Access Control and Privacy Policies (11)

Email: christian.urban at kcl.ac.uk Office: S1.27 (1st floor Strand Building) Slides: KEATS (also homework is there) • Imagine you have an completely innocent email message, like birthday wishes to your grandmother? Why should you still encrypt this message and your grandmother take the effort to decrypt it?

(Hint: The answer has nothing to do with preserving the privacy of your grandmother and nothing to do with keeping her birthday wishes super-secret. Also nothing to do with you and grandmother testing the latest encryption technology, nor just for the sake of it.)



Interlock Protocol

Protocol between a car C and a key transponder T:

- C generates a random number N
- C calculates $(F, G) = \{N\}_K$

 $O C \to T: N, F$

- T calculates $(F', G') = \{N\}_K$
- T checks that**F**=**F**'
- $T \rightarrow C: N, G'$
- C checks that G = G'

Zero-Knowledge Proofs

Essentially every NP-problem can be used for ZKPs

 modular logarithms: Alice chooses public A, B, p; and private x

 $A^x \equiv B \bmod p$

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Modular Arithmetic

It is easy to calculate

 $? \equiv 46 \mod 12$

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Modular Arithmetic

It is easy to calculate

$10 \equiv 46 \mod 12$

A: 10

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Ordinary, non-modular logarithms:

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$$\Rightarrow log_{10}17 = 1.2304489... \Rightarrow 10^{1.2304489} = 16.9999999$$

Conclusion: 1.2304489 is very close to the *true* solution

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 $2^{28_{3}05_{819}} \equiv 88_{032151} \mod 973_{30327}$

I could be tempted to try 28305820...

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Lets say I found 28305819...I try

 $2^{28305819} \equiv 88032151 \mod 97330327$

I could be tempted to try 28305820...but the real answer is 12314.

Commitment Stage

- Alice generates z random numbers $r_1, ..., r_z$, all less than p 1.
- Alice sends Bob for all **1**..*z*

 $b_i = A^{r_i} \mod p$

- Solution Bob generates random bits $b_1, ..., b_z$ by flipping a coin
- For each bit b_i , Alice sends Bob an s_i where

 $b_i = 0$: $s_i = r_i$ $b_i = 1$: $s_i = (r_i - r_j) \mod (p - 1)$

where r_j is the lowest j with $b_j = 1$

Commitment Stage

- Alice generates z random numbers $r_1, ..., r_z$, all less than p 1.
- Alice sends Bob for all I $b_i = A$ Alice $r_i: 4 \quad 9 \quad I \quad 3$ $Bob \quad b_i: 0 \quad I \quad 0 \quad I$ f_j
- Solution Bob generates random bits $b_1, ..., b_z$ by flipping a coin
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Confirmation Stage

For each b_i Bob checks whether s_i conforms to the protocol

$$b_i = 0$$
: $A^{s_i} \equiv b_i \mod p$
 $b_i = 1$: $A^{s_i} \equiv b_i * b_j^{-1} \mod p$

Bob was sent

$$b_1, \ldots, b_z,$$

 $r_1 - r_j, r_2 - r_j, \ldots, r_z - r_j \mod p - 1$

where the corresponding bits were I; Bob does not know r_j , he does not know any r_i where the bit was I Confirmation
 For each b_i Bob checks the protocol

$$A^{s_i} = A^{r_i - r_j}$$

= $A^{r_i} * A^{-r_j}$
= $b_{r_i} * b_{r_j}^{-1} \mod p$

$$b_i = 0$$
: $A^{s_i} \equiv b_i \mod p$
 $b_i = 1$: $A^{s_i} \equiv b_i * b_j^{-1} \mod p$

Bob was sent

$$b_1, \ldots, b_z, r_1 - r_j, r_2 - r_j, \ldots, r_z - r_j \mod p - 1$$

where the corresponding bits were I; Bob does not know r_j , he does not know any r_i where the bit was I

Proving Stage

Alice proves she knows x, the discrete log of B she sends

$$s_{z+i} = (x - r_j)$$

Bob confirms

$$A^{s_{z+1}} \equiv B * b_j^{-1} \mod p$$

Proving Stage

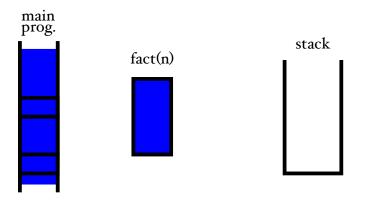
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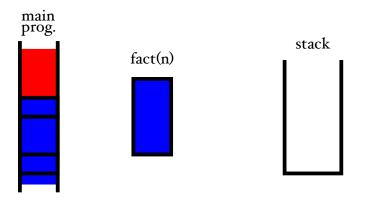
$$s_{z+i} = (x - r_j)$$

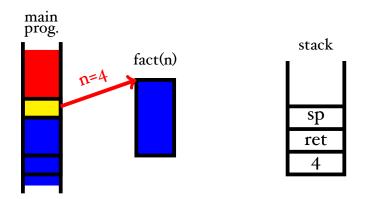
Bob confirms

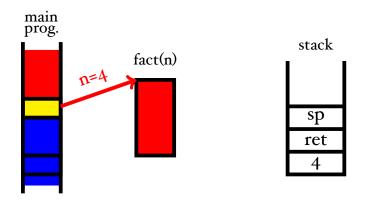
$$A^{s_{z+1}} \equiv B * b_j^{-1} \bmod p$$

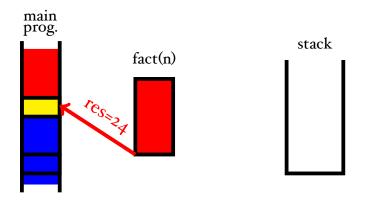
In order to cheat, Alice has to guess all bits in advance. She has only $\frac{1^{z}}{2}$ chance of doing so.

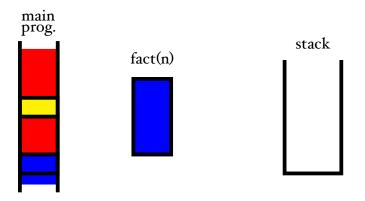


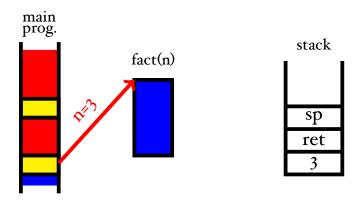


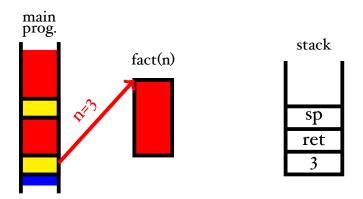


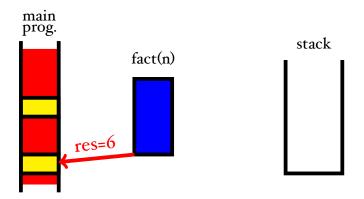


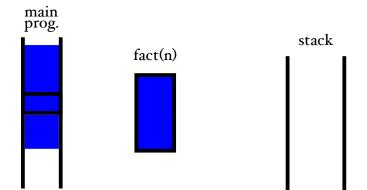


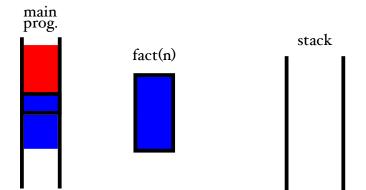


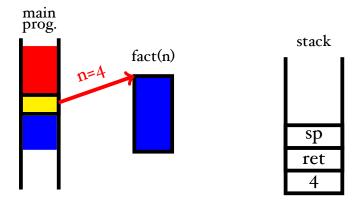


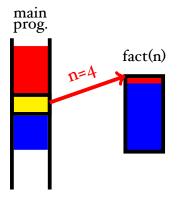


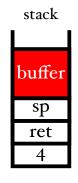












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