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Contents

theory *Myhill*
  imports *Myhill-2*
begin

1 Direction *regular language* ⇒*finite partition*

A *determinisitc finite automata (DFA)* M is a 5-tuple $(Q, \Sigma, \delta, s, F)$, where:

1. Q is a finite set of *states*, also denoted Q_M.
2. Σ is a finite set of *alphabets*, also denoted Σ_M.
3. δ is a *transition function* of type $Q \times \Sigma \Rightarrow Q$ (a total function), also denoted δ_M.
4. $s \in Q$ is a state called *initial state*, also denoted s_M.
5. $F \subseteq Q$ is a set of states named *accepting states*, also denoted F_M.

Therefore, we have $M = (Q_M, \Sigma_M, \delta_M, s_M, F_M)$. Every DFA M can be interpreted as a function assigning states to strings, denoted $\hat{\delta}_M$, the definition of which is as the following:

$$\begin{aligned} \hat{\delta}_M([]) &\equiv s_M \\ \hat{\delta}_M(xa) &\equiv \delta_M(\hat{\delta}_M(x), a) \end{aligned} \tag{1}$$

A string x is said to be *accepted* (or *recognized*) by a DFA M if $\hat{\delta}_M(x) \in F_M$. The language recognized by DFA M, denoted $L(M)$, is defined as:

$$L(M) \equiv \{x \mid \hat{\delta}_M(x) \in F_M\} \tag{2}$$

The standard way of specifying a laugage $\mathcal{L}$ as *regular* is by stipulating that: $\mathcal{L} = L(M)$ for some DFA M.

For any DFA M, the DFA obtained by changing initial state to another $p \in Q_M$ is denoted M_p, which is defined as:

$$M_p \equiv (Q_M, \Sigma_M, \delta_M, p, F_M) \tag{3}$$

Two states $p, q \in Q_M$ are said to be *equivalent*, denoted $p \approx_M q$, iff.

$$L(M_p) = L(M_q) \tag{4}$$

It is obvious that $\approx_M$ is an equivalent relation over Q_M. and the partition induced by $\approx_M$ has $|Q_M|$ equivalent classes. By overloading $\approx_M$, an equivalent relation over strings can be defined:

$$x \approx_M y \equiv \hat{\delta}_M(x) \approx_M \hat{\delta}_M(y) \tag{5}$$

It can be proved that the the partition induced by $\approx_M$ also has $|Q_M|$ equivalent classes. It is also easy to show that: if $x \approx_M y$, then $x \approx_{L(M)} y$, and this means $\approx_M$ is a more refined equivalent relation than $\approx_{L(M)}$. Since partition induced by $\approx_M$ is finite, the one induced by $\approx_{L(M)}$ must also be finite. Now, we get one direction of Myhill-Nerode Theorem:

Lemma 1 (Myhill-Nerode Theorem, Direction one)**.** *If a language $\mathcal{L}$ is regular (i.e. $\mathcal{L} = L(M)$ for some DFA M), then the partition induced by $\approx_{\mathcal{L}}$ is finite.*

The other direction is:

Lemma 2 (Myhill-Nerode Theorem, Direction two)**.** *If the partition induced by $\approx_{\mathcal{L}}$ is finite, then $\mathcal{L}$ is regular (i.e. $\mathcal{L} = L(M)$ for some DFA M).*

To prove this lemma, a DFA $M_{\mathcal{L}}$ is constructed out of $\approx_{\mathcal{L}}$ with:

$$Q_{M_{\mathcal{L}}} \equiv \{[\![x]\!]_{\approx_{\mathcal{L}}} \mid x \in \Sigma^*\} \tag{6a}$$
$$\Sigma_{M_{\mathcal{L}}} \equiv \Sigma_M \tag{6b}$$
$$\delta_{M_{\mathcal{L}}} \equiv (\lambda([\![x]\!]_{\approx_{\mathcal{L}}}, a).[\![xa]\!]_{\approx_{\mathcal{L}}}) \tag{6c}$$
$$s_{M_{\mathcal{L}}} \equiv [\![[]]\!]_{\approx_{\mathcal{L}}} \tag{6d}$$
$$F_{M_{\mathcal{L}}} \equiv \{[\![x]\!]_{\approx_{\mathcal{L}}} \mid x \in \mathcal{L}\} \tag{6e}$$

From the assumption of lemma 2, we have that $\{[\![x]\!]_{\approx_{\mathcal{L}}} \mid x \in \Sigma^*\}$ is finite It can be proved that $\mathcal{L} = L(M_{\mathcal{L}})$.

end

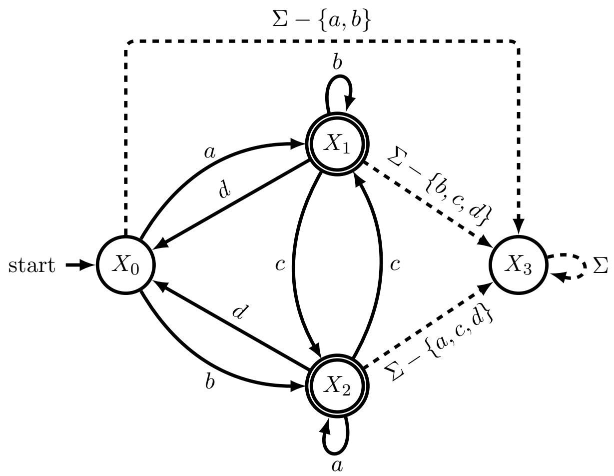

Figure 1: The relationship between automata and finite partition