Handout (5)

November 21, 2006

The original algorithm of Damas and Milner:

- Input: a context Γ and an expression e.
- Output: a substitution and a type.

- **Case Variables:**
$$W(\Gamma, x) = (\varepsilon, T[X_1 := Y_1, \ldots, X_n := Y_n])$$
where $(x : \forall\{X_1, \ldots, X_n\}.T) \in \Gamma$ and the Y_i are distinct and fresh (w.r.t. Γ and T).

- **Case Lambdas:** calculate first
$$W((x : \forall\{\}.Y, \Gamma), e) = (\theta, T_1)$$
where Y is fresh (w.r.t. Γ). Then return
$$W(\Gamma, \lambda x.e) = (\theta, \theta(Y) \to T_1).$$

- **Case Applications:** Calculate first
$$W(\Gamma, e_1) = (\theta_1, T_1)$$
then
$$W(\theta_1(\Gamma), e_2) = (\theta_2, T_2)$$
and then
$$\text{unify } \{\theta_2(T_1) =^? T_2 \to Y\} = \theta_3$$
where Y is fresh (w.r.t. Γ). Finally return
$$W(\Gamma, e_1\ e_2) = (\theta_3 \circ \theta_2 \circ \theta_1, \theta_3(Y)).$$

- **Case Lets:** Calculate first
$$W(\Gamma, e_1) = (\theta_1, T_1)$$
then
$$A = \text{tv}(T_1) - \text{ftv}(\theta_1(\Gamma))$$
and then
$$W((x : \forall A.T_1, \theta_1(\Gamma)), e_2) = (\theta_2, T_2)$$
Finally return
$$W(\Gamma, \text{let}\, x = e_1 \,\text{in}\, e_2) = (\theta_2 \circ \theta_1, T_2)$$