Homework 9

1. Describe what is meant by *eliminating tail recursion*, when such an optimization can be applied and why it is a benefit?

2. It is true (I confirmed it) that

$$\text{if } \varnothing \text{ does not occur in } r \quad \text{then} \quad L(r) \neq \{\}$$

holds, or equivalently

$$L(r) = \{\} \quad \text{implies} \quad \varnothing \text{ occurs in } r.$$

You can prove either version by induction on r. The best way to make more formal what is meant by '$\varnothing$ occurs in r', you can define the following function:

$$\begin{array}{lcl} \mathit{occurs}(\varnothing) & \stackrel{\text{def}}{=} & \mathit{true} \\ \mathit{occurs}(\epsilon) & \stackrel{\text{def}}{=} & \mathit{false} \\ \mathit{occurs}(c) & \stackrel{\text{def}}{=} & \mathit{false} \\ \mathit{occurs}(r_1 + r_2) & \stackrel{\text{def}}{=} & \mathit{occurs}(r_1) \vee \mathit{occurs}(r_2) \\ \mathit{occurs}(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & \mathit{occurs}(r_1) \vee \mathit{occurs}(r_2) \\ \mathit{occurs}(r^*) & \stackrel{\text{def}}{=} & \mathit{occurs}(r) \end{array}$$

Now you can prove

$$L(r) = \{\} \quad \text{implies} \quad \mathit{occurs}(r).$$

The interesting cases are $r_1 + r_2$ and r^*. The other direction is not true, that is if $\mathit{occurs}(r)$ then $L(r) = \{\}$. A counter example is $\varnothing + a$: although $\varnothing$ occurs in this regular expression, the corresponding language is not empty. The obvious extension to include the not-regular expression, $\sim r$, also leads to an incorrect statement. Suppose we add the clause

$$\mathit{occurs}(\sim r) \stackrel{\text{def}}{=} \mathit{occurs}(r)$$

to the definition above, then it will not be true that

$$L(r) = \{\} \quad \text{implies} \quad \mathit{occurs}(r).$$

Assume the alphabet contains just a and b, find a counter example to this property.