Compilers and Formal Languages

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Slides & Progs:	KEATS (also homework is there)

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The Goal of this Course

Write a compiler

Today a lexer.

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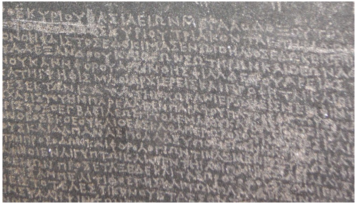

lexing ⇒ recognising words (Stone of Rosetta)

Regular Expressions

In programming languages they are often used to recognise:

- operands, digits
- identifiers
- numbers (non-leading zeros)
- keywords
- comments

`http://www.regexper.com`

Lexing: Test Case

```
write "Fib";
read n;
minus1 := 0;
minus2 := 1;
while n > 0 do {
       temp := minus2;
       minus2 := minus1 + minus2;
       minus1 := temp;
       n := n - 1
};
write "Result";
write minus2
```

`"if true then then 42 else +"`

KEYWORD:

`if`, `then`, `else`,

WHITESPACE:

`" "`, $\backslash$`n`,

IDENTIFIER:

$\texttt{LETTER} \cdot (\texttt{LETTER} + \texttt{DIGIT} + \_)^*$

NUM:

$(\texttt{NONZERODIGIT} \cdot \texttt{DIGIT}^*) + \texttt{0}$

OP:

`+`, `-`, `*`, `%`, `<`, `<=`

COMMENT:

$/* \cdot \sim(\texttt{ALL}^* \cdot (*/) \cdot \texttt{ALL}^*) \cdot */$

```
"if true then then 42 else +"
```

```
KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)
```

```
"if true then then 42 else +"
```

```
KEYWORD(if),
IDENT(true),
KEYWORD(then),
KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)
```

There is one small problem with the tokenizer. How should we tokenize...?

```
"x-3"
```

```
ID: ...
OP:
  "+", "-"
NUM:
  (NONZERODIGIT · DIGIT*) + ''0''
NUMBER:
  NUM + ("-" · NUM)
```

The same problem with

$$(ab + a) \cdot (c + bc)$$

and the string abc.

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$$(ab + a) \cdot (c + bc)$$

and the string abc.

Or, keywords are **`if`** etc and identifiers are letters followed by "letters + numbers + _"*

```
if      iffoo
```

POSIX: Two Rules

- Longest match rule ("maximal munch rule"): The longest initial substring matched by any regular expression is taken as the next token.

- Rule priority: For a particular longest initial substring, the first regular expression that can match determines the token.

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`http://www.haskell.org/haskellwiki/Regex_Posix`

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most posix matchers are buggy
`http://www.haskell.org/haskellwiki/Regex_Posix`
traditional lexers are fast, but hairy

Sulzmann & Lu Matcher

We want to match the string *abc* using r_1:

$$r_1 \xrightarrow{der\ a} r_2$$

Sulzmann & Lu Matcher

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$$r_1 \xrightarrow{der\,a} r_2 \xrightarrow{der\,b} r_3$$

Sulzmann & Lu Matcher

We want to match the string *abc* using r_1:

$$r_1 \xrightarrow{der\,a} r_2 \xrightarrow{der\,b} r_3 \xrightarrow{der\,c} r_4$$

Sulzmann & Lu Matcher

We want to match the string *abc* using r_1:

$$r_1 \xrightarrow{der\ a} r_2 \xrightarrow{der\ b} r_3 \xrightarrow{der\ c} r_4 \ nullable?$$

Sulzmann & Lu Matcher

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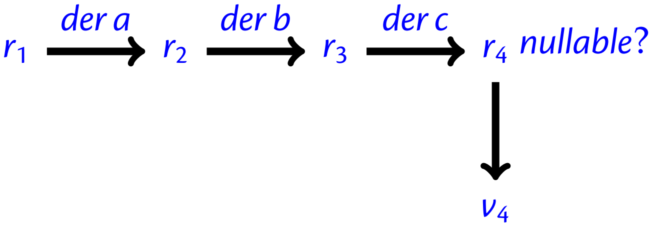

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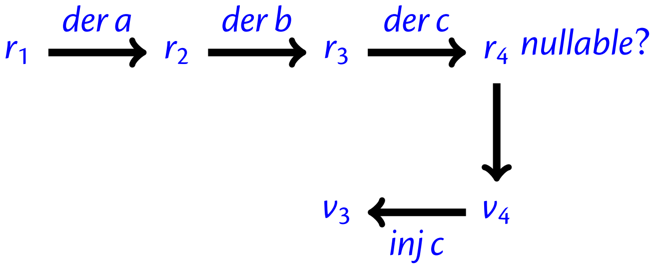

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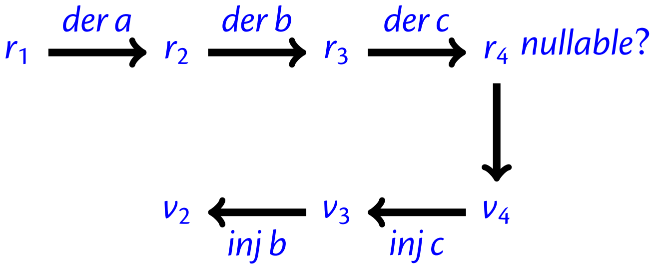

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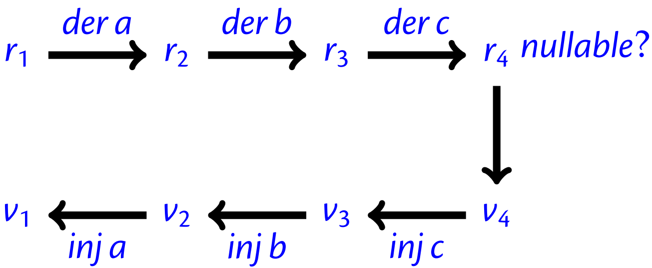

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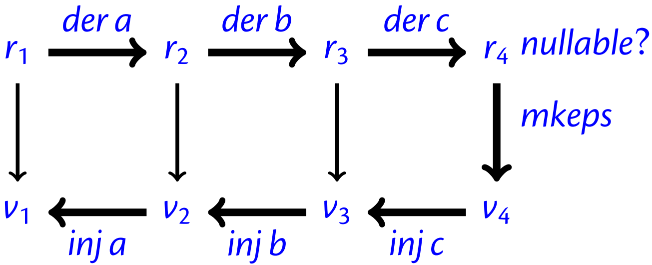


Regexes and Values

Regular expressions and their corresponding values:

$$
\begin{array}{lcl}
r & ::= & \mathbf{0} \\
 & | & \mathbf{1} \\
 & | & c \\
 & | & r_1 \cdot r_2 \\
 & | & r_1 + r_2 \\
 & & \\
 & | & r^*
\end{array}
\qquad
\begin{array}{lcl}
v & ::= & \\
 & & \mathit{Empty} \\
 & | & \mathit{Char}(c) \\
 & | & \mathit{Seq}(v_1, v_2) \\
 & | & \mathit{Left}(v) \\
 & | & \mathit{Right}(v) \\
 & | & \mathit{Stars}\,[] \\
 & | & \mathit{Stars}\,[v_1, \ldots\, v_n]
\end{array}
$$

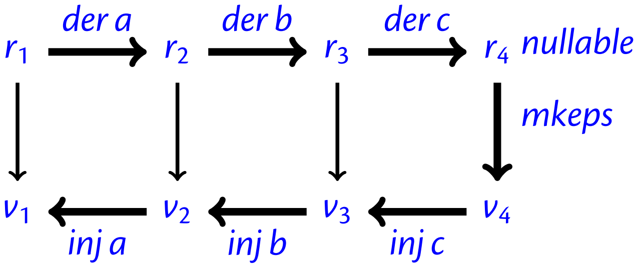
```scala
abstract class Rexp
case object ZERO extends Rexp
case object ONE extends Rexp
case class CHAR(c: Char) extends Rexp
case class ALT(r1: Rexp, r2: Rexp) extends Rexp
case class SEQ(r1: Rexp, r2: Rexp) extends Rexp
case class STAR(r: Rexp) extends Rexp

abstract class Val
case object Empty extends Val
case class Chr(c: Char) extends Val
case class Sequ(v1: Val, v2: Val) extends Val
case class Left(v: Val) extends Val
case class Right(v: Val) extends Val
case class Stars(vs: List[Val]) extends Val
```

r_1: $a \cdot (b \cdot c)$

r_2: $\mathbf{1} \cdot (b \cdot c)$

r_3: $(\mathbf{0} \cdot (b \cdot c)) + (\mathbf{1} \cdot c)$

r_4: $(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1})$



r_1: $a \cdot (b \cdot c)$
r_2: $\mathbf{1} \cdot (b \cdot c)$
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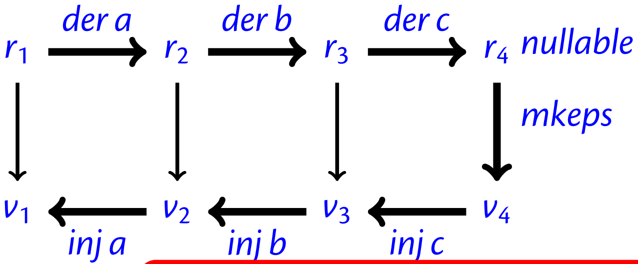


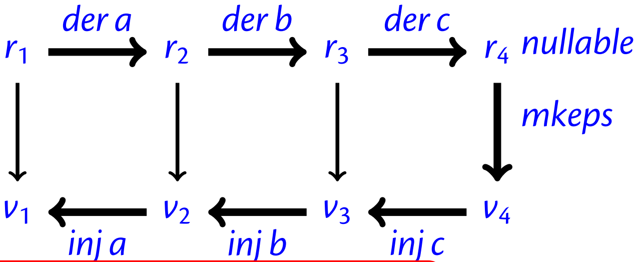
v_1: $Seq(Char(a), Seq(Char(b), Char(c)))$
v_2: $Seq(Empty, Seq(Char(b), Char(c)))$
v_3: $Right(Seq(Empty, Char(c)))$
v_4: $Right(Right(Empty))$

Flatten

Obtaining the string underlying a value:

$$\begin{array}{lcl}
|Empty| & \stackrel{\text{def}}{=} & [] \\
|Char(c)| & \stackrel{\text{def}}{=} & [c] \\
|Left(v)| & \stackrel{\text{def}}{=} & |v| \\
|Right(v)| & \stackrel{\text{def}}{=} & |v| \\
|Seq(v_1, v_2)| & \stackrel{\text{def}}{=} & |v_1| \mathbin{@} |v_2| \\
|Stars\,[v_1, \ldots, v_n]| & \stackrel{\text{def}}{=} & |v_1| \mathbin{@} \ldots \mathbin{@} |v_n|
\end{array}$$

r_1: $a \cdot (b \cdot c)$
r_2: $\mathbf{1} \cdot (b \cdot c)$
r_3: $(\mathbf{0} \cdot (b \cdot c)) + (\mathbf{1} \cdot c)$
r_4: $(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1})$



v_1: $Seq(Char(a), Seq(Char(b), Char(c)))$
v_2: $Seq(Empty, Seq(Char(b), Char(c)))$
v_3: $Right(Seq(Empty, Char(c)))$
v_4: $Right(Right(Empty))$

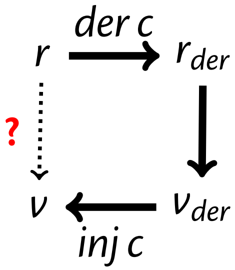
$|v_1|$: abc
$|v_2|$: bc
$|v_3|$: c
$|v_4|$: $[]$

Mkeps

Finding a (posix) value for recognising the empty string:

$$
\begin{aligned}
mkeps\,(\mathbf{1}) &\stackrel{\text{def}}{=} Empty\\
mkeps\,(r_1 + r_2) &\stackrel{\text{def}}{=} \text{if } nullable(r_1)\\
&\qquad \text{then } Left(mkeps(r_1))\\
&\qquad \text{else } Right(mkeps(r_2))\\
mkeps\,(r_1 \cdot r_2) &\stackrel{\text{def}}{=} Seq(mkeps(r_1), mkeps(r_2))\\
mkeps\,(r^*) &\stackrel{\text{def}}{=} Stars\,[]
\end{aligned}
$$

Inject



Inject

Injecting ("Adding") a character to a value

$$
\begin{array}{lcl}
inj\ (c)\ c\ (Empty) & \stackrel{\text{def}}{=} & Char\ c\\
inj\ (r_1 + r_2)\ c\ (Left(v)) & \stackrel{\text{def}}{=} & Left(inj\ r_1\ c\ v)\\
inj\ (r_1 + r_2)\ c\ (Right(v)) & \stackrel{\text{def}}{=} & Right(inj\ r_2\ c\ v)\\
inj\ (r_1 \cdot r_2)\ c\ (Seq(v_1, v_2)) & \stackrel{\text{def}}{=} & Seq(inj\ r_1\ c\ v_1, v_2)\\
inj\ (r_1 \cdot r_2)\ c\ (Left(Seq(v_1, v_2))) & \stackrel{\text{def}}{=} & Seq(inj\ r_1\ c\ v_1, v_2)\\
inj\ (r_1 \cdot r_2)\ c\ (Right(v)) & \stackrel{\text{def}}{=} & Seq(mkeps(r_1), inj\ r_2\ c\ v)\\
inj\ (r^*)\ c\ (Seq(v, Stars\ vs)) & \stackrel{\text{def}}{=} & Stars\ (inj\ r\ c\ v\ ::\ vs)
\end{array}
$$

inj: 1st arg $\mapsto$ a rexp; 2nd arg $\mapsto$ a character; 3rd arg $\mapsto$ a value
result $\mapsto$ a value

$$inj\,(c)\,c\,(Empty) \stackrel{\text{def}}{=} Char\,c$$

$$inj\ (r_1 + r_2)\ c\ (Left(v)) \stackrel{\text{def}}{=} Left(inj\ r_1\ c\ v)$$
$$inj\ (r_1 + r_2)\ c\ (Right(v)) \stackrel{\text{def}}{=} Right(inj\ r_2\ c\ v)$$

$$
\begin{array}{lcl}
inj\;(r_1 \cdot r_2)\;c\;(Seq(v_1, v_2)) & \stackrel{\text{def}}{=} & Seq(inj\;r_1\;c\;v_1, v_2)\\
inj\;(r_1 \cdot r_2)\;c\;(Left(Seq(v_1, v_2))) & \stackrel{\text{def}}{=} & Seq(inj\;r_1\;c\;v_1, v_2)\\
inj\;(r_1 \cdot r_2)\;c\;(Right(v)) & \stackrel{\text{def}}{=} & Seq(mkeps(r_1), inj\;r_2\;c\;v)
\end{array}
$$

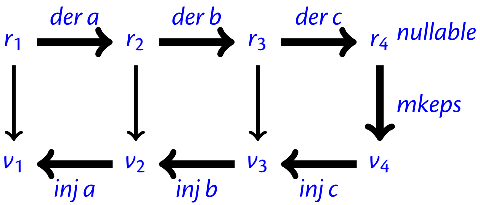
$$der\;c\;(r_1 \cdot r_2) \stackrel{\text{def}}{=} \textbf{if}\;nullable(r_1)\;\textbf{then}\;(der\;c\;r_1) \cdot r_2 + der\;c\;r_2\;\textbf{else}\;(der\;c\;r_1) \cdot r_2$$

$$inj\,(r^*)\;c\;(Seq(v, Stars\;vs)) \stackrel{\text{def}}{=} Stars\;(inj\;r\;c\;v\;::\;vs)$$

Lexing

$$
\begin{aligned}
lex\ r\ [\,] &\stackrel{\text{def}}{=} \text{if } nullable(r) \text{ then } mkeps(r) \text{ else } error \\
lex\ r\ a :: s &\stackrel{\text{def}}{=} inj\ r\ a\ lex(der(a,r), s)
\end{aligned}
$$

lex: returns a value



Records

- new regex: $(x : r)$ new value: $Rec(x, v)$

$(id : r_{id})$
$(key : r_{key})$

Records

- new regex: $(x : r)$ new value: $Rec(x, v)$
- $nullable(x : r) \stackrel{\text{def}}{=} nullable(r)$
- $der\,c\,(x : r) \stackrel{\text{def}}{=} der\,c\,r$
- $mkeps(x : r) \stackrel{\text{def}}{=} Rec(x, mkeps(r))$
- $inj\,(x : r)\,c\,v \stackrel{\text{def}}{=} Rec(x, inj\,r\,c\,v)$

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Records

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- $inj\, (x : r)\, c\, v \stackrel{\text{def}}{=} Rec(x, inj\, r\, c\, v)$

for extracting subpatterns $(z : ((x : ab) + (y : ba))$

$(id : r_{id})$
$(key : r_{key})$

- A regular expression for email addresses

$$(name: [a\text{-}z0\text{-}9\_\,.-]^{+}) \cdot @ \cdot (domain: [a\text{-}z0\text{-}9\,-]^{+}) \cdot . \cdot (top\_level: [a\text{-}z\,.]^{\{2,6\}})$$

`christian.urban@kcl.ac.uk`

- the result environment:

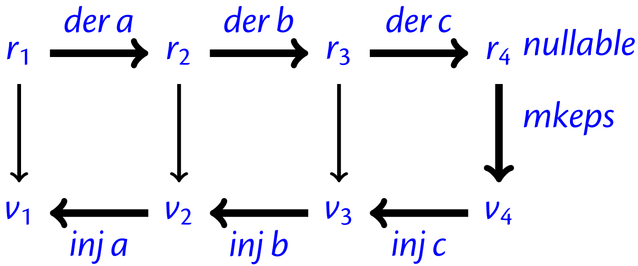
$$[(name : \texttt{christian.urban}), (domain : \texttt{kcl}), (top\_level : \texttt{ac.uk})]$$

While Tokens

$$\texttt{WHILE\_REGS} \stackrel{\text{def}}{=} \big((\texttt{"k"} : \texttt{KEYWORD}) + (\texttt{"i"} : \texttt{ID}) + (\texttt{"o"} : \texttt{OP}) + (\texttt{"n"} : \texttt{NUM}) + (\texttt{"s"} : \texttt{SEMI}) + (\texttt{"p"} : (\texttt{LPAREN} + \texttt{RPAREN})) + (\texttt{"b"} : (\texttt{BEGIN} + \texttt{END})) + (\texttt{"w"} : \texttt{WHITESPACE})\big)^*$$

Simplification

- If we simplify after the derivative, then we are building the value for the simplified regular expression, but **not** for the original regular expression.



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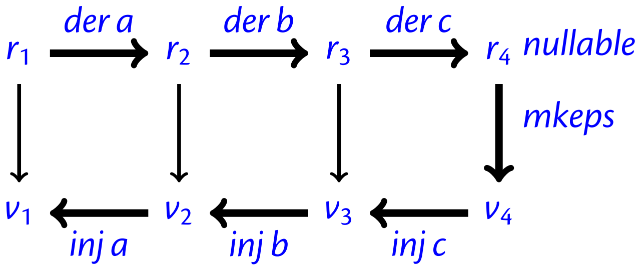

$$(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1}) \mapsto \mathbf{1}$$

Normally we would have

$$(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1})$$

and answer how this regular expression matches the empty string with the value

$$Right(Right(Empty))$$

But now we simplify this to $\mathbf{1}$ and would produce *Empty* (see *mkeps*).

Rectification

rectification functions:

$$
\begin{array}{lcll}
r \cdot \mathbf{0} & \mapsto & \mathbf{0} & \\
\mathbf{0} \cdot r & \mapsto & \mathbf{0} & \\
r \cdot \mathbf{1} & \mapsto & r & \lambda f_1\, f_2\, v.\, Seq(f_1\, v, f_2\, Empty) \\
\mathbf{1} \cdot r & \mapsto & r & \lambda f_1\, f_2\, v.\, Seq(f_1\, Empty, f_2\, v) \\
r + \mathbf{0} & \mapsto & r & \lambda f_1\, f_2\, v.\, Left(f_1\, v) \\
\mathbf{0} + r & \mapsto & r & \lambda f_1\, f_2\, v.\, Right(f_2\, v) \\
r + r & \mapsto & r & \lambda f_1\, f_2\, v.\, Left(f_1\, v)
\end{array}
$$

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r \cdot \mathbf{0} & \mapsto & \mathbf{0} & \\
\mathbf{0} \cdot r & \mapsto & \mathbf{0} & \\
r \cdot \mathbf{1} & \mapsto & r & \lambda f_1 f_2 v.\, Seq(f_1\, v, f_2\, Empty) \\
\mathbf{1} \cdot r & \mapsto & r & \lambda f_1 f_2 v.\, Seq(f_1\, Empty, f_2\, v) \\
r + \mathbf{0} & \mapsto & r & \lambda f_1 f_2 v.\, Left(f_1\, v) \\
\mathbf{0} + r & \mapsto & r & \lambda f_1 f_2 v.\, Right(f_2\, v) \\
r + r & \mapsto & r & \lambda f_1 f_2 v.\, Left(f_1\, v)
\end{array}
$$

old *simp* returns a rexp;
new *simp* returns a rexp and a rectification function.

Rectification $\_ + \_$

$simp(r)$:

case $r = r_1 + r_2$

let $(r_{1s}, f_{1s}) = simp(r_1)$

$(r_{2s}, f_{2s}) = simp(r_2)$

case $r_{1s} = \mathbf{0}$: return $(r_{2s}, \lambda v. \, Right(f_{2s}(v)))$

case $r_{2s} = \mathbf{0}$: return $(r_{1s}, \lambda v. \, Left(f_{1s}(v)))$

case $r_{1s} = r_{2s}$: return $(r_{1s}, \lambda v. \, Left(f_{1s}(v)))$

otherwise: return $(r_{1s} + r_{2s}, f_{alt}(f_{1s}, f_{2s}))$

$$
\begin{aligned}
f_{alt}(f_1, f_2) \stackrel{\text{def}}{=} \\
\lambda v. \ &\text{case } v = Left(v'):\quad \text{return } Left(f_1(v')) \\
&\text{case } v = Right(v'): \text{return } Right(f_2(v'))
\end{aligned}
$$

```
def simp(r: Rexp): (Rexp, Val => Val) = r match {
  case ALT(r1, r2) => {
    val (r1s, f1s) = simp(r1)
    val (r2s, f2s) = simp(r2)
    (r1s, r2s) match {
      case (ZERO, _) => (r2s, F_RIGHT(f2s))
      case (_, ZERO) => (r1s, F_LEFT(f1s))
      case _ =>
         if (r1s == r2s) (r1s, F_LEFT(f1s))
         else (ALT (r1s, r2s), F_ALT(f1s, f2s))
    }
  }
  ...
}
def F_RIGHT(f: Val => Val) = (v:Val) => Right(f(v))
def F_LEFT(f: Val => Val) = (v:Val) => Left(f(v))
def F_ALT(f1: Val => Val, f2: Val => Val) =
  (v:Val) => v match {
    case Right(v) => Right(f2(v))
    case Left(v) => Left(f1(v)) }
```

Rectification $\_ \cdot \_$

$simp(r)$:...

case $r = r_1 \cdot r_2$

let $(r_{1s}, f_{1s}) = simp(r_1)$

$(r_{2s}, f_{2s}) = simp(r_2)$

case $r_{1s} = \mathbf{0}$: return $(\mathbf{0}, f_{error})$

case $r_{2s} = \mathbf{0}$: return $(\mathbf{0}, f_{error})$

case $r_{1s} = \mathbf{1}$: return $(r_{2s}, \lambda v.\, Seq(f_{1s}(Empty), f_{2s}(v)))$

case $r_{2s} = \mathbf{1}$: return $(r_{1s}, \lambda v.\, Seq(f_{1s}(v), f_{2s}(Empty)))$

otherwise: return $(r_{1s} \cdot r_{2s}, f_{seq}(f_{1s}, f_{2s}))$

$f_{seq}(f_1, f_2) \stackrel{\text{def}}{=}$

$\lambda v.$ case $v = Seq(v_1, v_2)$: return $Seq(f_1(v_1), f_2(v_2))$

```scala
def simp(r: Rexp): (Rexp, Val => Val) = r match {
  case SEQ(r1, r2) => {
    val (r1s, f1s) = simp(r1)
    val (r2s, f2s) = simp(r2)
    (r1s, r2s) match {
      case (ZERO, _) => (ZERO, F_ERROR)
      case (_, ZERO) => (ZERO, F_ERROR)
      case (ONE, _) => (r2s, F_SEQ_Empty1(f1s, f2s))
      case (_, ONE) => (r1s, F_SEQ_Empty2(f1s, f2s))
      case _ => (SEQ(r1s,r2s), F_SEQ(f1s, f2s))
    }
  }
  ...}
def F_SEQ_Empty1(f1: Val => Val, f2: Val => Val) =
  (v:Val) => Sequ(f1(Empty), f2(v))
def F_SEQ_Empty2(f1: Val => Val, f2: Val => Val) =
  (v:Val) => Sequ(f1(v), f2(Empty))
def F_SEQ(f1: Val => Val, f2: Val => Val) =
  (v:Val) => v match {
    case Sequ(v1, v2) => Sequ(f1(v1), f2(v2)) }
```

Rectification Example

$$(b \cdot c) + (\mathbf{0} + \mathbf{1}) \mapsto (b \cdot c) + \mathbf{1}$$

Rectification Example

$$(\underline{b \cdot c}) + (\underline{\mathbf{0} + \mathbf{1}}) \mapsto (b \cdot c) + \mathbf{1}$$

Rectification Example

$$(\underline{b \cdot c}) + (\underline{\mathbf{0} + \mathbf{1}}) \mapsto (b \cdot c) + \mathbf{1}$$

$$\begin{array}{lcl} f_{s1} & = & \lambda v.v \\ f_{s2} & = & \lambda v.Right(v) \end{array}$$

Rectification Example

$$\underline{(b \cdot c) + (\mathbf{0} + \mathbf{1})} \mapsto (b \cdot c) + \mathbf{1}$$

$$\begin{array}{lcl} f_{s1} & = & \lambda v.v \\ f_{s2} & = & \lambda v.Right(v) \end{array}$$

$$\begin{array}{l} f_{alt}(f_{s1}, f_{s2}) \stackrel{\text{def}}{=} \\ \quad \lambda v.\ \text{case } v = Left(v'): \quad \text{return } Left(f_{s1}(v')) \\ \qquad\quad \text{case } v = Right(v'): \text{return } Right(f_{s2}(v')) \end{array}$$

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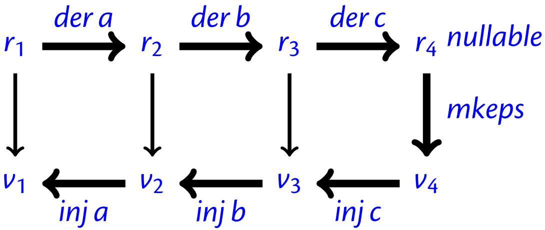
mkeps simplified case: $Right(Empty)$

rectified case: $Right(Right(Empty))$

Lexing with Simplification

$$lex\, r\, [] \stackrel{\text{def}}{=} \text{if } nullable(r) \text{ then } mkeps(r) \text{ else } error$$

$$lex\, r\, c :: s \stackrel{\text{def}}{=} \text{let } (r', frect) = simp(der(c, r))$$
$$inj\, r\, c\, (frect(lex(r', s)))$$



Environments

Obtaining the "recorded" parts of a value:

$$
\begin{aligned}
env(Empty) &\stackrel{\text{def}}{=} [] \\
env(Char(c)) &\stackrel{\text{def}}{=} [] \\
env(Left(v)) &\stackrel{\text{def}}{=} env(v) \\
env(Right(v)) &\stackrel{\text{def}}{=} env(v) \\
env(Seq(v_1, v_2)) &\stackrel{\text{def}}{=} env(v_1) \,@\, env(v_2) \\
env(Stars\,[v_1, \ldots, v_n]) &\stackrel{\text{def}}{=} env(v_1) \,@ \ldots @\, env(v_n) \\
env(Rec(x : v)) &\stackrel{\text{def}}{=} (x : |v|) :: env(v)
\end{aligned}
$$

While Tokens

$$
\begin{array}{rl}
\texttt{WHILE\_REGS} \stackrel{\text{def}}{=} & ((\texttt{"k"} : \texttt{KEYWORD}) + \\
& (\texttt{"i"} : \texttt{ID}) + \\
& (\texttt{"o"} : \texttt{OP}) + \\
& (\texttt{"n"} : \texttt{NUM}) + \\
& (\texttt{"s"} : \texttt{SEMI}) + \\
& (\texttt{"p"} : (\texttt{LPAREN} + \texttt{RPAREN})) + \\
& (\texttt{"b"} : (\texttt{BEGIN} + \texttt{END})) + \\
& (\texttt{"w"} : \texttt{WHITESPACE}))^*
\end{array}
$$

```
"if true then then 42 else +"
```

```
KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)
```

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"if true then then 42 else +"
```

```
KEYWORD(if),
IDENT(true),
KEYWORD(then),
KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)
```

Lexer: Two Rules

- Longest match rule ("maximal munch rule"): The longest initial substring matched by any regular expression is taken as next token.

- Rule priority: For a particular longest initial substring, the first regular expression that can match determines the token.

Environments

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```

Are dfas completed by definition as in do they have a to have transitions for every char at every state?

How can you tell if a language will be regular or irregular?