Automata and Formal Languages (10)

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There are more problems, than there are programs.

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There must be a problem for which there is no program.

Revision: Proofs

Subsets

$$A \subseteq B$$

$$\forall e.\ e \in A \Rightarrow e \in B$$

Subsets

$A \subseteq B$ and $B \subseteq A$

then $A = B$

Injective Function

f is an injective function iff

$$\forall xy.\ f(x) = f(y) \Rightarrow x = y$$

Cardinality

$|A| \stackrel{\text{def}}{=}$ "how many elements"

$A \subseteq B \Rightarrow |A| \leq |B|$

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$|A| \stackrel{\text{def}}{=}$ "how many elements"

$A \subseteq B \Rightarrow |A| \leq |B|$

if there is an injective function $f : A \rightarrow B$ then $|A| \leq |B|$

Natural Numbers

$$\mathbb{N} \stackrel{\text{def}}{=} \{0, 1, 2, 3, \ldots\ldots\}$$

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$$\mathbb{N} \stackrel{\text{def}}{=} \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots\ldots\}$$

A is countable iff $|A| \leq |\mathbb{N}|$

First Question

$$|\mathbb{N} - \{\mathbf{0}\}| \quad ? \quad |\mathbb{N}|$$

$\geq$ or $\leq$ or $=$

$$|\mathbb{N} - \{\mathbf{0}, \mathbf{1}\}| \quad \mathbf{?} \quad |\mathbb{N}|$$

$$|\mathbb{N} - \{0, 1\}| \quad ? \quad |\mathbb{N}|$$

$$|\mathbb{N} - \mathbb{O}| \quad ? \quad |\mathbb{N}|$$

$\mathbb{O} \stackrel{\text{def}}{=}$ odd numbers $\{1, 3, 5.....\}$

$$|\mathbb{N} - \{0, 1\}| \quad ? \quad |\mathbb{N}|$$

$$|\mathbb{N} - \mathbb{O}| \quad ? \quad |\mathbb{N}|$$

$\mathbb{O} \stackrel{\text{def}}{=}$ odd numbers $\{1, 3, 5 \ldots\ldots\}$

$\mathbb{E} \stackrel{\text{def}}{=}$ even numbers $\{0, 2, 4 \ldots\ldots\}$

$$|\mathbb{N} \cup -\mathbb{N}| \quad \textcolor{red}{?} \quad |\mathbb{N}|$$

$\mathbb{N} \stackrel{\text{def}}{=}$ positive numbers $\{0, 1, 2, 3, \ldots\ldots\}$

$-\mathbb{N} \stackrel{\text{def}}{=}$ negative numbers $\{0, -1, -2, -3, \ldots\ldots\}$

A is countable if there exists an injective $f : A \rightarrow \mathbb{N}$

A is uncountable if there does not exist an injective $f : A \rightarrow \mathbb{N}$

countable: $|A| \leq |\mathbb{N}|$
uncountable: $|A| > |\mathbb{N}|$

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Does there exist such an A ?

Halting Problem

Assume a program H that decides for all programs A and all input data D whether

- $H(A, D) \overset{\text{def}}{=} 1$ iff $A(D)$ terminates
- $H(A, D) \overset{\text{def}}{=} 0$ otherwise

Halting Problem (2)

Given such a program H define the following program C: for all programs A

- $C(A) \stackrel{\text{def}}{=} \mathbf{0}$ iff $H(A, A) = \mathbf{0}$
- $C(A) \stackrel{\text{def}}{=}$ loops otherwise

Contradiction

$H(C,C)$ is either $\mathbf{0}$ or $\mathbf{1}$.

- $H(C,C) = 1 \overset{\text{def } H}{\Rightarrow} C(C)\downarrow \overset{\text{def } C}{\Rightarrow} H(C,C) = 0$
- $H(C,C) = 0 \overset{\text{def } H}{\Rightarrow} C(C)$ loops $\overset{\text{def } C}{\Rightarrow}$ $H(C,C) = 1$

Contradiction in both cases. So H cannot exist.