Handout (5)

November 21, 2006

The original algorithm of Damas and Milner:

- Input: a context $\mathit{\Gamma}$ and an expression e.
- Output: a substitution and a type.

- **Case Variables:**
$$W(\mathit{\Gamma}, x) = (\varepsilon, T[X_1 := Y_1, \ldots, X_n := Y_n])$$
where $(x : \forall\{X_1, \ldots, X_n\}.T) \in \mathit{\Gamma}$ and the Y_i are distinct and fresh (w.r.t. $\mathit{\Gamma}$ and T).

- **Case Lambdas:** calculate first
$$W((x : \forall\{\}.Y, \mathit{\Gamma}), e) = (\theta, T_1)$$
where Y is fresh (w.r.t. $\mathit{\Gamma}$). Then return
$$W(\mathit{\Gamma}, \lambda x.e) = (\theta, \theta(Y) \to T_1).$$

- **Case Applications:** Calculate first
$$W(\mathit{\Gamma}, e_1) = (\theta_1, T_1)$$
then
$$W(\theta_1(\mathit{\Gamma}), e_2) = (\theta_2, T_2)$$
and then
$$\text{unify}\ \{\theta_2(T_1) =^? T_2 \to Y\} = \theta_3$$
where Y is fresh (w.r.t. $\mathit{\Gamma}$). Finally return
$$W(\mathit{\Gamma}, e_1\ e_2) = (\theta_3 \circ \theta_2 \circ \theta_1, \theta_3(Y)).$$

- **Case Lets:** Calculate first
$$W(\mathit{\Gamma}, e_1) = (\theta_1, T_1)$$
then
$$A = \text{tv}(T_1) - \text{ftv}(\theta_1(\mathit{\Gamma}))$$
and then
$$W((x : \forall A.T_1, \theta_1(\mathit{\Gamma})), e_2) = (\theta_2, T_2)$$
Finally return
$$W(\mathit{\Gamma}, \text{let}\, x = e_1\ \text{in}\, e_2) = (\theta_2 \circ \theta_1, T_2)$$