Access Control and Privacy Policies (11)

Email: christian urban at kcl ac.uk. Office: S_{1.27} (1st floor Strand Building) Slides: KEATS (also homework is there) • Imagine you have an completely innocent email message, like birthday wishes to your grandmother? Why should you still encrypt this message and your grandmother take the effort to decrypt it?

(Hint: The answer has nothing to do with preserving the privacy of your grandmother and nothing to do with keeping her birthday wishes super-secret. Also nothing to do with you and grandmother testing the latest encryption technology, nor just for the sake of it.)

Interlock Protocol

Protocol between a car *C* and a key transponder *T*:

- ¹ *C* generates a random number *N*
- \odot *C* calculates $(F, G) = \{N\}_K$

3 $C \rightarrow T$: *N*, *F*

- $\mathbf{F} \cdot \mathbf{T}$ calculates $(F', G') = \{N\}_K$
- **5** *T* checks that $F = F'$
- $T \rightarrow C: N, G'$
- Ω *C* checks that $G = G'$

Zero-Knowledge Proofs

Essentially every NP-problem can be used for ZKPs

modular logarithms: Alice chooses public *A*, *B*, *p*; and private *x*

 $A^x \equiv B \mod p$

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Modular Arithmetic

It is easy to calculate

 $? \equiv 46 \mod 12$

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It is easy to calculate

$10 \equiv 46 \mod 12$

 $A: \mathbf{I} \Omega$

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Ordinary, non-modular logarithms:

$$
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Conclusion: 1.2304489 is very close to the *true* solution

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I could be tempted to try 28305820…but the real answer is 12314.

Commitment Stage

- **1** Alice generates z random numbers $r_1, ..., r_z$, all less than $p - I$.
- 2 Alice sends Bob for all \overline{L} .

 $h_i = A^{r_i}$ *mod p*

- \bullet Bob generates random bits $b_1, ..., b_z$ by flipping a coin
- ⁴ For each bit *bⁱ* , Alice sends Bob an *sⁱ* where

 $b_i = 0$: $s_i = r_i$ $b_i = 1$: $s_i = (r_i - r_i) \text{ mod } (p - 1)$

where r_j is the lowest j with $b_j = r_j$

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- 2 Alice sends Bob for all 1<mark>.</mark>.. $h_i = A$ Alice r_i : 4 9 1 3 $Bob \, b_i$; or or *↑ j*
- \bullet Bob generates random bits $b_1, ..., b_z$ by flipping a coin
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Confirmation Stage

¹ For each *bⁱ* Bob checks whether *sⁱ* conforms to the protocol

$$
b_i = \text{o}: \ \ A^{s_i} \equiv b_i \bmod p
$$

$$
b_i = \text{i}: \ \ A^{s_i} \equiv b_i * b_j^{-1} \bmod p
$$

Bob was sent

 $b_{1}, \ldots, b_{z},$ *r*¹ *− r^j* , *r*² *− r^j* , …, *r^z − r^j mod p −* 1 where the corresponding bits were \mathbf{I} ; Bob does not know *r^j* , he does not know any *rⁱ* where the hit was **1**

 \bullet For each b_i Bob checks the protocol

Confirmat

 $A^{s_i} = A^{r_i - r_j}$ $= A^{r_i} * A^{-r_j}$ $= b_{r_i} * b_{r_j}^{-1}$ *rj mod p*

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Proving Stage

¹ Alice proves she knows *x*, the discrete log of *B* she sends

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s_{z+1}=(x-r_j)
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² Bob confirms

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In order to cheat, Alice has to guess all bits in advance. She has only $\frac{1}{2}$ *z* chance of doing so.

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