Automata and Formal Languages (9)

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Slides: KEATS (also home work is there)

Functional Programming

```
def fib(n) = if n == 0 then 0
             else if n == 1 then 1
             else fib(n - 1) + fib(n - 2);

def fact(n) = if n == 0 then 1 else n * fact(n - 1);

def ack(m, n) = if m == 0 then n + 1
                else if n == 0 then ack(m - 1, 1)
                else ack(m - 1, ack(m, n - 1));

def gcd(a, b) = if b == 0 then a else gcd(b, a % b);
```

$$
\begin{array}{lcl}
\mathit{Exp} & \rightarrow & \mathit{Var} \mid \mathit{Num} \\
& \mid & \mathit{Exp} + \mathit{Exp} \mid \ldots \mid (\,\mathit{Exp}\,) \\
& \mid & \texttt{if}\ \mathit{BExp}\ \texttt{then}\ \mathit{Exp}\ \texttt{else}\ \mathit{Exp} \\
& \mid & \texttt{write}\ \mathit{Exp} \\
& \mid & \mathit{Exp}\ \texttt{;}\ \mathit{Exp} \\
& \mid & \mathit{FunName}\ (\mathit{Exp},\ldots,\mathit{Exp}) \\
\mathit{BExp} & \rightarrow & \ldots \\
\mathit{Decl} & \rightarrow & \mathit{Def}\ \ \texttt{;}\ \ \mathit{Decl} \\
& \mid & \mathit{Exp} \\
\mathit{Def} & \rightarrow & \texttt{def}\ \mathit{FunName}(x_1,\ldots,x_n) = \mathit{Exp}
\end{array}
$$

Abstract Syntax Tree

```
abstract class Exp
abstract class BExp
abstract class Decl

case class
  Def(name: String, args: List[String], body: Exp)
                                              extends Decl
case class Main(e: Exp) extends Decl

case class Call(name: String, args: List[Exp]) extends Exp
case class If(a: BExp, e1: Exp, e2: Exp) extends Exp
case class Write(e: Exp) extends Exp
case class Var(s: String) extends Exp
case class Num(i: Int) extends Exp
case class Aop(o: String, a1: Exp, a2: Exp) extends Exp
case class Sequ(e1: Exp, e2: Exp) extends Exp
case class Bop(o: String, a1: Exp, a2: Exp) extends BExp
```

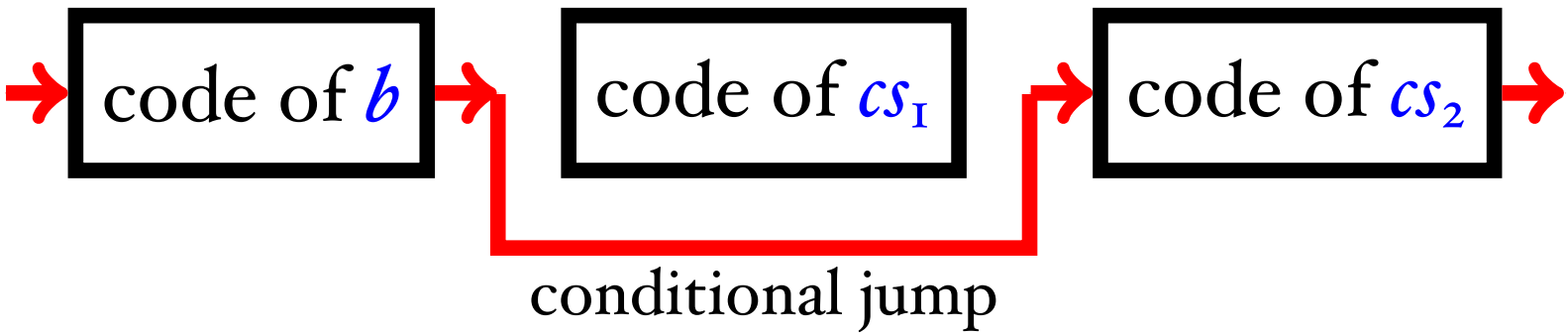
Mathematical Functions

Compilation of some mathematical functions:

```
Aop("+", a1, a2)  ⇒  ...iadd
Aop("-", a1, a2)  ⇒  ...isub
Aop("*", a1, a2)  ⇒  ...imul
Aop("/", a1, a2)  ⇒  ...idiv
Aop("%", a1, a2)  ⇒  ...irem
```

Boolean Expressions

Compilation of boolean expressions:




```
Bop("==", a1, a2)  ⇒  ...if_icmpne...
Bop("!=", a1, a2)  ⇒  ...if_icmpeq...
Bop("<", a1, a2)   ⇒  ...if_icmpge...
Bop("<=", a1, a2)  ⇒  ...if_icmpgt...
```

Sequences

Compiling `arg1 ; arg2`:

```
...arg1...
pop
...arg1...
```

Write

Compiling `write(arg)`:

```
...arg...
dup
invokestatic XXX/XXX/write(I)V
```

```
case Write(a1) => {
    compile_exp(a1, env) ++
    List("dup\n",
         "invokestatic XXX/XXX/write(I)V\n")
  }
```

Functions

```
.method public static write(I)V
   .limit locals 5
   .limit stack 5
   iload 0
   getstatic java/lang/System/out Ljava/io/PrintStream;
   swap
   invokevirtual java/io/PrintStream/println(I)V
   return
.end method
```

We will need for definitions

```
.method public static f (I...I)I
  .limit locals ??
  .limit stack ??
  ??
.end method
```

Stack Estimation

```
def max_stack_exp(e: Exp): Int = e match {
  case Call(_, args) => args.map(max_stack_exp).sum
  case If(a, e1, e2) => max_stack_bexp(a) +
  ^^I(List(max_stack_exp(e1), max_stack_exp(e1)).max)
  case Write(e) => max_stack_exp(e) + 1
  case Var(_) => 1
  case Num(_) => 1
  case Aop(_, a1, a2) =>
  ^^Imax_stack_exp(a1) + max_stack_exp(a2)
  case Sequ(e1, e2) =>
  ^^IList(max_stack_exp(e1), max_stack_exp(e2)).max
}

def max_stack_bexp(e: BExp): Int = e match {
  case Bop(_, a1, a2) =>
  ^^Imax_stack_exp(a1) + max_stack_exp(a2)
}
```

Successor

```
.method public static suc(I)I
.limit locals 1
.limit stack 3
  iload 0
  ldc 1
  iadd
  ireturn
.end method
```

```
def suc(x) = x + 1;
```

Addition

```
.method public static add(II)I
.limit locals 2
.limit stack 6
  iload 0
  ldc 0
  if_icmpne If_else_2
  iload 1
  goto If_end_3
If_else_2:
  iload 0
  ldc 1
  isub
  iload 1
  invokestatic defs/defs/add(II)I
  invokestatic defs/defs/suc(I)I
If_end_3:
  ireturn
.end method
```

```
def add(x, y) =
    if x == 0 then y
    else suc(add(x - 1, y));
```

Factorial

```
.method public static facT(II)I
.limit locals 2
.limit stack 6
  iload 0
  ldc 0
  if_icmpne If_else_2
  iload 1
  goto If_end_3
If_else_2:
  iload 0
  ldc 1
  isub
  iload 0
  iload 1
  imul
  invokestatic fact/fact/facT(II)I
If_end_3:
  ireturn
.end method
```

```
def facT(n, acc) =
  if n == 0 then acc
  else facT(n - 1, n * acc);
```

```
.method public static facT(II)I
.limit locals 2
.limit stack 7
facT_Start:
  iload 0
  ldc 0
  if_icmpne If_else_2
  iload 1
  goto If_end_3
If_else_2:
  iload 0
  ldc 1
  isub
  iload 0
  iload 1
  imul
  istore 1
  istore 0
  goto facT_Start
If_end_3:
  ireturn
.end method
```

```
def facT(n, acc) =
  if n == 0 then acc
  else facT(n - 1, n * acc);
```

Tail Recursion

A call to `f(args)` is usually compiled as

```
args onto stack
invokestatic  .../f
```

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```
args onto stack
invokestatic  .../f
```

A call is in tail position provided:

- `if Bexp then Exp else Exp`
- `Exp ; Exp`
- `Exp op Exp`

then a call `f(args)` can be compiled as

```
prepare environment
jump to start of function
```

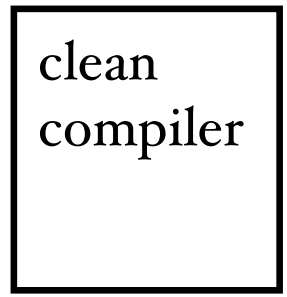
Tail Recursive Call

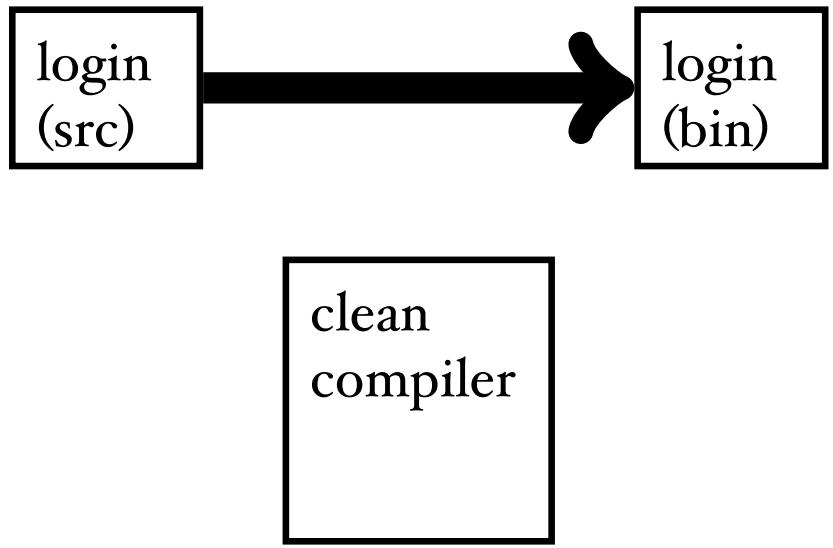
```
def compile_expT(a: Exp, env: Mem, name: String): Instrs =
  ...
  case Call(n, args) => if (name == n)
  {
    val stores = args.zipWithIndex.map
       { case (x, y) => "istore " + y.toString + "\n" }
    args.flatMap(a => compile_expT(a, env, "")) ++
    stores.reverse ++
    List ("goto " + n + "_Start\n")
  }
  else
  {
    val is = "I" * args.length
    args.flatMap(a => compile_expT(a, env, "")) ++
    List ("invokestatic XXX/XXX/" + n + "(" + is + ")I\n")
  }
```

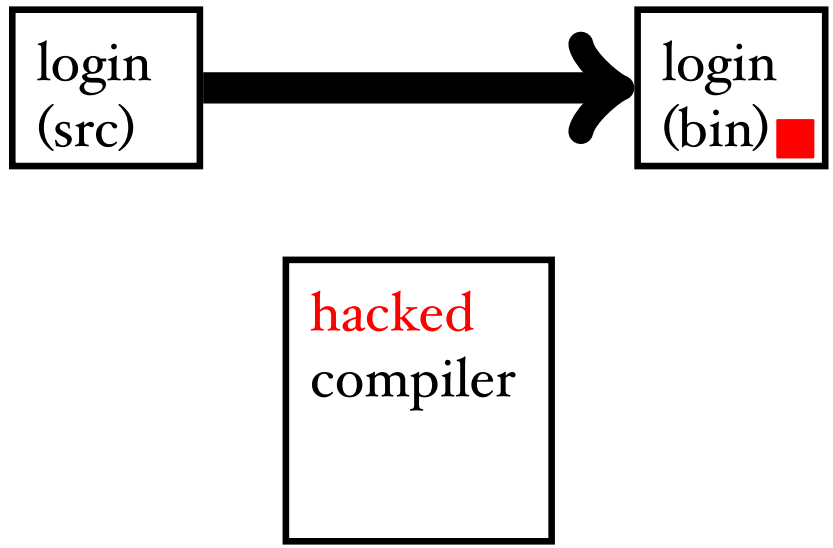
Using a compiler,
how can you mount the
perfect attack against a system?

What is a perfect attack?

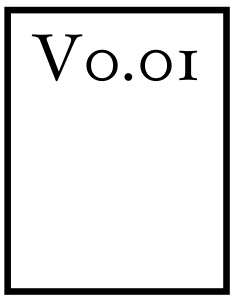
1. you can potentially completely take over a target system
2. your attack is (nearly) undetectable
3. the victim has (almost) no chance to recover


clean
compiler


login
(src)
login
(bin)
clean
compiler


login
(src)
login
(bin)
hacked
compiler

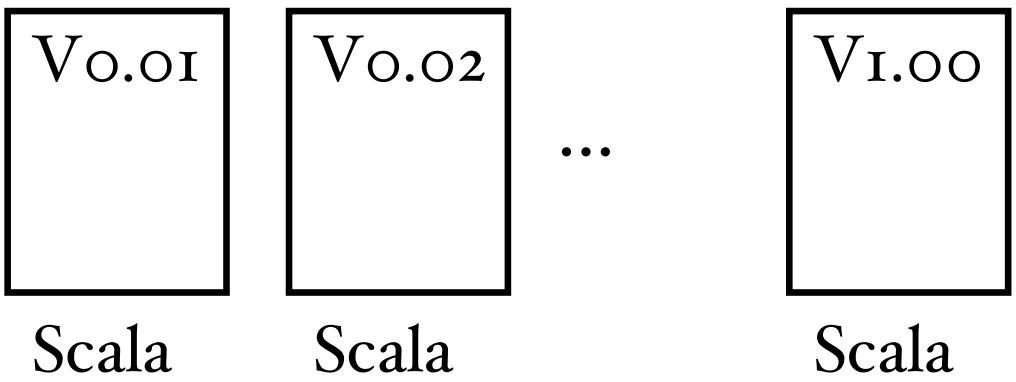
my compiler (src)



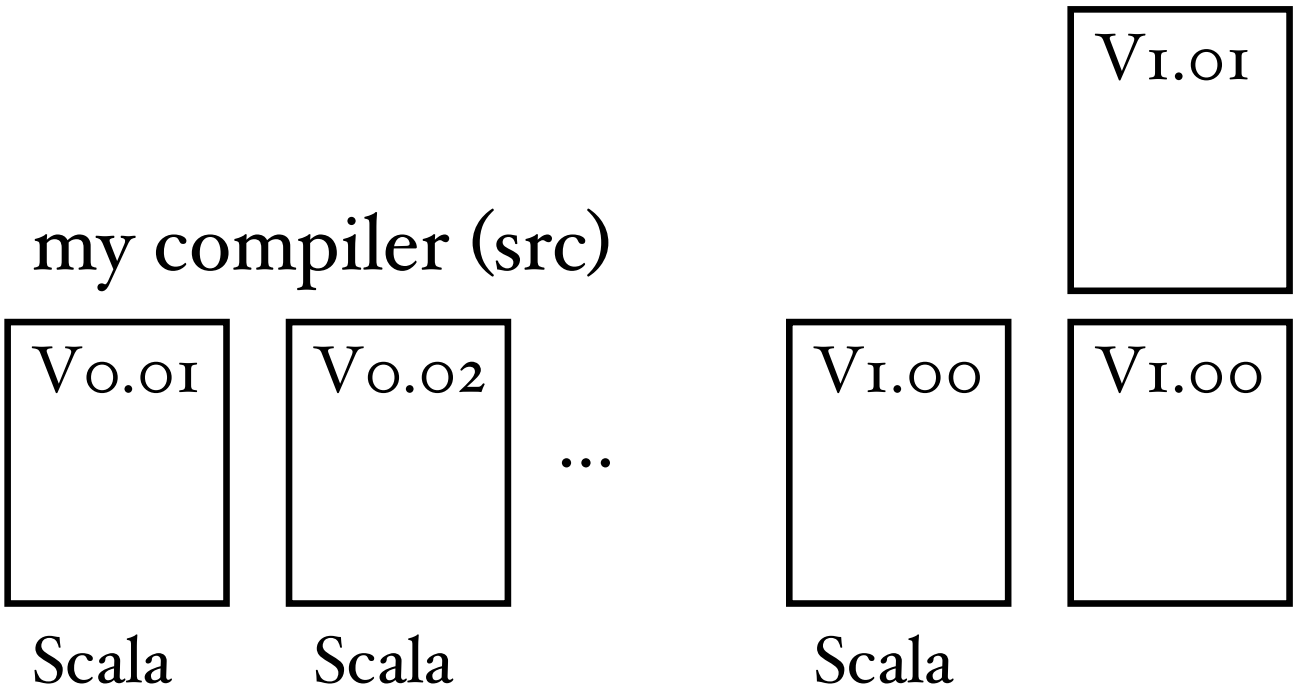
Scala

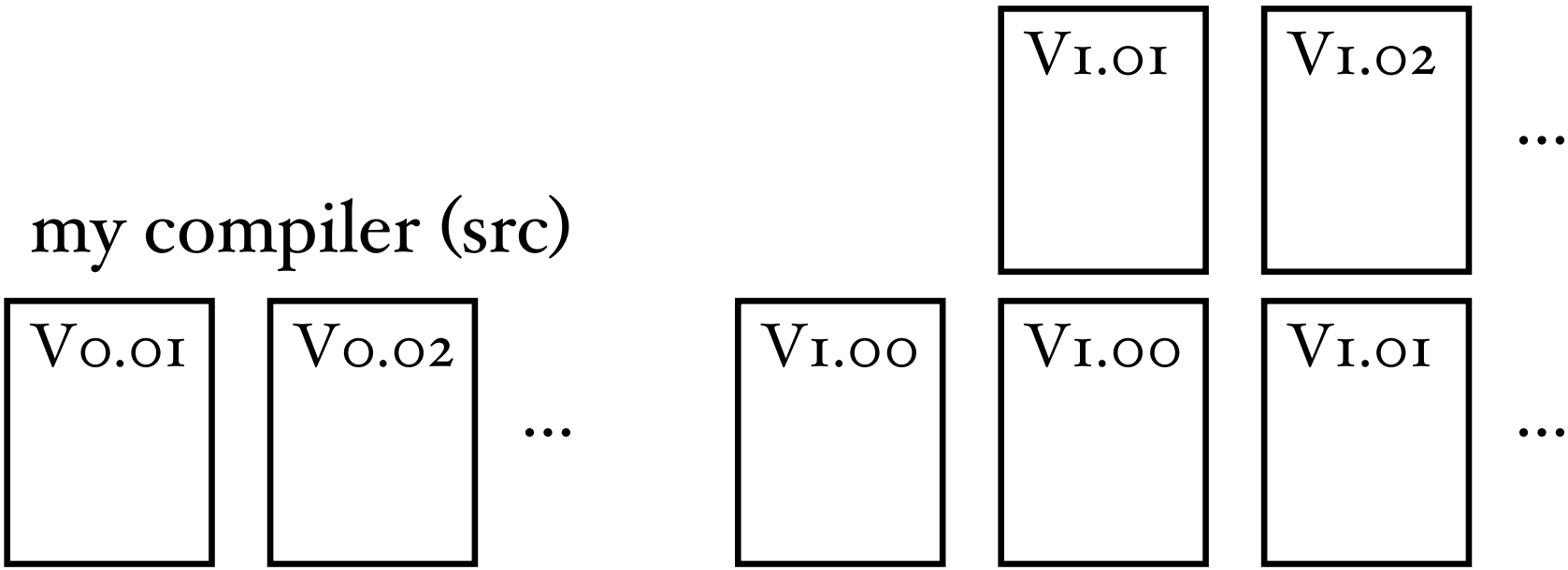
host language

my compiler (src)



host language


V1.01
my compiler (src)
V0.01
V0.02
...
V1.00
V1.00
Scala
Scala
Scala
host language


my compiler (src)
V0.01
V0.02
...
Scala
Scala
host language
V1.00
Scala
V1.01
V1.00
V1.02
V1.01
...
...
no host language
needed

Hacking Compilers

Ken Thompson
Turing Award, 1983

Ken Thompson showed how to hide a Trojan Horse in a compiler without leaving any traces in the source code.

No amount of source level verification will protect you from such Thompson-hacks.

Therefore in safety-critical systems it is important to rely on only a very small TCB.

Hacking Compilers

Ken Thompson
Turing Award, 198

1) *Assume you ship the compiler as binary and also with sources.*
2) *Make the compiler aware when it compiles itself.*
3) *Add the Trojan horse.*
4) *Compile.*
5) *Delete Trojan horse from the sources of the compiler.*
6) *Go on holiday for the rest of your life. ;o)*

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