CSCI 742 - Compiler Construction

Lecture 11

Grammar Transformations

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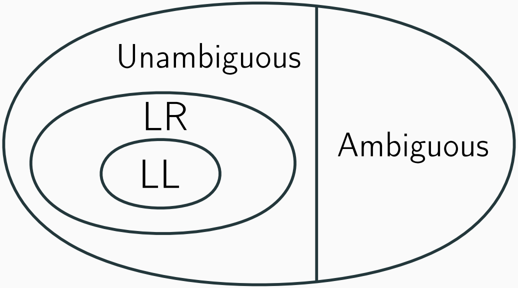
Space of Context-free Grammars

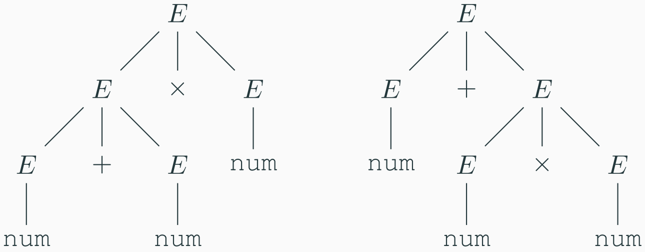


Context-Free Grammars (CFG)

Recap: Ambiguity

- Ambiguous grammar: a word has more than one parse tree
- There is no algorithm to decide if a grammar is ambiguous

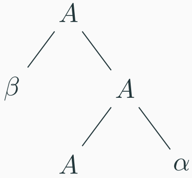
$$\boxed{\begin{aligned} E &\to E + E \\ E &\to E \times E \\ E &\to \mathrm{num} \end{aligned}}$$

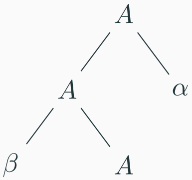


Two parse trees for $\mathrm{num} + \mathrm{num} \times \mathrm{num}$

Ambiguity Patterns

- There are common ambiguity patterns in context-free grammars
- Example:
- If a non-terminal $A \in N$ is both left-recursive and right-recursive then G is ambiguous
  - Left-recursive: it has a derivation $A \Rightarrow^+ A\alpha \quad (\alpha \in (N \cup T)^+)$
  - Right-recursive: it has a derivation $A \Rightarrow^+ \beta A \quad (\beta \in (N \cup T)^+)$

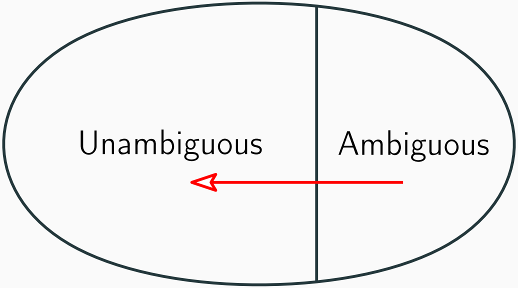




Resolving Ambiguity

- Designing unambiguous grammars is usually tricky
- Sometimes it is possible to eliminate ambiguity by rewriting grammar
    - similar to Chomsky normal form conversion
- Occasionally more natural grammar is the ambiguous one
- Parser generators allow disambiguating declarations for ambiguous grammars

Eliminating Ambiguity



Goal: transform an ambiguous grammar to an equivalent unambiguous grammar

Equivalent Grammars

- Grammars G_1 and G_2 are equivalent if they generate the same language
$$L(G_1) = L(G_2)$$
- In other words if a sentence can be derived from one of the grammars it can be derived also from other grammar

Common Examples of Ambiguity

- Operators with different priorities

$$x + y \times z = t$$

- Associativity of operators of the same priority

$$x + y - z + t$$

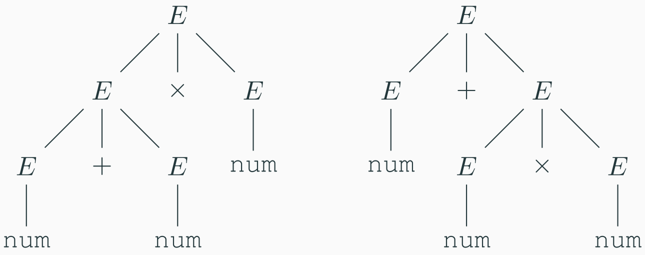
- Dangling `else`

```
if a then if b then s1 else s2
```

Example Ambiguity (Priority)

Two parse trees for $\texttt{num} + \texttt{num} \times \texttt{num}$

$$\boxed{\begin{aligned} E &\to E + E \\ E &\to E \times E \\ E &\to \texttt{num} \end{aligned}}$$



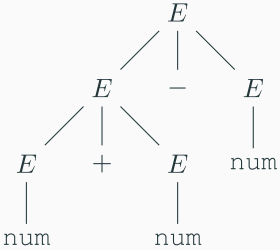
priority(+) > priority(×)   priority(×) > priority(+)

Multiplication should take precedence over addition

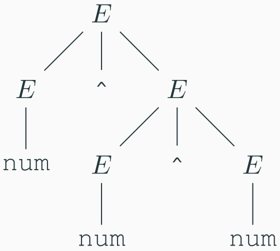
Example Ambiguity (Associativity)

How do operators with same priority associate in a sequence?

$$E \to E + E$$
$$E \to E - E$$
$$E \to E \mathbin{\hat{}} E$$
$$E \to \mathrm{num}$$



Left-associative



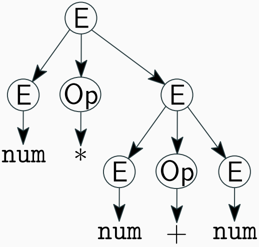
Right-associative

Example

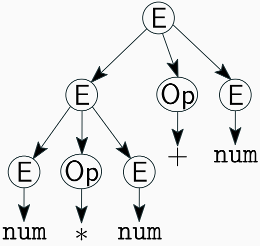
- This grammar is ambiguous

$$E \rightarrow E \text{ Op } E \mid \texttt{num}$$
$$\text{Op} \rightarrow + \mid - \mid * \mid /$$

- Two parse trees for `num * num + num`



`num * (num + num)`



`(num * num) + num`

Resolving Ambiguity

Example

- This grammar is ambiguous

$$\text{E} \rightarrow \text{E Op E} \mid \texttt{num}$$
$$\text{Op} \rightarrow + \mid - \mid * \mid /$$

- Grammar does not consider operator precedence
- We can eliminate ambiguity by rewriting it to a new grammar

Resolving Ambiguity: Rewriting

$$\overbrace{\texttt{num} \; * \; \texttt{num} \; * \; \texttt{num}} \; + \; \overbrace{\texttt{num} \; / \; \texttt{num}} \; - \; \overbrace{\texttt{num}}$$

- **Intuition:** since * and / bind more tightly than + and -, think of an expression as a series of “blocks” of terms multiplied and divided together joined by +s and -s

Resolving Ambiguity: Rewriting

$$\overbrace{\texttt{num} \mid * \mid \texttt{num} \mid * \mid \texttt{num}} \mid + \mid \overbrace{\texttt{num} \mid / \mid \texttt{num}} \mid - \mid \overbrace{\texttt{num}}$$

Force a construction order where

- First decide how many "blocks" will be of terms joined by + and -
- Then expand those blocks by filling in the integers multiplied and divided together
- A possible grammar:

$$S \to T \mid S + T \mid S - T$$
$$T \to \mathrm{num} \mid T * \mathrm{num} \mid T/\mathrm{num}$$

- Grammar is left recursive: makes operators left associative

Left Factoring

Question:

- Is the following grammar ambiguous? If yes how can we fix it?

$$S \to \texttt{if}\ E\ \texttt{then}\ S\ \texttt{else}\ S \mid \texttt{if}\ E\ \texttt{then}\ S \mid \cdots$$
$$E \to \cdots$$

Left Factoring

Question:

- Is the following grammar ambiguous? If yes how can we fix it?

$$S \to \texttt{if } E \texttt{ then } S \texttt{ else } S \mid \texttt{if } E \texttt{ then } S \mid \cdots$$
$$E \to \cdots$$

Possible Solution:

- On expanding S we cannot choose between productions when the next token is `if`
- We can solve this problem by factoring out the common parts
- This is called left-factoring

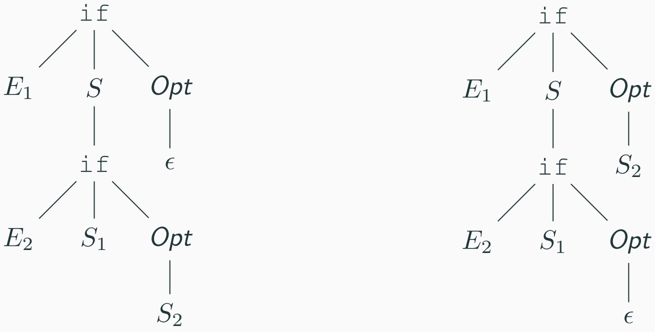
$$S \to \texttt{if } E \texttt{ then } S \; \textit{Opt}$$
$$\textit{Opt} \to \texttt{else } S \mid \epsilon$$

- Is the grammar still ambiguous?

Ambiguous `if-then-else` Grammar

$$S \to \texttt{if}\ E\ \texttt{then}\ S\ \textit{Opt}$$
$$\textit{Opt} \to \texttt{else}\ S \mid \epsilon$$

- Which `if` is the `else` attached to?



$$\texttt{if}\ E_1\ \texttt{then}\ \texttt{if}\ E_2\ \texttt{then}\ S_1\ \texttt{else}\ S_2$$

Grammar for Closest-**`if`** Rule

- Want to rule out $\texttt{if}\ E\ \texttt{then}\ \boxed{\texttt{if}\ E\ \texttt{then}\ S}\ \texttt{else}\ S$
- Impose that unmatched `if` statements occur only in the `else` clauses

$$
\begin{aligned}
S &\to \textit{Matched} \mid \textit{Unmatched} \\
\textit{Matched} &\to \texttt{if}\ E\ \texttt{then}\ \textit{Matched}\ \texttt{else}\ \textit{Matched} \\
\textit{Unmatched} &\to \texttt{if}\ E\ \texttt{then}\ S \\
&\mid \quad \texttt{if}\ E\ \texttt{then}\ \textit{Matched}\ \texttt{else}\ \textit{Unmatched}
\end{aligned}
$$

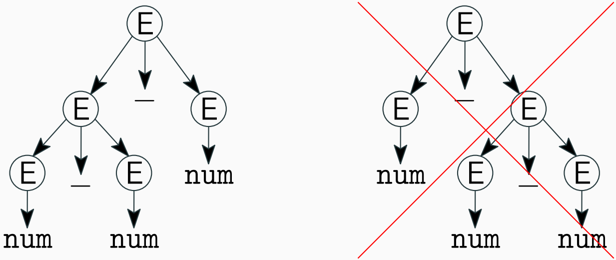
Resolving Ambiguity: Precedence & Associativity

- Instead of rewriting the grammar,
  use the more natural ambiguous grammars
- Use disambiguating declarations to disambiguate grammars
- Most parser generators allow precedence and associativity declarations to disambiguate grammars

Associativity Declarations

- Consider ambiguous grammar:

$$E \to E - E \mid \text{num}$$

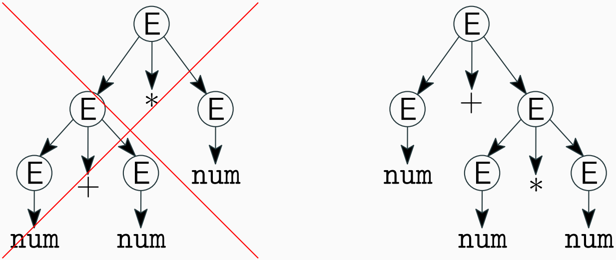


- Left associativity declaration:
  %left –

Precedence Declarations

- Consider ambiguous grammar:

$$E \rightarrow E + E \mid E * E \mid \mathrm{num}$$

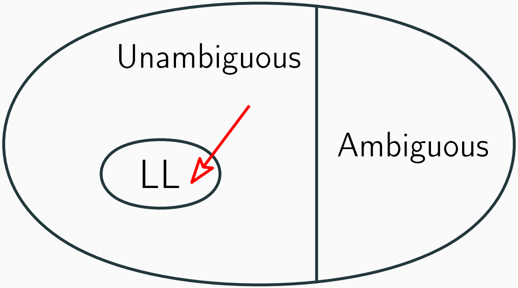


- Precedence declarations (order of precedence is low to high):

  %left $+$

  %left $*$

Create Equivalent LL Grammar



Remove Left Recursion

- Left recursion poses problems for LL parsers (e.g. $S \rightarrow Sa$)
- If a non-terminal can expand to a string with itself on the left, parser may expand that non-terminal forever without actually parsing anything
- It is possible to eliminate left recursion by transformation to right recursion
- For a left-recursive pair of rules:

$$A \rightarrow A\alpha \mid \beta$$

- Replace with the following rules:

$$A \rightarrow \beta A'$$
$$A' \rightarrow \alpha A' | \epsilon$$

Exercise

Question

Eliminate left recursion from the following grammar

$$S \to T \mid S + T \mid S - T$$
$$T \to \mathtt{num} \mid T * \mathtt{num} \mid T/\mathtt{num}$$

non-LL Grammar

- Consider the grammar: $$S \to E + E \mid E$$ $$E \to \mathrm{num} \mid (E)$$

- and the two derivations

$$S \Rightarrow E \qquad \Rightarrow (E) \qquad \Rightarrow (\textsf{num})$$
$$S \Rightarrow E + E \Rightarrow (E) + E \Rightarrow (\textsf{num}) + E \Rightarrow (\textsf{num}) + \textsf{num}$$

- **Question.** Can we decide between

$$S \Rightarrow E$$
$$S \Rightarrow E + E$$

  as the first derivation step based on finite number of lookahead tokens?
- **Answer.** No. Grammar is not LL(k) for any number of k

Making a Grammar LL

- **Problem:** can't decide which S production to apply until we see symbol after first expression
- **Left-factoring:** Factor common prefix E, add new non-terminal E' for what follows that prefix

$$\begin{aligned} S &\to E + E \mid E \\ E &\to \text{num} \mid (E) \end{aligned} \quad\Longrightarrow\quad \begin{aligned} S &\to EE' \\ E' &\to +E \mid \epsilon \\ E &\to \text{num} \mid (E) \end{aligned}$$

Making a Grammar LL

- An LL grammar does not have left recursion
- Conversion to LL:

1) First step: remove left recursion from grammar

$$\begin{aligned} \color{red}{A} &\rightarrow \color{red}{A}\alpha \\ &\mid \ \beta \end{aligned}$$

2) Second step: left factor the grammar

$$\begin{aligned} \color{red}{A} &\rightarrow \color{red}{\alpha}\ \beta_1 \\ &\mid \color{red}{\alpha}\ \beta_2 \end{aligned}$$

- This procedure does not necessarily convert any CFG to LL

Exercise

Question:

Left factor the following grammar:

$$
\begin{aligned}
A \rightarrow\ & XA \\
\mid\ & XB \\
\mid\ & X \\
\mid\ & Y \\
\mid\ & Z
\end{aligned}
$$