Homework 3

1. What is a regular language? Are there alternative ways to define this notion? If yes, give an explanation why they define the same notion.

2. Why is every finite set of strings a regular language?

3. Assume you have an alphabet consisting of the letters a, b and c only. (1) Find a regular expression that recognises the two strings ab and ac. (2) Find a regular expression that matches all strings *except* these two strings. Note, you can only use regular expressions of the form

$$r ::= \varnothing \mid \epsilon \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

4. Define the function *zeroable* which takes a regular expression as argument and returns a boolean. The function should satisfy the following property:

$$\textit{zeroable}(r) \text{ if and only if } L(r) = \varnothing$$

5. Given the alphabet $\{a, b\}$. Draw the automaton that has two states, say q_0 and q_1. The starting state is q_0 and the final state is q_1. The transition function is given by

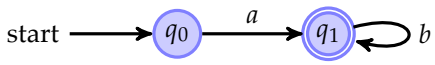
$$\begin{array}{l}(q_0, a) \to q_0\\ (q_0, b) \to q_1\\ (q_1, b) \to q_1\end{array}$$

   What is the language recognised by this automaton?

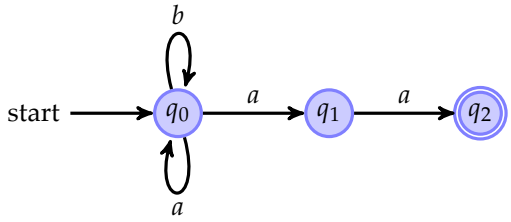
6. Give a non-deterministic finite automaton that can recognise the language $L(a \cdot (a + b)^* \cdot c)$.

7. Given a deterministic finite automata $A(Q, q_0, F, \delta)$, define which language is recognised by this automaton. Can you define also the language defined by a non-deterministic automaton?

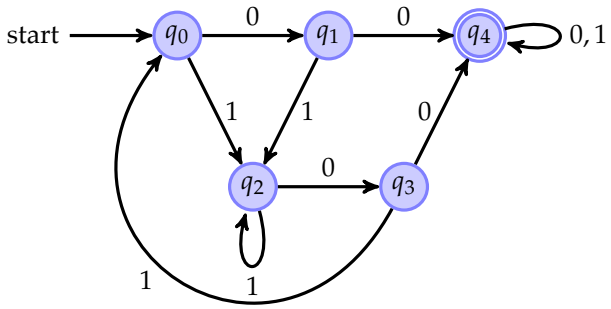
8. Given the following deterministic finite automata over the alphabet $\{a, b\}$, find an automaton that recognises the complement language. (Hint: Recall that for the algorithm from the lectures, the automaton needs to be in completed form, that is have a transition for every letter from the alphabet.)



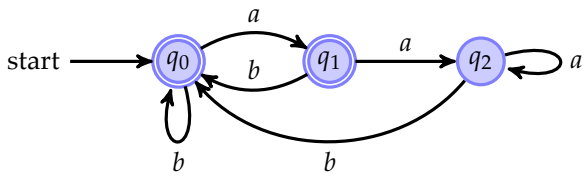
9. Given the following non-deterministic finite automaton over the alphabet $\{a, b\}$, find a deterministic finite automaton that recognises the same language:



10. Given the following deterministic finite automaton over the alphabet $\{0, 1\}$, find the corresponding minimal automaton. In case states can be merged, state clearly which states can be merged.



11. Given the following finite deterministic automaton over the alphabet $\{a, b\}$:



Give a regular expression that can recognise the same language as this automaton. (Hint: If you use Brzozwski's method, you can assume Arden's lemma which states that an equation of the form $q = q \cdot r + s$ has the unique solution $q = s \cdot r^*$.)

12. If a non-deterministic finite automaton (NFA) has n states. How many states does a deterministic automaton (DFA) that can recognise the same language as the NFA maximal need?