Homework 2

Please submit your solutions via email. Please submit only ASCII text or PDFs. Every solution should be preceeded by the corresponding question, like:

Qn:	**…a difficult question from me…**
A:	**…an answer from you …**
Q$n+1$	**…another difficult question…**
A:	**…another brilliant answer from you…**

Solutions will only be accepted until 20th December! Please send only one homework per email.

1. What is the language recognised by the regular expressions $(\mathbf{0}^*)^*$.

2. Review the first handout about sets of strings and read the second handout. Assuming the alphabet is the set $\{a, b\}$, decide which of the following equations are true in general for arbitrary languages A, B and C:

$$\begin{array}{rcl} (A \cup B)@C & =^? & A@C \cup B@C \\ A^* \cup B^* & =^? & (A \cup B)^* \\ A^*@A^* & =^? & A^* \\ (A \cap B)@C & =^? & (A@C) \cap (B@C) \end{array}$$

   In case an equation is true, give an explanation; otherwise give a counter-example.

3. Given the regular expressions $r_1 = \mathbf{1}$ and $r_2 = \mathbf{0}$ and $r_3 = a$. How many strings can the regular expressions r_1^*, r_2^* and r_3^* each match?

4. Give regular expressions for (a) decimal numbers and for (b) binary numbers. (Hint: Observe that the empty string is not a number. Also observe that leading 0s are normally not written.)

5. Decide whether the following two regular expressions are equivalent $(\mathbf{1} + a)^* \equiv^? a^*$ and $(a \cdot b)^* \cdot a \equiv^? a \cdot (b \cdot a)^*$.

6. Given the regular expression $r = (a \cdot b + b)^*$. Compute what the derivative of r is with respect to a, b and c. Is r nullable?

7. Prove that for all regular expressions r we have

$$\mathit{nullable}(r) \quad \text{if and only if} \quad [] \in L(r)$$

Write down clearly in each case what you need to prove and what are the assumptions.

8. Define what is meant by the derivative of a regular expressions with respect to a character. (Hint: The derivative is defined recursively.)

9. Assume the set *Der* is defined as

$$Der\, c\, A \stackrel{\text{def}}{=} \{s \mid c :: s \in A\}$$

   What is the relation between *Der* and the notion of derivative of regular expressions?

10. Give a regular expression over the alphabet $\{a, b\}$ recognising all strings that do not contain any substring bb and end in a.

11. Do $(a + b)^* \cdot b^+$ and $(a^* \cdot b^+) + (b^* \cdot b^+)$ define the same language?

12. Define the function *zeroable* by recursion over regular expressions. This function should satisfy the property

$$zeroable(r) \;\text{ if and only if }\; L(r) = \{\} \qquad (*)$$

   The function *nullable* for the not-regular expressions can be defined by

$$nullable(\sim r) \stackrel{\text{def}}{=} \neg(nullable(r))$$

   Unfortunately, a similar definition for *zeroable* does not satisfy the property in $(*)$:

$$zeroable(\sim r) \stackrel{\text{def}}{=} \neg(zeroable(r))$$

   Find out why?

13. Give a regular expressions that can recognise all strings from the language $\{a^n \mid \exists k.\; n = 3k + 1\}$.

14. Give a regular expression that can recognise an odd number of as or an even number of bs.

15. **(Optional)** This question is for you to provide regular feedback to me: for example what were the most interesting, least interesting, or confusing parts in this lecture? Any problems with my Scala code? Please feel free to share any other questions or concerns.