

Compilers and Formal Languages (6)

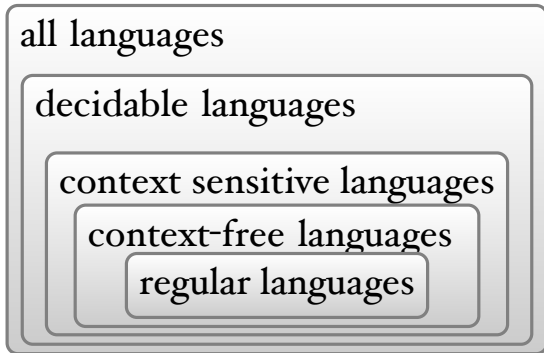
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Slides: KEATS (also home work is there)

Hierarchy of Languages

Recall that languages are sets of strings.



Parser Combinators

Atomic parsers, for example

$$I :: rest \Rightarrow \{(I, rest)\}$$

- you consume one or more tokens from the input (stream)
- also works for characters and strings

Alternative parser (code $p \parallel q$)

- apply p and also q ; then combine the outputs

$$p(\text{input}) \cup q(\text{input})$$

Sequence parser (code $p \sim q$)

- apply first p producing a set of pairs
- then apply q to the unparsed parts
- then combine the results:

$((\text{output}_1, \text{output}_2), \text{unparsed part})$

$$\{((o_1, o_2), u_2) \mid (o_1, u_1) \in p(\text{input}) \wedge (o_2, u_2) \in q(u_1)\}$$

Function parser (code $p \Rightarrow f$)

- apply p producing a set of pairs
- then apply the function f to each first component

$$\{(f(o_I), u_I) \mid (o_I, u_I) \in p(\text{input})\}$$

Types of Parsers

- **Sequencing:** if p returns results of type T , and q returns results of type S , then $p \sim q$ returns results of type

$$T \times S$$

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- **Semantic Action:** if p returns results of type T and f is a function from T to S , then $p \Rightarrow f$ returns results of type

$$S$$

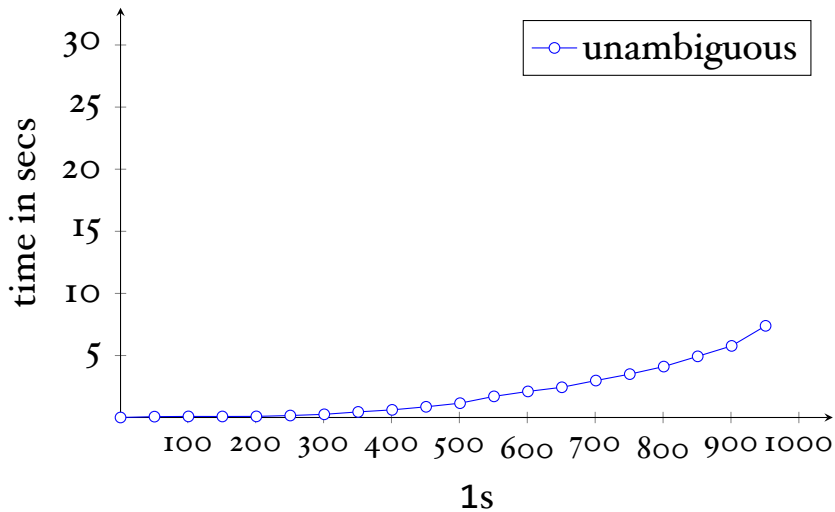
Two Grammars

Which languages are recognised by the following two grammars?

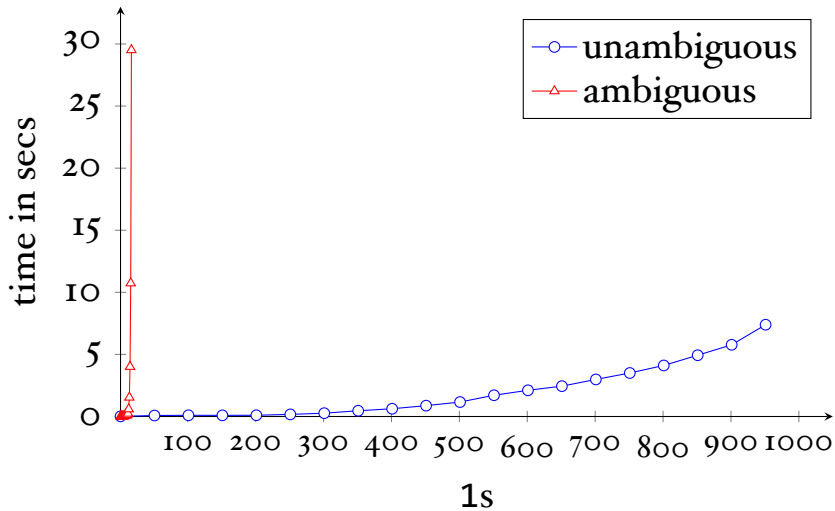
$$S ::= I \cdot S \cdot S \mid \epsilon$$

$$U ::= I \cdot U \mid \epsilon$$

Ambiguous Grammars



Ambiguous Grammars



Arithmetic Expressions

A grammar for arithmetic expressions and numbers:

$$E ::= E \cdot + \cdot E \mid E \cdot * \cdot E \mid (\cdot E \cdot) \mid N$$

$$N ::= N \cdot N \mid 0 \mid 1 \mid \dots \mid 9$$

Unfortunately it is left-recursive (and ambiguous).

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Numbers

$$N ::= N \cdot N \mid 0 \mid 1 \mid \dots \mid 9$$

A non-left-recursive, non-ambiguous grammar for numbers:

$$N ::= 0 \cdot N \mid 1 \cdot N \mid \dots \mid 0 \mid 1 \mid \dots \mid 9$$

Removing Left-Recursion

The rule for numbers is directly left-recursive:

$$N ::= N \cdot N \mid 0 \mid 1 \quad (\dots)$$

Translate

$$\begin{array}{l} N ::= N \cdot \alpha \\ \quad | \beta \end{array} \Rightarrow \begin{array}{l} N ::= \beta \cdot N' \\ N' ::= \alpha \cdot N' \\ \quad | \epsilon \end{array}$$

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Which means in this case:

$$\begin{array}{l} N \rightarrow 0 \cdot N' \mid 1 \cdot N' \\ N' \rightarrow N \cdot N' \mid \epsilon \end{array}$$

Operator Precedences

To disambiguate

$$\mathbf{E} ::= \mathbf{E} \cdot + \cdot \mathbf{E} \mid \mathbf{E} \cdot * \cdot \mathbf{E} \mid (\cdot \mathbf{E} \cdot) \mid \mathbf{N}$$

Decide on how many precedence levels, say
highest for $()$, medium for $*$, lowest for $+$

$$\begin{aligned} \mathbf{E}_{low} & ::= \mathbf{E}_{med} \cdot + \cdot \mathbf{E}_{low} \mid \mathbf{E}_{med} \\ \mathbf{E}_{med} & ::= \mathbf{E}_{hi} \cdot * \cdot \mathbf{E}_{med} \mid \mathbf{E}_{hi} \\ \mathbf{E}_{hi} & ::= (\cdot \mathbf{E}_{low} \cdot) \mid \mathbf{N} \end{aligned}$$

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What happens with $1 + 3 + 4$?

Chomsky Normal Form

All rules must be of the form

$$A ::= a$$

or

$$A ::= B \cdot C$$

No rule can contain ϵ .

ϵ -Removal

- 1 If $A ::= \alpha \cdot B \cdot \beta$ and $B ::= \epsilon$ are in the grammar, then add $A ::= \alpha \cdot \beta$ (iterate if necessary).
- 2 Throw out all $B ::= \epsilon$.

$$N ::= o \cdot N' \mid I \cdot N'$$
$$N' ::= N \cdot N' \mid \epsilon$$

$$N ::= o \cdot N' \mid I \cdot N' \mid o \mid I$$
$$N' ::= N \cdot N' \mid N \mid \epsilon$$

$$N ::= o \cdot N' \mid I \cdot N' \mid o \mid I$$
$$N' ::= N \cdot N' \mid N$$

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CYK Algorithm

If grammar is in Chomsky normalform ...

$S ::= N \cdot P$

$P ::= V \cdot N$

$N ::= N \cdot N$

$N ::= \text{students} \mid \text{Jeff} \mid \text{geometry} \mid \text{trains}$

$V ::= \text{trains}$

Jeff trains geometry students

CYK Algorithm

- fastest possible algorithm for recognition problem
- runtime is $O(n^3)$
- grammars need to be transformed into CNF

The Goal of this Course

Write a Compiler



We have a lexer and a parser...

Stmt ::= skip
| *Id* := ***AExp***
| if ***BExp*** then ***Block*** else ***Block***
| while ***BExp*** do ***Block***
| read *Id*
| write *Id*
| write *String*

Stmts ::= ***Stmt*** ; ***Stmts***
| ***Stmt***

Block ::= { ***Stmts*** }
| ***Stmt***

AExp ::= ...

BExp ::= ...

```
write "Fib";  
read n;  
minus1 := 0;  
minus2 := 1;  
while n > 0 do {  
    temp := minus2;  
    minus2 := minus1 + minus2;  
    minus1 := temp;  
    n := n - 1  
};  
write "Result";  
write minus2
```

An Interpreter

```
{  
  x := 5;  
  y := x * 3;  
  y := x * 4;  
  x := u * 3  
}
```

- the interpreter has to record the value of x before assigning a value to y

An Interpreter

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   $x := 5;$   
   $y := x * 3;$   
   $y := x * 4;$   
   $x := u * 3$   
}
```

- the interpreter has to record the value of x before assigning a value to y
- `eval(stmt, env)`

An Interpreter

$$\text{eval}(n, E) \stackrel{\text{def}}{=} n$$

$$\text{eval}(x, E) \stackrel{\text{def}}{=} E(x) \quad \text{lookup } x \text{ in } E$$

$$\text{eval}(a_1 + a_2, E) \stackrel{\text{def}}{=} \text{eval}(a_1, E) + \text{eval}(a_2, E)$$

$$\text{eval}(a_1 - a_2, E) \stackrel{\text{def}}{=} \text{eval}(a_1, E) - \text{eval}(a_2, E)$$

$$\text{eval}(a_1 * a_2, E) \stackrel{\text{def}}{=} \text{eval}(a_1, E) * \text{eval}(a_2, E)$$

$$\text{eval}(a_1 = a_2, E) \stackrel{\text{def}}{=} \text{eval}(a_1, E) = \text{eval}(a_2, E)$$

$$\text{eval}(a_1 \neq a_2, E) \stackrel{\text{def}}{=} \neg(\text{eval}(a_1, E) = \text{eval}(a_2, E))$$

$$\text{eval}(a_1 < a_2, E) \stackrel{\text{def}}{=} \text{eval}(a_1, E) < \text{eval}(a_2, E)$$

An Interpreter (2)

$$\text{eval}(\text{skip}, E) \stackrel{\text{def}}{=} E$$

$$\text{eval}(x := a, E) \stackrel{\text{def}}{=} E(x \mapsto \text{eval}(a, E))$$

$$\begin{aligned} \text{eval}(\text{if } b \text{ then } cs_1 \text{ else } cs_2, E) &\stackrel{\text{def}}{=} \\ &\text{if } \text{eval}(b, E) \text{ then } \text{eval}(cs_1, E) \\ &\text{else } \text{eval}(cs_2, E) \end{aligned}$$

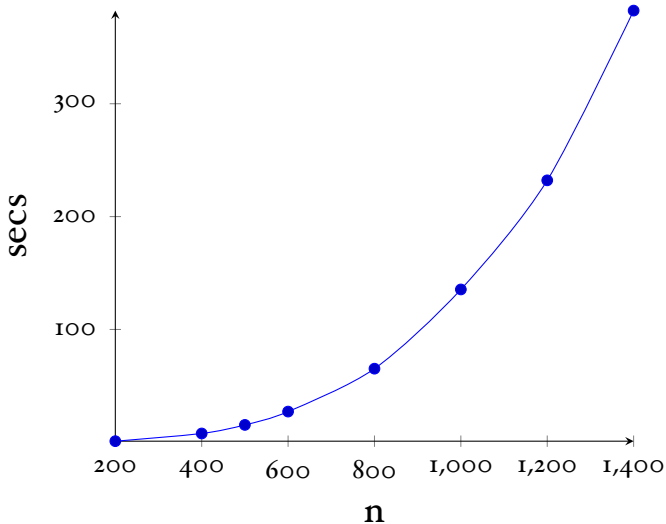
$$\begin{aligned} \text{eval}(\text{while } b \text{ do } cs, E) &\stackrel{\text{def}}{=} \\ &\text{if } \text{eval}(b, E) \\ &\text{then } \text{eval}(\text{while } b \text{ do } cs, \text{eval}(cs, E)) \\ &\text{else } E \end{aligned}$$

$$\text{eval}(\text{write } x, E) \stackrel{\text{def}}{=} \{ \text{println}(E(x)) ; E \}$$

Test Program

```
start := 1000;
x := start;
y := start;
z := start;
while 0 < x do {
  while 0 < y do {
    while 0 < z do { z := z - 1 };
    z := start;
    y := y - 1
  };
  y := start;
  x := x - 1
}
```


Interpreted Code



Java Virtual Machine

- introduced in 1995
- is a stack-based VM (like Postscript, CLR of .Net)
- contains a JIT compiler
- many languages take advantage of JVM's infrastructure (JRE)
- is garbage collected \Rightarrow no buffer overflows
- some languages compile to the JVM: Scala, Clojure...

Coursework: MkEps

$$mkeps([c_1 c_2 \dots c_n]) \stackrel{\text{def}}{=} \textit{undefined}$$

$$mkeps(r^*) \stackrel{\text{def}}{=} \textit{Stars} []$$

$$mkeps(r^{\{n\}}) \stackrel{\text{def}}{=} \textit{Stars} (mkeps(r))^n$$

$$mkeps(r^{\{n..\}}) \stackrel{\text{def}}{=} \textit{Stars} (mkeps(r))^n$$

$$mkeps(r^{\{..n\}}) \stackrel{\text{def}}{=} \textit{Stars} []$$

$$mkeps(r^{\{n..m\}}) \stackrel{\text{def}}{=} \textit{Stars} (mkeps(r))^n$$

$$mkeps(r^+) \stackrel{\text{def}}{=} mkeps(r^{\{1..\}})$$

$$mkeps(r^?) \stackrel{\text{def}}{=} mkeps(r^{\{..1\}})$$

Coursework: Inj

$inj([c_1 c_2 \dots c_n]) c$	$Empty$	$\stackrel{\text{def}}{=} Cbr\ c$
$inj(r^*) c$	$(Seq\ v\ (Stars\ vs))$	$\stackrel{\text{def}}{=} Stars\ (inj\ r\ c\ v :: vs)$
$inj(r^{\{n\}}) c$	$(Seq\ v\ (Stars\ vs))$	$\stackrel{\text{def}}{=} Stars\ (inj\ r\ c\ v :: vs)$
$inj(r^{\{n..\}}) c$	$(Seq\ v\ (Stars\ vs))$	$\stackrel{\text{def}}{=} Stars\ (inj\ r\ c\ v :: vs)$
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$inj(r^+) c\ v$		$\stackrel{\text{def}}{=} inj(r^{\{1..\}}) c\ v$
$inj(r^?) c\ v$		$\stackrel{\text{def}}{=} inj(r^{\{..\ 1\}}) c\ v$