Priority Inheritance Protocol Proved Correct

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- Verification of PCP in PVS (2000)
  - A related protocol
  - Priority Ceiling Protocol

Our Motivation

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Some excerpts

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"I observed in the kernel code (to my disgust), the Linux PIP implementation is a nightmare: extremely heavy weight, involving maintenance of a full wait-for graph, and requiring updates for a range of events, including priority changes and interruptions of wait operations."

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- Formal specification at appropriate abstract level, convenient for:
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- Theorems usable to guide implementation, critical point:
  - Understanding the relationship with real OS code
  - Not yet formalized

Real-Time OSes

- Purpose: gurantee the most urgent task to be processed in time
- Processes have priorities
- Resources can be locked and unlocked

Problem

High-priority process

Low-priority process

Problem

High-priority process

Medium-priority process

Low-priority process

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- Priority Inversion $\stackrel{\text{def}}{=} H < L$

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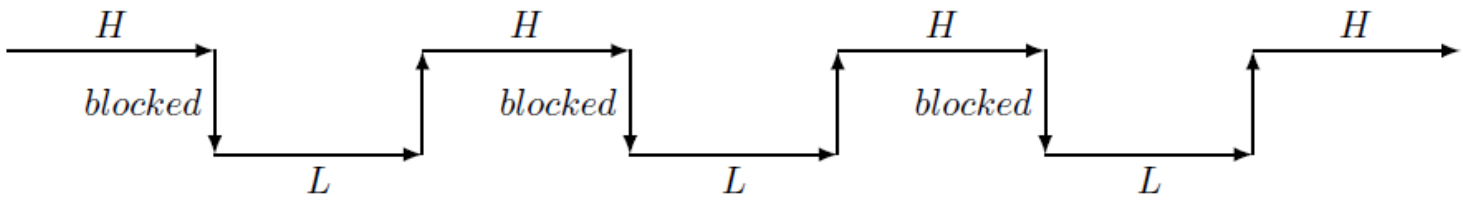
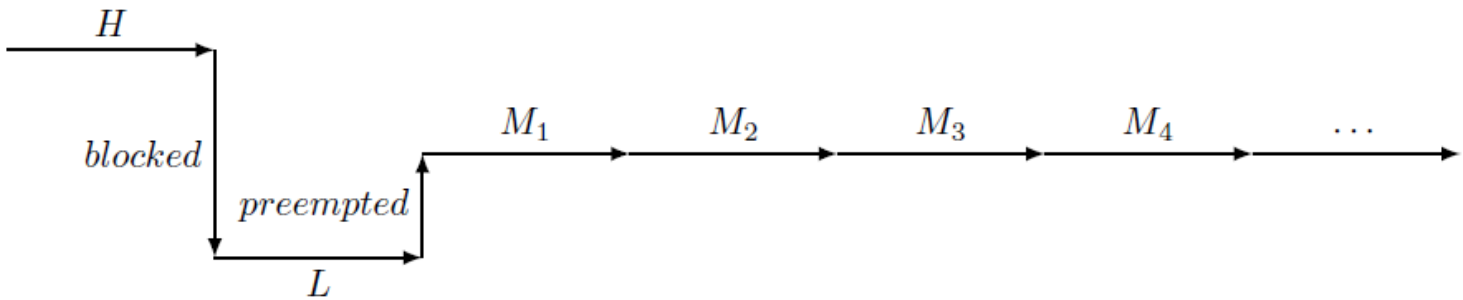
Low-priority process

- Priority Inversion $\stackrel{\text{def}}{=} H < L$
- avoid indefinite priority inversion

Priority Inversion

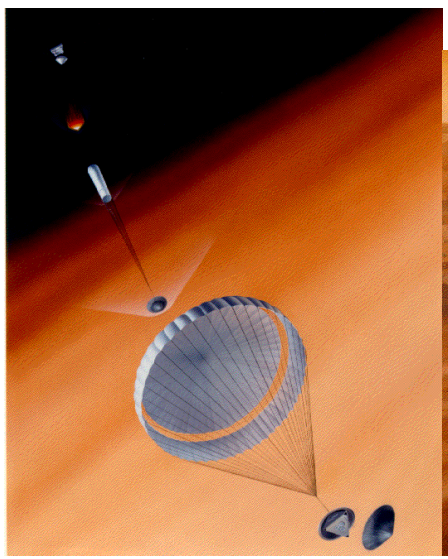
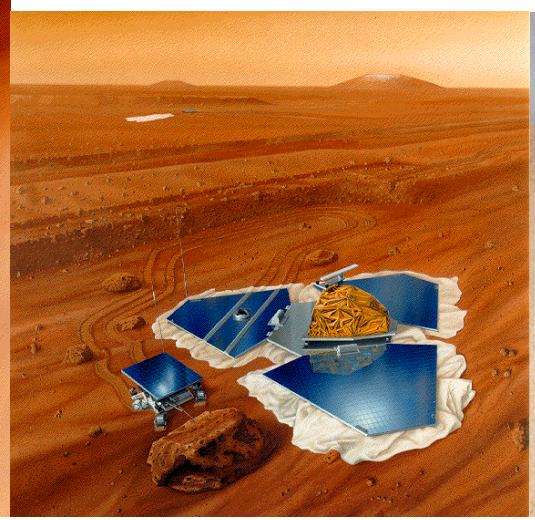
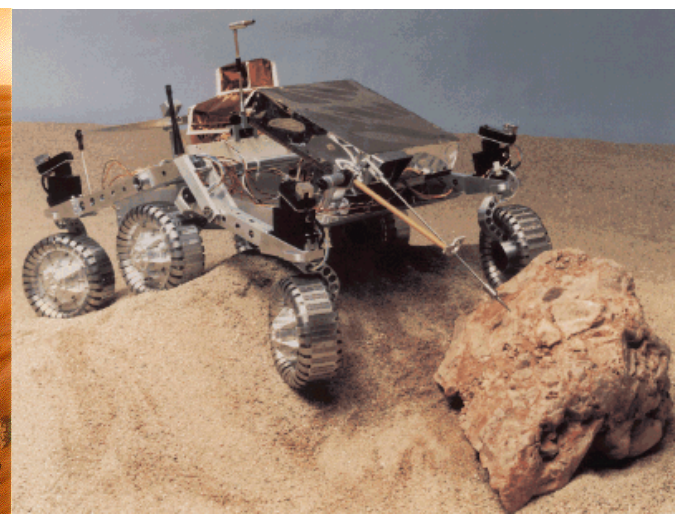


Prioirty Inversion



Indefinite Priority Inversion

Mars Pathfinder Mission 1997

Solution

Priority Inheritance Protocol (PIP):

High-priority process

Medium-priority process

Low-priority process

(temporarily raise its priority)

A Correctness "Proof" in 1990

- a paper* in 1990 "proved" the correctness of an algorithm for PIP

  ...after the thread (whose priority has been raised) completes its critical section and releases the lock, it "returns to its original priority level".

* in IEEE Transactions on Computers

High-priority process 1

High-priority process 2

Low-priority process

High-priority process 1

High-priority process 2

Low-priority process

- Solution:
  Return to highest remaining priority

Event Abstraction

- Use Inductive Approach of L. Paulson

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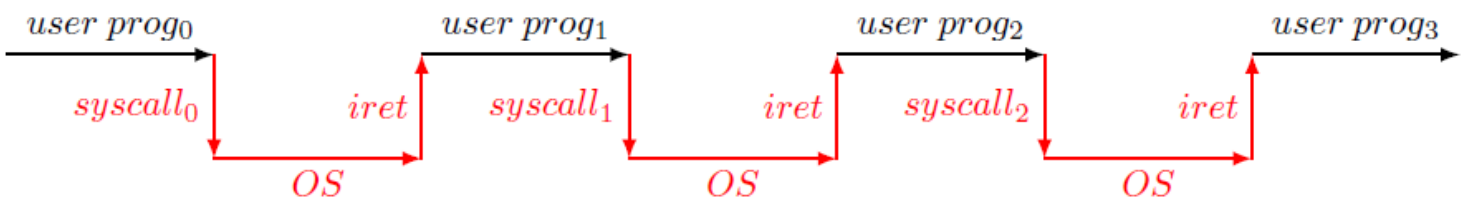
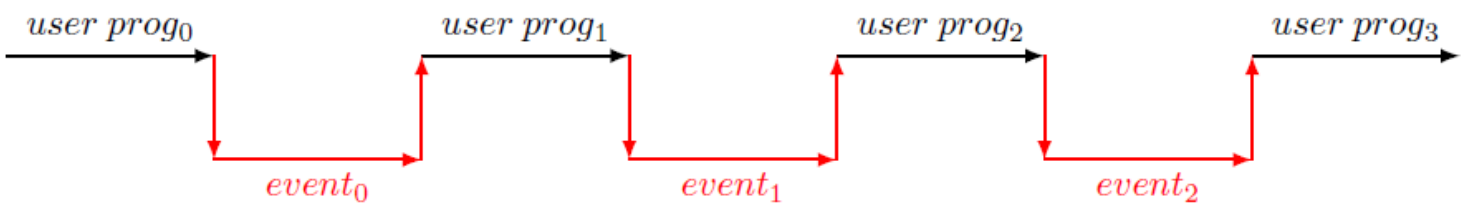


Execution of OS



Abstracted as events

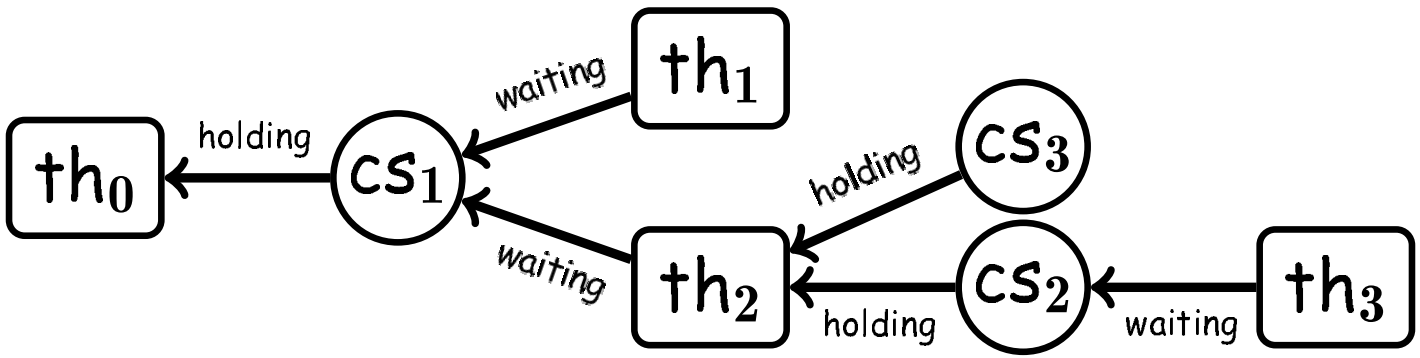
Events

Create thread priority
Exit thread
Set thread priority
Lock thread cs
Unlock thread cs

Precedences

$$\text{prec th s} \stackrel{\text{def}}{=} (\text{priority th s}, \text{last\_set th s})$$

RAGs



$$\text{RAG wq} \stackrel{\text{def}}{=} \{(\text{T th}, \text{C cs}) \mid \text{waits wq th cs}\}$$
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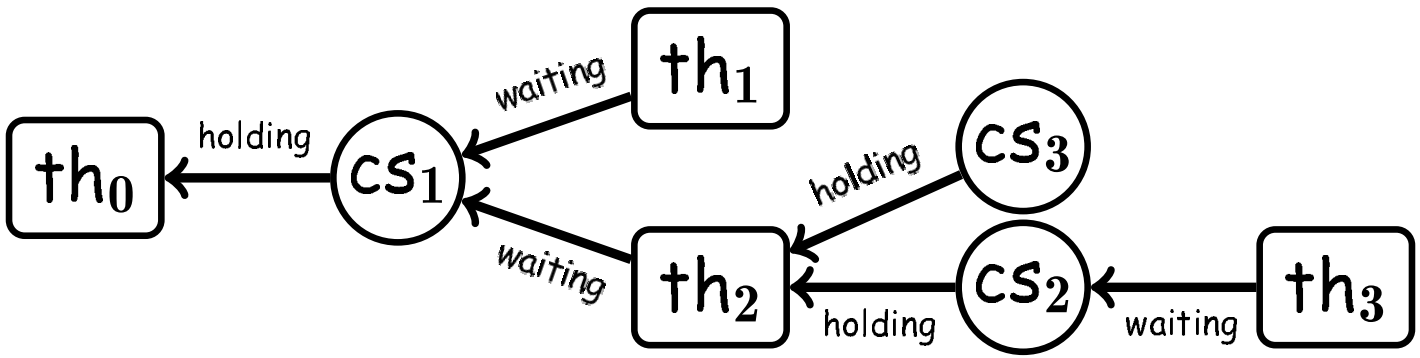

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Good Next Events

$$\frac{th \notin threads\ s}{step\ s\ (Create\ th\ prio)}$$

$$\frac{th \in running\ s \quad resources\ s\ th = \varnothing}{step\ s\ (Exit\ th)}$$

$$\frac{th \in running\ s}{step\ s\ (Set\ th\ prio)}$$

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$$\frac{th \in running\ s \quad (C\ cs, T\ th) \notin (RAG\ s)^+}{step\ s\ (P\ th\ cs)}$$

$$\frac{th \in running\ s \quad holds\ s\ th\ cs}{step\ s\ (V\ th\ cs)}$$

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* modulo some further assumptionsIt does not matter which process gets a released lock.

Implementation

s = current state; s' = next state = $e\#s$

When e = Create th prio, Exit th

- RAG s' = RAG s
- No precedence needs to be recomputed

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s = current state; s' = next state = $e\#s$

When e = Set th prio

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When e = Unlock th cs where there is a thread to take over

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  - Refinement to real code and relation between implementations

Questions?

- Thank you for listening!