Types in Programming Languages (7)

Christian Urban

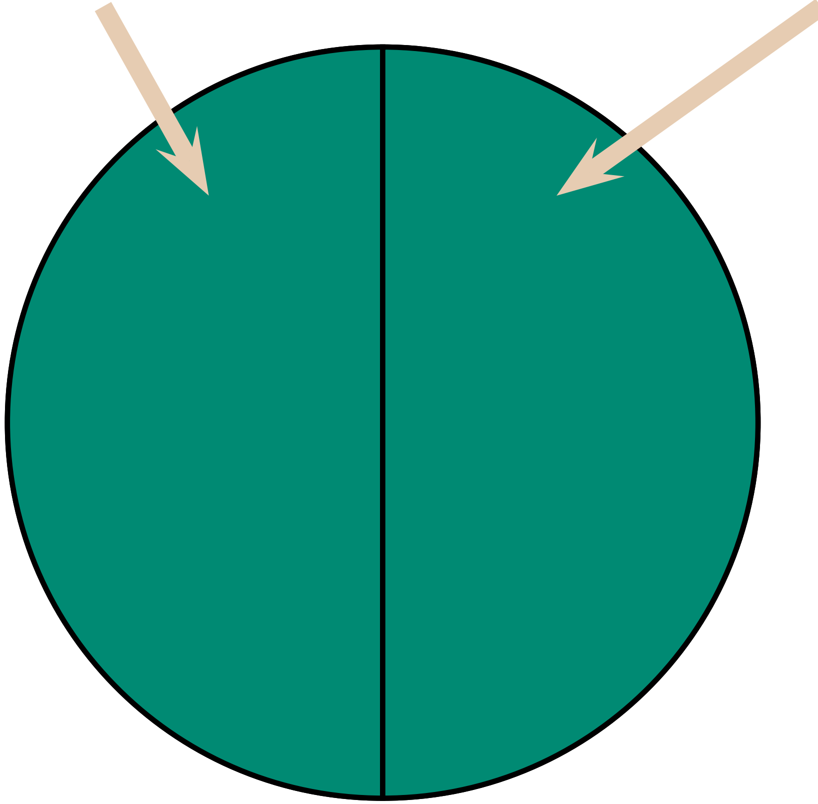
http://www4.in.tum.de/lehre/vorlesungen/types/WS0607/

Previously

untrapped errors

e.g. access of an array outside its bounds; jumping to a legal address

evil



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e.g. division by zero; jumping to an illegal address

annoying

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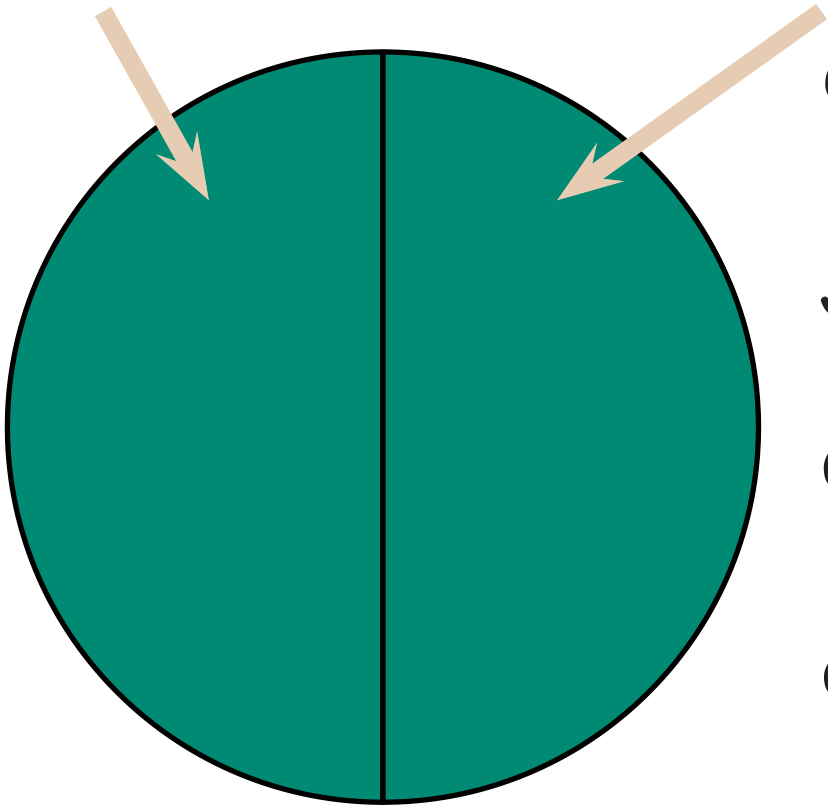

trapped errors

e.g. division by zero; jumping to an illegal address

annoying

A programming language is called **safe** if no untrapped errors can occur. Safety can be achieved by run-time checks or static checks.

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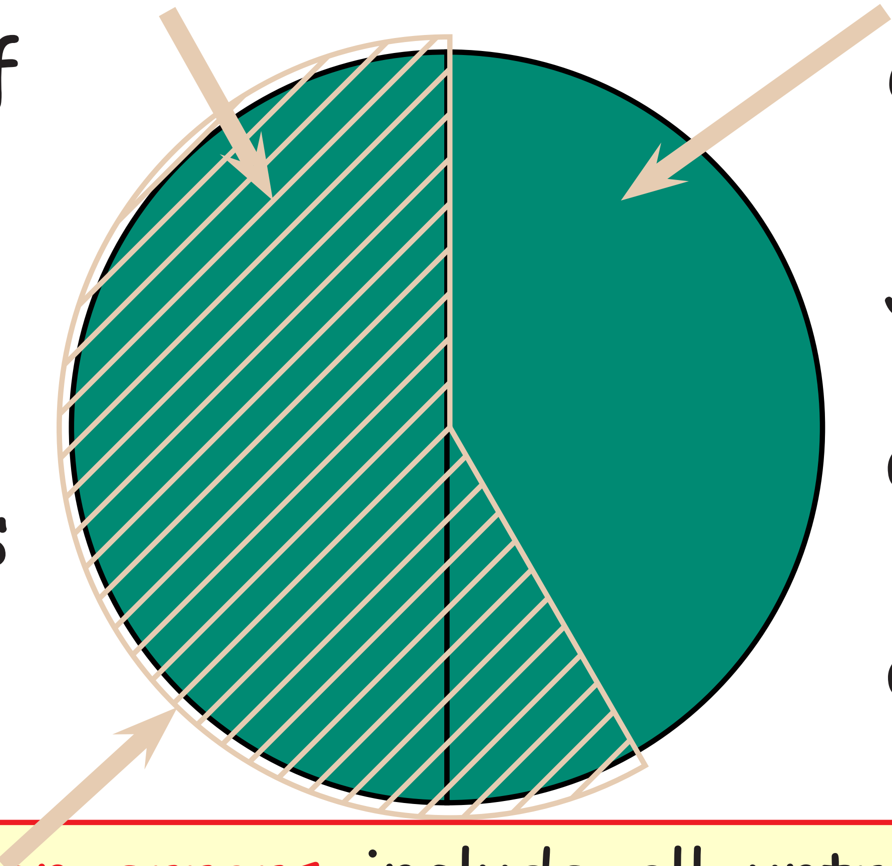

trapped errors

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annoying

Forbidden errors include all untrapped errors and some trapped ones. A strongly typed programming language prevents all forbidden errors.

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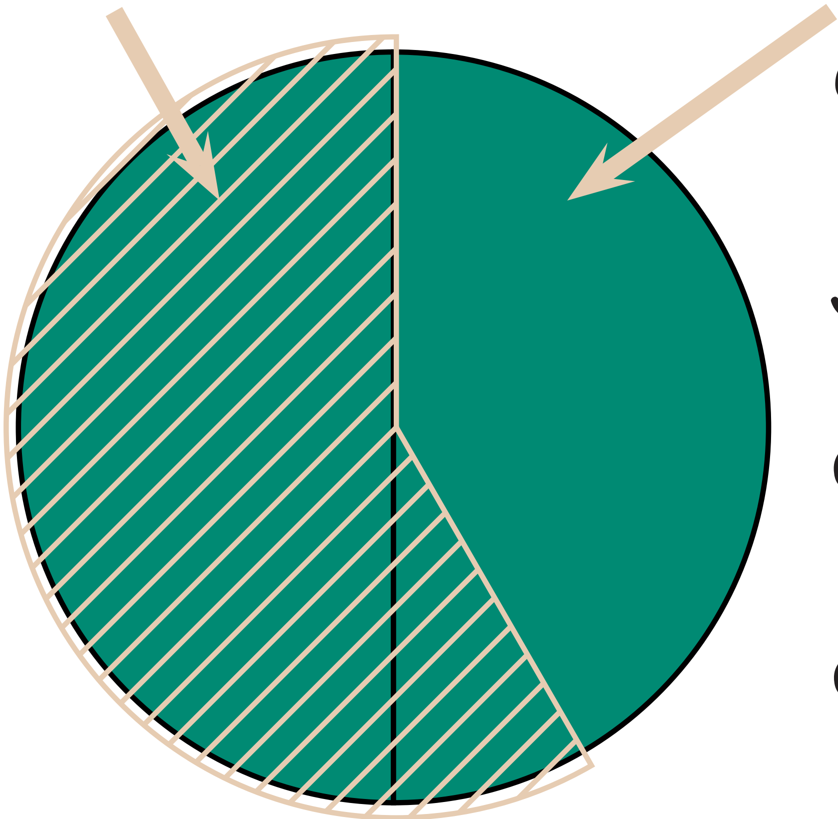

trapped errors

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annoying

A weakly typed programming language prevents some untrapped errors, but **not** all; C, C++ have features that make them weakly typed.

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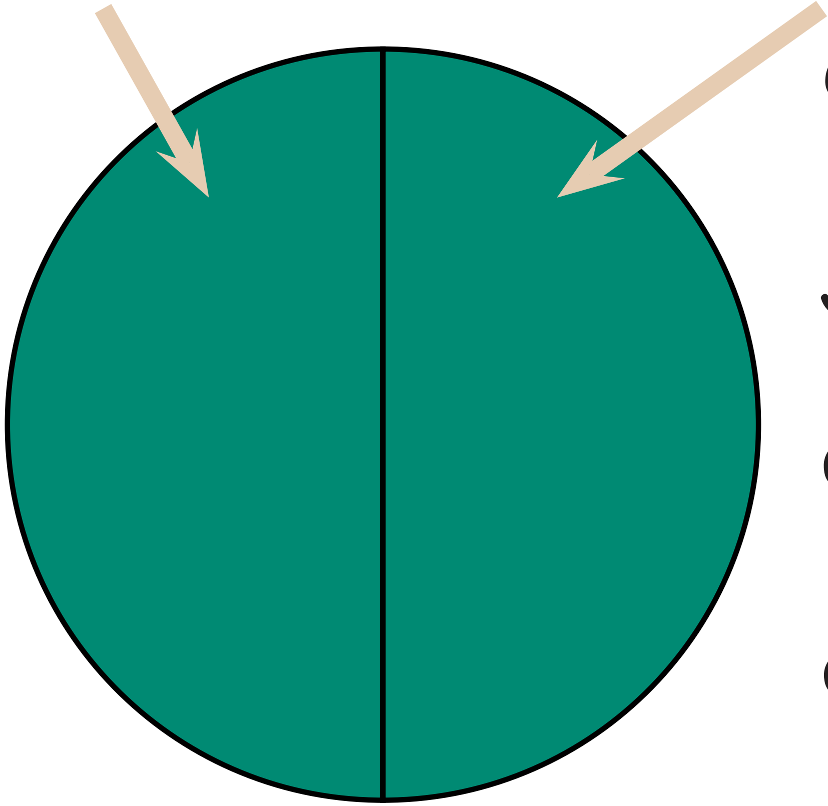

trapped errors

e.g. division by zero; jumping to an illegal address

annoying

	Typed	Untyped
Safe	SML, Java	LISP
Unsafe	C, C++	Assembler

Real World-Compilers

- So far we said that a program should type-check, and then we forget about types (not always possible because of dynamic checks)
- This is however **not** what happens in practice:
  - an optimising compiler for a high-level language might make as many as 20 passes over a single program
  - many optimisations require type-information to succeed (direct register allocation for integer operations)
  - a compiler often translates between many intermediate languages (type-information helps to stay sane)

Safety in the Target Lang.

- The target language (e.g. Java bytecode or Microsoft's Common Language infrastructure) might be typed.
- In Java bytecode the types of the parameters of all instructions are known and the verifier ensures they are correct.
- This ensures there are no operand stack overflows or underflows; pointer arithmetic is not arbitrary.
- Only when the bytecode is run, most checks are not needed anymore.
- The ultimate goal is that you can run untrusted code on your machine.

Example We Shall Look At

- We want to ensure the property of control-flow safety of “assembler programs”:

  A program cannot jump to an arbitrary address, but only to a well-defined subset of possible entry points.

- Greg Morrisett calls this language TAL-0 (Typed Assembly Language) and describes it in the book on advanced topics on types and programming languages.

Language

- Registers

$$r ::= r_1 \mid \ldots \mid r_k$$

- Operands

$$\begin{array}{lcll} v & ::= & n & \text{integer literal} \\ & \mid & l & \text{label or pointer} \\ & \mid & r & \text{register} \end{array}$$

- Instructions

$$\begin{array}{lcl} i & ::= & r := v \\ & \mid & r := r + v \\ & \mid & \text{if } r \text{ jump } v \end{array}$$

- Programs

$$\begin{array}{lcl} p & ::= & \text{jump } v \\ & \mid & i; p \end{array}$$

Example

- The calculation of the product of r_1 and r_2, placing the result in r_3; return to an address assumed to be in r_4:

prod:	$r_3 := 0$	% $res := 0$
	jump loop	
loop:	if r_1 jump done	% if $a = 0$ goto done
	$r_3 := r_2 + r_3$	% $res := res + b$
	$r_1 := r_1 + (-1)$	% $a := a - 1$
	jump loop	
done:	jump r_4	% return

Machine States

- Machine states are triples (H, R, p)
- Heaps

$$\{l_1 := p_1, \ldots, l_m := p_m\}$$

Heaps

$$p_{\mathsf{prod}} \left\{ \begin{array}{ll} \text{prod:} & r_3 := 0 \\ & \text{jump loop} \end{array} \right.$$

$$p_{\mathsf{loop}} \left\{ \begin{array}{ll} \text{loop:} & \text{if } r_1 \text{ jump done} \\ & r_3 := r_2 + r_3 \\ & r_1 := r_1 + (-1) \\ & \text{jump loop} \end{array} \right.$$

$$p_{\mathsf{done}} \left\{ \begin{array}{ll} \text{done:} & \text{jump } r_4 \end{array} \right.$$

$$H = \{\mathsf{prod} = p_{\mathsf{prod}}, \mathsf{loop} = p_{\mathsf{loop}}, \mathsf{done} = p_{\mathsf{done}}\}$$

Machine States

- Machine states are triples (H, R, p)
- Heaps

$$\{l_1 := p_1, \ldots, l_m := p_m\}$$

- Register files

$$\{r_1 := v_1, \ldots, r_n := v_n\}$$

- Safety property is that no machine state is stuck (for example jump 42 is stuck).

Transitions

- Jump

$$\frac{H(v) = p}{(H, R, \text{jump } v) \to (H, R, p)}$$

- Mov

$$(H, R, r := v; p) \to (H, R[r := v], p)$$

- Add

$$\frac{R(r') = n \quad R(v) = n'}{(H, R, r := r' + v; p) \to (H, R[r := r' + v], p)}$$

- If-eq

$$\frac{R(r) = 0 \quad H(v) = p'}{(H, R, \text{if } r \text{ jump } v; p) \to (H, R, p')}$$

- If-neq

$$\frac{R(r) \neq 0}{(H, R, \text{if } r \text{ jump } v; p) \to (H, R, p)}$$

Type System

- Any well-typed "machine" cannot get stuck (remember jump 42 should not be a well-typed program).
- Types

$$\begin{array}{rcll} T & ::= & \mathsf{int} & \\ & \mid & X & \text{type-variables} \\ & \mid & \forall X.T & \text{polymorphic types} \\ & \mid & \mathsf{code}(\Gamma) & \text{code labels} \\ & \mid & & \end{array}$$

- $\Gamma ::= \{r_1 : T_1, \ldots, r_n : T_n\}$: these are register file types (in a minute)

Example

prod: $r_3 := 0$
jump loop

loop: if r_1 jump done
$r_3 := r_2 + r_3$
$r_1 := r_1 + (-1)$
jump loop

done: jump r_4

- Γ contains the "assumptions" we make about the code

$$\{r_1, r_2, r_3 : \mathsf{int}, r_4 : \forall X.\mathsf{code}\{r_1, r_2, r_3 : \mathsf{int}, r_4 : X\}\}$$

- They will be recorded in Δ, for example

$$\{\mathsf{prod} : \mathsf{code}(\Gamma), \mathsf{loop} : \mathsf{code}(\Gamma), \mathsf{done} : \mathsf{code}(\Gamma)\}$$

Judgements (I)

- We will have **several** kinds of judgments:

- Integer literal

$$\Delta \vdash n : \mathsf{int}$$

- Label

$$\frac{(l : T) \in \Delta}{\Delta \vdash l : T}$$

(We want to have unique labels.)

Judgements (II)

- Register

$$\frac{(r : T) \in \Gamma}{\Delta; \Gamma \vdash r : T}$$

- Value (non-register)

$$\frac{\Delta \vdash v : T}{\Delta; \Gamma \vdash v : T}$$

- Type-instantiation

$$\frac{\Delta; \Gamma \vdash v : \forall X.T}{\Delta; \Gamma \vdash v : T[X := T']}$$

Judgements (III)

- Instructions will be dealt with by

$$\Delta \vdash i : \Gamma_{\mathsf{in}} \rightarrow \Gamma_{\mathsf{out}}$$

- Mov

$$\frac{\Delta; \Gamma \vdash v : T}{\Delta \vdash r := v : \Gamma \rightarrow \Gamma[r : T]}$$

- Add

$$\frac{\Delta; \Gamma \vdash r' : \mathsf{int} \quad \Delta; \Gamma \vdash v : \mathsf{int}}{\Delta \vdash r := r' + v : \Gamma \rightarrow \Gamma[r : \mathsf{int}]}$$

- If

$$\frac{\Delta; \Gamma \vdash r : \mathsf{int} \quad \Delta; \Gamma \vdash v : \mathsf{code}(\Gamma)}{\Delta \vdash \mathsf{if}\ r\ \mathsf{jump}\ v : \Gamma \rightarrow \Gamma}$$

Judgements (IV)

- programs (instruction sequences)

$$\Delta \vdash p : \mathsf{code}(\Gamma)$$

- Jump

$$\frac{\Delta; \Gamma \vdash v : \mathsf{code}(\Gamma)}{\Delta \vdash \mathsf{jump}\ v : \mathsf{code}(\Gamma)}$$

- Seq

$$\frac{\Gamma \vdash i : \Gamma \to \Gamma' \quad \Delta \vdash p : \mathsf{code}(\Gamma')}{\Delta; \Gamma \vdash i; p : \mathsf{code}(\Gamma)}$$

Examples

- Let Γ be

$$\{r_1, r_2, r_3 : \mathsf{int}, r_4 : \forall X.\mathsf{code}\{r_1, r_2, r_3 : \mathsf{int}, r_4 : X\}\}$$

- Let Δ be

$$\{\mathsf{prod} : \mathsf{code}(\Gamma), \mathsf{loop} : \mathsf{code}(\Gamma), \mathsf{done} : \mathsf{code}(\Gamma)\}$$

- Derivable judgements:
  - $\Delta \vdash \mathsf{if}\ r_1\ \mathsf{jump\ done} : \Gamma \to \Gamma$
  - $\Delta \vdash r_3 := r_2 + r_3 : \Gamma \to \Gamma$
  - $\Delta \vdash r_1 := r_1 + (-1) : \Gamma \to \Gamma$
  - $\Delta \vdash \mathsf{jump\ loop} : \mathsf{code}(\Gamma)$
- So we showed $\Delta \vdash p_{\mathsf{loop}} : \mathsf{code}(\Gamma)$

Loose Ends

- A register file is well-typed, written $\Delta \vdash R : \Gamma$, if for all $(r : T)$ in R

$$\Delta; \Gamma \vdash r : T$$

- A heap is well-typed, written $\vdash H : \Delta$, if for all $l : T$ in Δ

$$\Delta \vdash H(l) : T$$

  and the T does not contain any free type-variables.

Well-Typedness

- The types avoid to jump to an integer or an undefined label — however the situation is more complicated than is solvable by tags.
- We can have

$$
\begin{array}{ll}
\text{foo:} & r_1 := \text{bar} \\
 & \text{jump } r_1 \\
 & \\
\text{bar:} & \ldots
\end{array}
$$

Well-Typedness

- Polymorphism even allows us

$$\{r_1 : \mathsf{int}, \ldots\}$$
$$\mathsf{jump\ bar}$$

$$\{r_1 : \mathsf{code}(\ldots), \ldots\}$$
$$\mathsf{jump\ bar}$$

where the type of bar is

$$\forall X.\mathsf{code}(r_1 : X, \ldots).$$

Next Time

- We can show that given a well-typed machine state M then M cannot get stuck (i.e. jump to an integer or an undefined label).
- Proof-Outline: M is not immediately stuck and if $M \rightarrow M'$ then M' is also well-typed.

- Question: given a machine state $M = (H, R, p)$ can one find a Δ and Γ such that $\Delta \vdash p : \mathsf{code}(\Gamma)$ etc?
- Answer: We do not know. (Likely not.)
- The compiler has to give enough information during the compilation process so that the bytecode only needs to be "type-verified" — type-inference is too hard.

More Next Week

- Slides at the end of

  http://www4.in.tum.de/lehre/vorlesungen/types/WS0607/

  There is also an appraisal form where you can complain anonymously.

- You can say whether the lecture was too easy, too quiet, too hard to follow, too chaotic and so on. You can also comment on things I should repeat.