

Reading a File

$$\frac{\begin{array}{l} slev(\text{File}) < slev(\text{Bob}) \Rightarrow \\ \qquad\qquad \text{Bob controls Permitted (File, read)} \\ \text{Bob says Permitted (File, read)} \\ slev(\text{File}) < slev(\text{Bob}) \end{array}}{\text{Permitted (File, read)}}$$

Reading a File

$$\frac{\begin{array}{l} slev(\text{File}) < slev(\text{Bob}) \Rightarrow \\ \qquad\qquad \text{Bob controls Permitted (File, read)} \\ \text{Bob says Permitted (File, read)} \\ slev(\text{File}) = P \\ slev(\text{Bob}) = S \\ slev(P) < slev(S) \end{array}}{\text{Permitted (File, read)}}$$

Substitution Rule

$$\frac{\Gamma \vdash \boldsymbol{slev(P) = l_1} \quad \Gamma \vdash \boldsymbol{slev(Q) = l_2} \quad \Gamma \vdash \boldsymbol{l_1 < l_2}}{\Gamma \vdash \boldsymbol{slev(P) < slev(Q)}}$$

Substitution Rule

$$\frac{\Gamma \vdash \boldsymbol{slev(P) = l_1} \quad \Gamma \vdash \boldsymbol{slev(Q) = l_2} \quad \Gamma \vdash \boldsymbol{l_1 < l_2}}{\Gamma \vdash \boldsymbol{slev(P) < slev(Q)}}$$

- $\boldsymbol{slev}(\mathrm{Bob}) = \boldsymbol{S}$
- $\boldsymbol{slev}(\mathrm{File}) = \boldsymbol{P}$
- $\boldsymbol{slev(P) < slev(S)}$

Reading a File

$$\frac{\begin{array}{l} \boldsymbol{slev}(\text{File}) < \boldsymbol{slev}(\text{Bob}) \Rightarrow \\ \qquad\qquad \text{Bob controls Permitted (File, read)} \\ \text{Bob says Permitted (File, read)} \\ \boldsymbol{slev}(\text{File}) = \boldsymbol{P} \\ \boldsymbol{slev}(\text{Bob}) = \boldsymbol{TS} \\ ? \end{array}}{\text{Permitted (File, read)}}$$

Reading a File

$$\frac{\begin{array}{l} \boldsymbol{slev}(\text{File}) < \boldsymbol{slev}(\text{Bob}) \Rightarrow \\ \qquad\qquad \text{Bob controls Permitted (File, read)} \\ \text{Bob says Permitted (File, read)} \\ \boldsymbol{slev}(\text{File}) = \boldsymbol{P} \\ \boldsymbol{slev}(\text{Bob}) = \boldsymbol{TS} \\ \boldsymbol{slev}(\boldsymbol{P}) < \boldsymbol{slev}(\boldsymbol{S}) \\ \boldsymbol{slev}(\boldsymbol{S}) < \boldsymbol{slev}(\boldsymbol{TS}) \end{array}}{\text{Permitted (File, read)}}$$

Transitivity Rule

$$\frac{\Gamma \vdash \boldsymbol{l}_1 < \boldsymbol{l}_2 \qquad \Gamma \vdash \boldsymbol{l}_2 < \boldsymbol{l}_3}{\Gamma \vdash \boldsymbol{l}_1 < \boldsymbol{l}_3}$$

- $\boldsymbol{slev}(\boldsymbol{P}) < \boldsymbol{slev}(\boldsymbol{S})$
- $\boldsymbol{slev}(\boldsymbol{S}) < \boldsymbol{slev}(\boldsymbol{TS})$

  $\boldsymbol{slev}(\boldsymbol{P}) < \boldsymbol{slev}(\boldsymbol{TS})$

Reading Files

- Access policy for reading

$$\frac{\begin{array}{l}\forall f.\, slev(f) < slev(\text{Bob}) \Rightarrow \\ \qquad\qquad \text{Bob controls Permitted } (f, \text{read}) \\ \text{Bob says Permitted (File, read)} \\ slev(\text{File}) = P \\ slev(\text{Bob}) = TS \\ slev(P) < slev(S) \\ slev(S) < slev(TS)\end{array}}{\text{Permitted (File, read)}}$$

Reading Files

- Access policy for reading

$$\begin{array}{c}
\forall f.\ \mathit{slev}(f) \leq \mathit{slev}(\text{Bob}) \Rightarrow \\
\qquad\qquad \text{Bob controls Permitted } (f, \text{read}) \\
\text{Bob says Permitted (File, read)} \\
\mathit{slev}(\text{File}) = \mathit{TS} \\
\mathit{slev}(\text{Bob}) = \mathit{TS} \\
\mathit{slev}(P) < \mathit{slev}(S) \\
\mathit{slev}(S) < \mathit{slev}(\mathit{TS}) \\
\hline
\text{Permitted (File, read)}
\end{array}$$

Writing Files

- Access policy for writing

$$\frac{\begin{array}{l}\forall f.\, slev(\text{Bob}) \leq slev(f) \Rightarrow \\ \qquad\qquad \text{Bob controls Permitted}\ (f, \text{write}) \\ \text{Bob says Permitted (File, write)} \\ slev(\text{File}) = TS \\ slev(\text{Bob}) = S \\ slev(P) < slev(S) \\ slev(S) < slev(TS)\end{array}}{\text{Permitted (File, write)}}$$

Sending Rule

$$\frac{\Gamma \vdash P \; \textit{says} \; F \quad \Gamma \vdash P \; \textit{sends} \; Q : F}{\Gamma \vdash Q \; \textit{says} \; F}$$

Sending Rule

$$\frac{\Gamma \vdash P \; \textit{says} \; F \quad \Gamma \vdash P \; \textit{sends} \; Q : F}{\Gamma \vdash Q \; \textit{says} \; F}$$

$$P \; \text{sends} \; Q : F \stackrel{\text{def}}{=}$$
$$(P \; \text{says} \; F) \Rightarrow (Q \; \text{says} \; F)$$

Trusted Third Party

$$
\begin{array}{l}
A \text{ sends } S : \mathit{Connect}(A, B)\\
S \text{ says } (\mathit{Connect}(A, B) \Rightarrow\\
\qquad\qquad \{K_{AB}\}_{K_{AS}} \wedge \{\{K_{AB}\}_{K_{BS}}\}_{K_{AS}})\\
S \text{ sends } A : \{K_{AB}\}_{K_{AS}} \wedge \{\{K_{AB}\}_{K_{BS}}\}_{K_{AS}}\\
A \text{ sends } B : \{K_{AB}\}_{K_{BS}}\\
A \text{ sends } B : \{m\}_{K_{AB}}
\end{array}
$$

Trusted Third Party

$$
\begin{array}{l}
A \text{ sends } S : \mathit{Connect}(A, B) \\
S \text{ says } (\mathit{Connect}(A, B) \Rightarrow \\
\qquad\qquad \{K_{AB}\}_{K_{AS}} \wedge \{\{K_{AB}\}_{K_{BS}}\}_{K_{AS}}) \\
S \text{ sends } A : \{K_{AB}\}_{K_{AS}} \wedge \{\{K_{AB}\}_{K_{BS}}\}_{K_{AS}} \\
A \text{ sends } B : \{K_{AB}\}_{K_{BS}} \\
A \text{ sends } B : \{m\}_{K_{AB}}
\end{array}
$$

$\Gamma \vdash B \text{ says } m$?