Compilers and Formal Languages

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Slides & Progs: KEATS (also homework is there)

1 Introduction, Languages	6 While-Language
2 Regular Expressions, Derivatives	7 Compilation, JVM
3 Automata, Regular Languages	8 Compiling Functional Languages
4 Lexing, Tokenising	9 Optimisations
5 Grammars, Parsing	10 LLVM

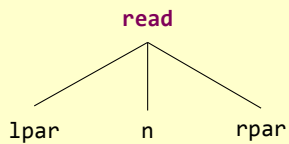
Parser

Parser

parser input: a sequence of tokens

```
key(read) lpar id(n) rpar semi
```

parser output: an abstract syntax tree



What Parsing is Not

Usually parsing does not check semantic correctness, e.g.

- whether a function is not used before it is defined
- whether a function has the correct number of arguments or are of correct type
- whether a variable can be declared twice in a scope

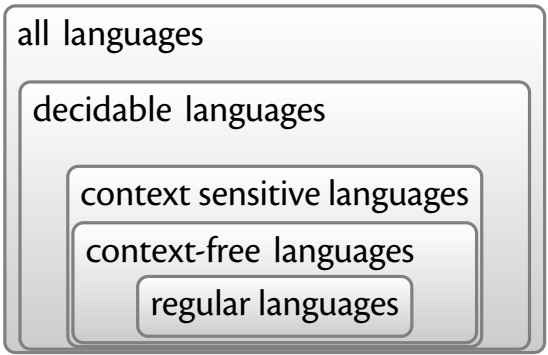
Regular Languages

While regular expressions are very useful for lexing, there is no regular expression that can recognise the language $a^n b^n$.

$$((((()))))(()) \quad \text{vs.} \quad ((((()))))(()))$$

So we cannot find out with regular expressions whether parentheses are matched or unmatched. Also regular expressions are not recursive, e.g. $(1 + 2) + 3$.

Hierarchy of Languages



Time flies like an arrow.

Fruit flies like bananas.

CFGs

A **context-free grammar** G consists of

a finite set of nonterminal symbols (e.g. A upper case)

a finite set terminal symbols or tokens (lower case)

a start symbol (which must be a nonterminal)

a set of rules

$$A ::= rhs$$

where rhs are sequences involving terminals and nonterminals, including the empty sequence ϵ.

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where rhs are sequences involving terminals and nonterminals, including the empty sequence ϵ.

We also allow rules

$$A ::= rhs_1 | rhs_2 | \ldots$$

Palindromes

A grammar for palindromes over the alphabet $\{a, b\}$:

$$\mathbf{S} ::= a \cdot \mathbf{S} \cdot a$$

$$\mathbf{S} ::= b \cdot \mathbf{S} \cdot b$$

$$\mathbf{S} ::= a$$

$$\mathbf{S} ::= b$$

$$\mathbf{S} ::= \epsilon$$

Palindromes

A grammar for palindromes over the alphabet $\{a, b\}$:

$$\mathbf{S} ::= a \cdot \mathbf{S} \cdot a \mid b \cdot \mathbf{S} \cdot b \mid a \mid b \mid \epsilon$$

Arithmetic Expressions

$$
\begin{aligned}
E ::= & \; 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\
& \mid E \cdot + \cdot E \\
& \mid E \cdot - \cdot E \\
& \mid E \cdot * \cdot E \\
& \mid (\cdot E \cdot)
\end{aligned}
$$

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$$

```
1 + 2 * 3 + 4
```

A CFG Derivation

Begin with a string containing only the start symbol, say S

Replace any nonterminal X in the string by the right-hand side of some production $X ::= rhs$

Repeat 2 until there are no nonterminals left

$$S \to \ldots \to \ldots \to \ldots \to \ldots$$

Example Derivation

$$S ::= \epsilon \mid a \cdot S \cdot a \mid b \cdot S \cdot b$$

$$\begin{aligned} S &\rightarrow aSa \\ &\rightarrow abSba \\ &\rightarrow abaSaba \\ &\rightarrow abaaba \end{aligned}$$

Example Derivation

$$
\begin{aligned}
E ::= {} & 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\
& \mid E \cdot + \cdot E \\
& \mid E \cdot - \cdot E \\
& \mid E \cdot * \cdot E \\
& \mid (\cdot E \cdot)
\end{aligned}
$$

$$
\begin{aligned}
E &\rightarrow E * E \\
&\rightarrow E + E * E \\
&\rightarrow E + E * E + E \\
&\rightarrow^{+} 1 + 2 * 3 + 4
\end{aligned}
$$

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$$
\begin{aligned}
E &\to E * E \\
&\to E + E * E \\
&\to E + E * E + E \\
&\to^{+} 1 + 2 * 3 + 4
\end{aligned}
\qquad
\begin{aligned}
E &\to E + E \\
&\to E + E + E \\
&\to E + E * E + E \\
&\to^{+} 1 + 2 * 3 + 4
\end{aligned}
$$

Context Sensitive Grammars

It is much harder to find out whether a string is parsed by a context sensitive grammar:

$$S ::= bSAA \mid \epsilon$$

$$A ::= a$$

$$bA ::= Ab$$

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Time flies like an arrow;
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```

Language of a CFG

Let G be a context-free grammar with start symbol $\mathbf{S}$.
Then the language $L(G)$ is:

$$\{c_1 \ldots c_n \mid \forall i.\ c_i \in T \wedge \mathbf{S} \rightarrow^* c_1 \ldots c_n\}$$

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Terminals, because there are no rules for replacing them.

Once generated, terminals are "permanent".

Terminals ought to be tokens of the language (but can also be strings).

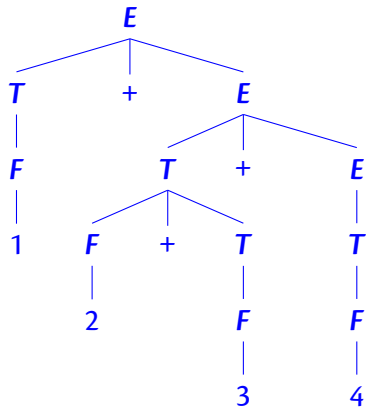
Parse Trees

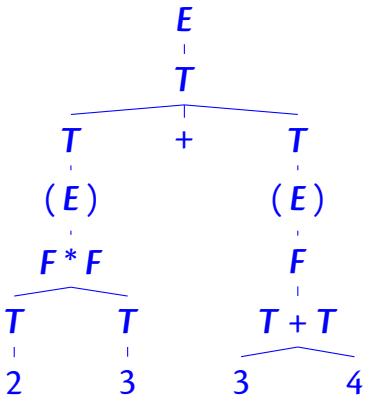
$E ::= T \mid T \cdot + \cdot E \mid T \cdot - \cdot E$

$T ::= F \mid F \cdot * \cdot T$

$F ::= 0 \ldots 9 \mid (\cdot E \cdot)$

1 + 2 * 3 + 4




E
T
T
+
T
(E)
(E)
F*F
F
T
T
T+T
2
3
3
4

Arithmetic Expressions

$$
\begin{array}{rcl}
E & ::= & 0..9 \\
 & | & E \cdot + \cdot E \\
 & | & E \cdot - \cdot E \\
 & | & E \cdot * \cdot E \\
 & | & (\cdot E \cdot)
\end{array}
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\mid {} & E \cdot + \cdot E \\
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\mid {} & E \cdot * \cdot E \\
\mid {} & (\cdot E \cdot)
\end{aligned}
$$

A CFG is **left-recursive** if it has a nonterminal E such that $E \rightarrow^{+} E \cdot \ldots$

Ambiguous Grammars

A grammar is **ambiguous** if there is a string that has at least two different parse trees.

$$
\begin{array}{rcl}
E & ::= & num\_token \\
 & | & E \cdot + \cdot E \\
 & | & E \cdot - \cdot E \\
 & | & E \cdot * \cdot E \\
 & | & (\cdot E \cdot)
\end{array}
$$

```
1 + 2 * 3 + 4
```

‘Dangling’ Else

Another ambiguous grammar:

$$
\begin{array}{rcl}
E & \rightarrow & \text{if } E \text{ then } E \\
 & | & \text{if } E \text{ then } E \text{ else } E \\
 & | & \ldots
\end{array}
$$

```
if a then if x then y else c
```

Parser Combinators

One of the simplest ways to implement a parser, see

https://vimeo.com/142341803

Parser combinators:

$$\underbrace{\text{list of tokens}}_{\text{input}} \Rightarrow \underbrace{\text{set of (parsed input, unparsed input)}}_{\text{output}}$$

- atomic parsers
- sequencing
- alternative
- semantic action

Atomic parsers, for example, number tokens

$$\texttt{Num(123)} :: \mathit{rest} \;\Rightarrow\; \{(\texttt{Num(123)}, \mathit{rest})\}$$

you consume one or more token from the input (stream)

also works for characters and strings

Alternative parser (code $p \mid\mid q$)

apply p and also q; then combine the outputs

$$p(\text{input}) \cup q(\text{input})$$

Sequence parser (code $p \sim q$)

apply first p producing a set of pairs
then apply q to the unparsed part
then combine the results:

$$((\text{output}_1, \text{output}_2), \text{unparsed part})$$

$$\{((o_1, o_2), u_2) \mid (o_1, u_1) \in p(\text{input}) \wedge (o_2, u_2) \in q(u_1)\}$$

Function parser (code $p \Rightarrow f$)

apply p producing a set of pairs

then apply the function f to each first component

$$\{(f(o_1), u_1) \mid (o_1, u_1) \in p(\text{input})\}$$

Function parser (code $p \Rightarrow f$)

apply p producing a set of pairs

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$$\{(f(o_1), u_1) \mid (o_1, u_1) \in p(\text{input})\}$$

f is the semantic action ("what to do with the parsed input")

Semantic Actions

Addition

$$T \sim + \sim E \Rightarrow \underbrace{f((x,y),z) \Rightarrow x + z}_{\text{semantic action}}$$

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Multiplication

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Parenthesis

$$(\sim E \sim) \Rightarrow f((x,y),z) \Rightarrow y$$

Types of Parsers

Sequencing: if p returns results of type T, and q results of type S, then $p \sim q$ returns results of type

$$T \times S$$

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$$T$$

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$$T$$

Semantic Action: if p returns results of type T and f is a function from T to S, then $p \Rightarrow f$ returns results of type

$$S$$

Input Types of Parsers

input: token list

output: set of (output_type, token list)

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input: token list

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actually it can be any input type as long as it is a kind of sequence (for example a string)

Scannerless Parsers

input: string

output: set of (output_type, string)

but using lexers is better because whitespaces or comments can be filtered out; then input is a sequence of tokens

Successful Parses

input: string

output: set of (output_type, string)

a parse is successful whenever the input has been fully "consumed" (that is the second component is empty)

Abstract Parser Class

```
abstract class Parser[I, T] {
  def parse(ts: I): Set[(T, I)]

  def parse_all(ts: I) : Set[T] =
    for ((head, tail) <- parse(ts);
         if (tail.isEmpty)) yield head
}
```

```
class AltParser[I, T](p: => Parser[I, T],
                      q: => Parser[I, T])
                          extends Parser[I, T] {
  def parse(sb: I) = p.parse(sb) ++ q.parse(sb)
}

class SeqParser[I, T, S](p: => Parser[I, T],
                         q: => Parser[I, S])
                             extends Parser[I, (T, S)] {
  def parse(sb: I) =
    for ((head1, tail1) <- p.parse(sb);
         (head2, tail2) <- q.parse(tail1))
           yield ((head1, head2), tail2)
}

class FunParser[I, T, S](p: => Parser[I, T], f: T => S)
                                    extends Parser[I, S] {
  def parse(sb: I) =
    for ((head, tail) <- p.parse(sb))
      yield (f(head), tail)
}
```

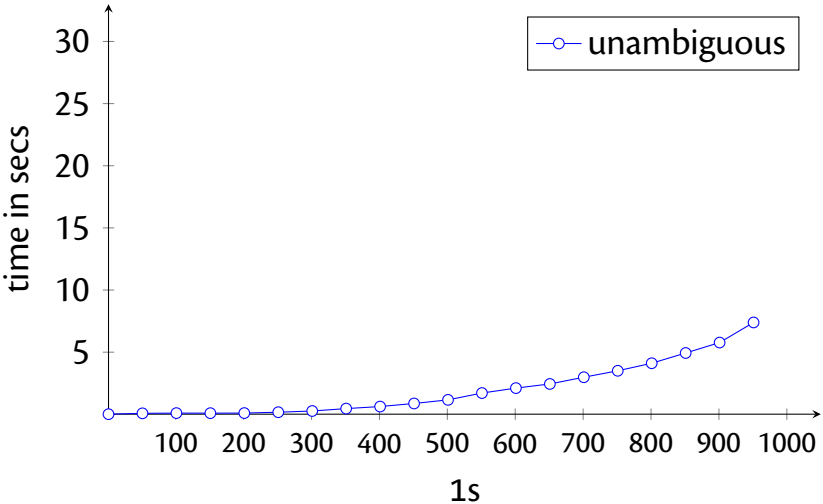
Two Grammars

Which languages are recognised by the following two grammars?

$$\begin{array}{rcl} S & \rightarrow & 1 \cdot S \cdot S \\ & | & \epsilon \end{array}$$

$$\begin{array}{rcl} U & \rightarrow & 1 \cdot U \\ & | & \epsilon \end{array}$$

Ambiguous Grammars



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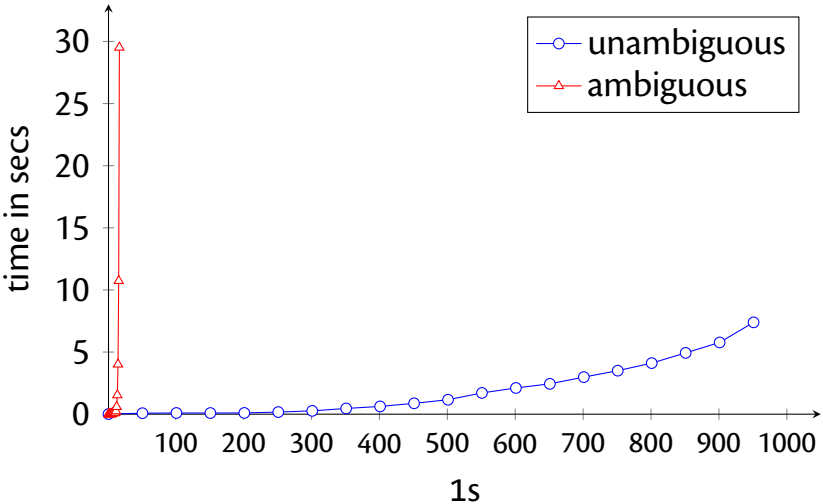

While-Language

$$
\begin{array}{rcl}
\textit{Stmt} & ::= & \text{skip} \\
& | & \textit{Id} := \textit{AExp} \\
& | & \text{if } \textit{BExp} \text{ then } \textit{Block} \text{ else } \textit{Block} \\
& | & \text{while } \textit{BExp} \text{ do } \textit{Block} \\
\textit{Stmts} & ::= & \textit{Stmt} \,;\, \textit{Stmts} \\
& | & \textit{Stmt} \\
\textit{Block} & ::= & \{\, \textit{Stmts} \,\} \\
& | & \textit{Stmt} \\
\textit{AExp} & ::= & \ldots \\
\textit{BExp} & ::= & \ldots
\end{array}
$$

An Interpreter

$$
\begin{array}{l}
\{ \\
\quad x := 5; \\
\quad y := x * 3; \\
\quad y := x * 4; \\
\quad x := u * 3 \\
\}
\end{array}
$$

the interpreter has to record the value of x before assigning a value to y

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$$

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`eval(stmt, env)`

Interpreter

$$
\begin{array}{lcl}
\mathrm{eval}(n, E) & \stackrel{\text{def}}{=} & n \\
\mathrm{eval}(x, E) & \stackrel{\text{def}}{=} & E(x) \quad \text{lookup } x \text{ in } E \\
\mathrm{eval}(a_1 + a_2, E) & \stackrel{\text{def}}{=} & \mathrm{eval}(a_1, E) + \mathrm{eval}(a_2, E) \\
\mathrm{eval}(a_1 - a_2, E) & \stackrel{\text{def}}{=} & \mathrm{eval}(a_1, E) - \mathrm{eval}(a_2, E) \\
\mathrm{eval}(a_1 * a_2, E) & \stackrel{\text{def}}{=} & \mathrm{eval}(a_1, E) * \mathrm{eval}(a_2, E) \\
 & & \\
\mathrm{eval}(a_1 = a_2, E) & \stackrel{\text{def}}{=} & \mathrm{eval}(a_1, E) = \mathrm{eval}(a_2, E) \\
\mathrm{eval}(a_1 != a_2, E) & \stackrel{\text{def}}{=} & \neg(\mathrm{eval}(a_1, E) = \mathrm{eval}(a_2, E)) \\
\mathrm{eval}(a_1 < a_2, E) & \stackrel{\text{def}}{=} & \mathrm{eval}(a_1, E) < \mathrm{eval}(a_2, E)
\end{array}
$$

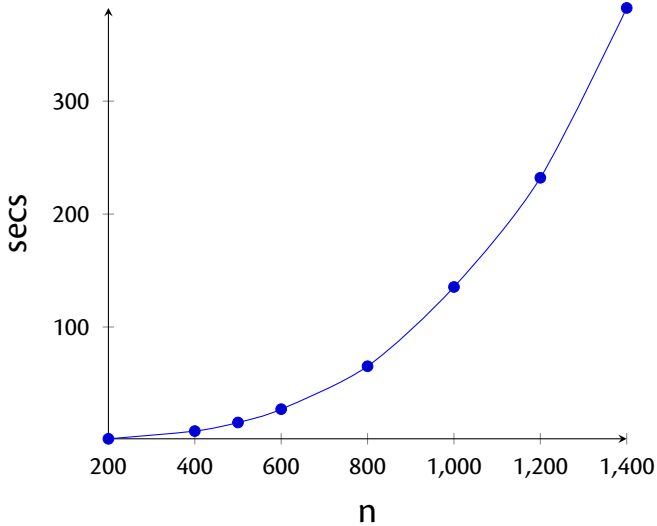
Interpreter (2)

$$\begin{array}{lcl}
\text{eval}(\text{skip}, E) & \stackrel{\text{def}}{=} & E \\
\text{eval}(x := a, E) & \stackrel{\text{def}}{=} & E(x \mapsto \text{eval}(a, E)) \\
\text{eval}(\text{if } b \text{ then } cs_1 \text{ else } cs_2, E) & \stackrel{\text{def}}{=} & \\
\multicolumn{3}{l}{\qquad \text{if eval}(b, E) \text{ then eval}(cs_1, E)} \\
\multicolumn{3}{l}{\qquad\qquad\qquad \text{else eval}(cs_2, E)} \\
\text{eval}(\text{while } b \text{ do } cs, E) & \stackrel{\text{def}}{=} & \\
\multicolumn{3}{l}{\qquad \text{if eval}(b, E)} \\
\multicolumn{3}{l}{\qquad \text{then eval}(\text{while } b \text{ do } cs, \text{eval}(cs, E))} \\
\multicolumn{3}{l}{\qquad \text{else } E} \\
\text{eval}(\text{write } x, E) & \stackrel{\text{def}}{=} & \{\ \text{println}(E(x))\ ;\ E\ \}
\end{array}$$

Test Program

??

Interpreted Code



Java Virtual Machine

introduced in 1995

is a stack-based VM (like Postscript, CLR of .Net)

contains a JIT compiler

many languages take advantage of JVM's infrastructure (JRE)

is garbage collected $\Rightarrow$ no buffer overflows

some languages compile to the JVM: Scala, Clojure...