

Priority Inheritance Protocol Proved Correct

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- Verification of PCP in PVS (2000)
 - A related protocol
 - Priority Ceiling Protocol

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- Little help was found in the literature
- Some mentioning the complication

Some excerpts

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"I observed in the kernel code (to my disgust), the Linux PIP implementation is a nightmare: extremely heavy weight, involving maintenance of a full wait-for graph, and requiring updates for a range of events, including priority changes and interruptions of wait operations."

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 - Understanding the relationship with real OS code
 - Not yet formalized

Real-Time OSes

- Purpose: gurantee the most urgent task be processed in time
- Processes have priorities
- Resources can be locked and unlocked

Problem

High-priority process

Low-priority process

Problem

H high-priority process

M medium-priority process

L low-priority process

Problem

High-priority process

Medium-priority process

Low-priority process

- Priority Inversion $\stackrel{\text{def}}{=} H < L$

Problem

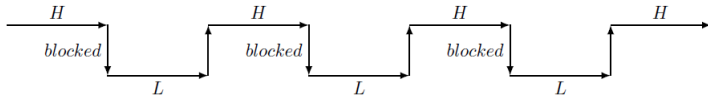
High-priority process

Medium-priority process

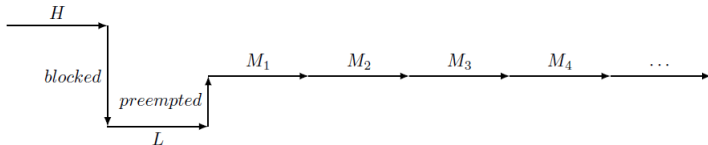
Low-priority process

- Priority Inversion $\stackrel{\text{def}}{=} H < L$
- avoid indefinite priority inversion

Priority Inversion

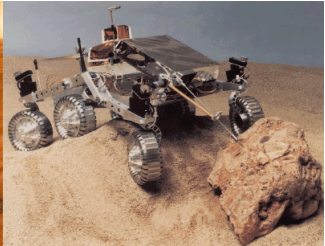
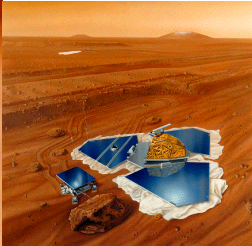
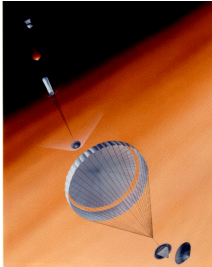


Priority Inversion



Indefinite Priority Inversion

Mars Pathfinder Mission 1997



Solution

Priority Inheritance Protocol (PIP):

High-priority process

Medium-priority process

Low-priority process

(temporarily raise its priority)

A Correctness “Proof” in 1990

- a paper* in 1990 “proved” the correctness of an algorithm for PIP

...after the thread (whose priority has been raised) completes its critical section and releases the lock, it “returns to its original priority level”.

* in IEEE Transactions on Computers

High-priority process 1

High-priority process 2

Low-priority process

High-priority process 1

High-priority process 2

Low-priority process

- Solution:
Return to highest **remaining**
priority

Event Abstraction

- Use Inductive Approach of L. Paulson

Event Abstraction

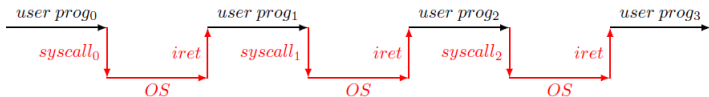
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Execution of OS



Abstracted as events

Events

Create thread priority

Exit thread

Set thread priority

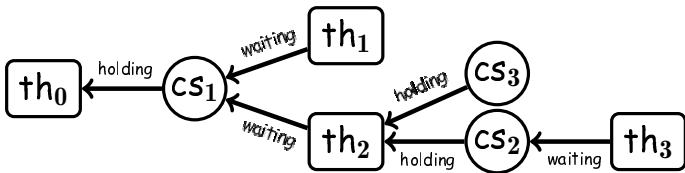
Lock thread cs

Unlock thread cs

Precedences

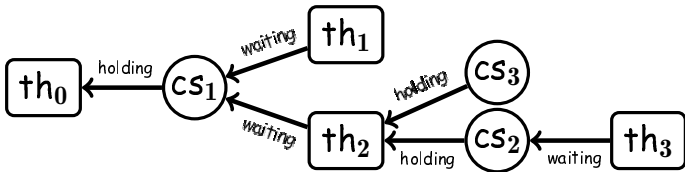
prec th s $\stackrel{\text{def}}{=}$ (priority th s, last_set th s)

RAGs



$$\text{RAG } wq \stackrel{\text{def}}{=} \{(T \text{ th}, C \text{ cs}) \mid \text{waits } wq \text{ th cs}\} \\ \cup \{(C \text{ cs}, T \text{ th}) \mid \text{holds } wq \text{ th cs}\}$$

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Good Next Events

$$\frac{th \notin \text{threads } s}{\text{step } s \text{ (Create } th \text{ prio)}}$$
$$\frac{th \in \text{running } s \quad \text{resources } s \text{ } th = \emptyset}{\text{step } s \text{ (Exit } th \text{)}}$$
$$\frac{th \in \text{running } s}{\text{step } s \text{ (Set } th \text{ prio)}}$$

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$$\frac{\text{th} \in \text{running } s \quad (C \text{ cs}, T \text{ th}) \notin (RAG \text{ s})^+}{\text{step } s (P \text{ th } \text{cs})}$$
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* modulo some further assumptions It does not matter which process gets a released lock.

Implementation

s = current state; s' = next state = $e\#s$

When e = Create th prio, Exit th

- RAG s' = RAG s
- No precedence needs to recalculate

Implementation

s = current state; s' = next state = $e\#s$

When e = Set th prio

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 - Refinement to real code and relation between implementations

Questions?

- Thank you for listening!