

## Homework 3

Please submit your solutions via email. Please submit only ASCII text or PDFs. Every solution should be preceded by the corresponding question, like:

**Q<sub>n</sub>:**     ...a difficult question from me...  
**A:**        ...an answer from you ...  
**Q<sub>n</sub> + 1**   ...another difficult question...  
**A:**        ...another brilliant answer from you...

**Solutions will only be accepted until 30th December! Please send only one homework per email.**

1. What is a regular language? Are there alternative ways to define this notion? If yes, give an explanation why they define the same notion.
2. Why is every finite set of strings a regular language?
3. Assume you have an alphabet consisting of the letters  $a$ ,  $b$  and  $c$  only.  
(1) Find a regular expression that recognises the two strings  $ab$  and  $ac$ . (2) Find a regular expression that matches all strings *except* these two strings. Note, you can only use regular expressions of the form

$$r ::= \mathbf{0} \mid \mathbf{1} \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

4. Define the function *zeroable* which takes a regular expression as argument and returns a boolean. The function should satisfy the following property:

$$\text{zeroable}(r) \text{ if and only if } L(r) = \{\}$$

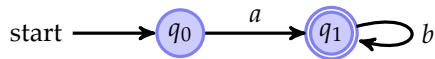
5. Given the alphabet  $\{a, b\}$ . Draw the automaton that has two states, say  $q_0$  and  $q_1$ . The starting state is  $q_0$  and the final state is  $q_1$ . The transition function is given by

$$\begin{aligned}(q_0, a) &\rightarrow q_0 \\(q_0, b) &\rightarrow q_1 \\(q_1, b) &\rightarrow q_1\end{aligned}$$

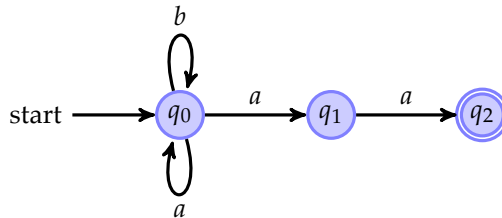
What is the language recognised by this automaton?

6. Give a non-deterministic finite automaton that can recognise the language  $L(a \cdot (a + b)^* \cdot c)$ .

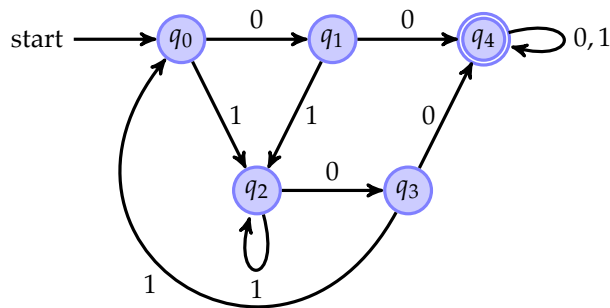
7. Given a deterministic finite automaton  $A(Q, q_0, F, \delta)$ , define which language is recognised by this automaton. Can you define also the language defined by a non-deterministic automaton?
8. Given the following deterministic finite automaton over the alphabet  $\{a, b\}$ , find an automaton that recognises the complement language. (Hint: Recall that for the algorithm from the lectures, the automaton needs to be in completed form, that is have a transition for every letter from the alphabet.)



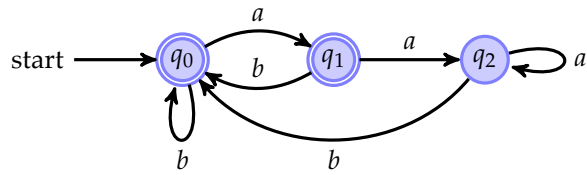
9. Given the following non-deterministic finite automaton over the alphabet  $\{a, b\}$ , find a deterministic finite automaton that recognises the same language:



10. Given the following deterministic finite automaton over the alphabet  $\{0, 1\}$ , find the corresponding minimal automaton. In case states can be merged, state clearly which states can be merged.



11. Given the following finite deterministic automaton over the alphabet  $\{a, b\}$ :



Give a regular expression that can recognise the same language as this automaton. (Hint: If you use Brzozwki's method, you can assume Arden's lemma which states that an equation of the form  $q = q \cdot r + s$  has the unique solution  $q = s \cdot r^*$ .)

12. If a non-deterministic finite automaton (NFA) has  $n$  states. How many states does a deterministic automaton (DFA) that can recognise the same language as the NFA maximal need?