Homework 3

Please submit your solutions via email. Please submit only ASCII text or PDFs. Every solution should be preceeded by the corresponding question text, like:

Qn:	**…a difficult question from me…**
A:	**…an answer from you …**
Q$n+1$	**…another difficult question…**
A:	**…another brilliant answer from you…**

Solutions will only be accepted until 20th December! Please send only one homework per email.

1. What is a regular language? Are there alternative ways to define this notion? If yes, give an explanation why they define the same notion.

2. Why is every finite set of strings a regular language?

3. Assume you have an alphabet consisting of the letters a, b and c only. (1) Find a regular expression that recognises the two strings ab and ac. (2) Find a regular expression that matches all strings *except* these two strings. Note, you can only use regular expressions of the form

$$r ::= \mathbf{0} \mid \mathbf{1} \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

4. Define the function *zeroable* which takes a regular expression as argument and returns a boolean. The function should satisfy the following property:

$$\textit{zeroable(r)} \text{ if and only if } L(r) = \{\}$$

5. Given the alphabet $\{a, b\}$. Draw the automaton that has two states, say q_0 and q_1. The starting state is q_0 and the final state is q_1. The transition function is given by

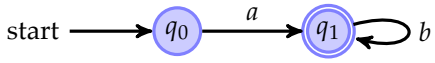
$$\begin{array}{l}(q_0, a) \to q_0\\ (q_0, b) \to q_1\\ (q_1, b) \to q_1\end{array}$$

   What is the language recognised by this automaton?

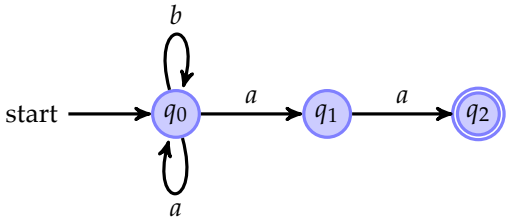
6. Give a non-deterministic finite automaton that can recognise the language $L(a \cdot (a + b)^* \cdot c)$.

7. Given a deterministic finite automaton $A(Q, q_0, F, \delta)$, define which language is recognised by this automaton. Can you define also the language defined by a non-deterministic automaton?

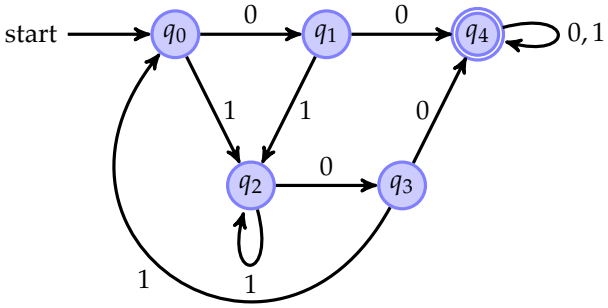
8. Given the following deterministic finite automaton over the alphabet $\{a, b\}$, find an automaton that recognises the complement language. (Hint: Recall that for the algorithm from the lectures, the automaton needs to be in completed form, that is have a transition for every letter from the alphabet.)



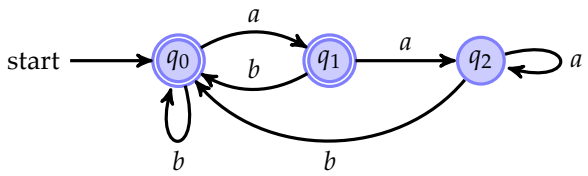
9. Given the following non-deterministic finite automaton over the alphabet $\{a, b\}$, find a deterministic finite automaton that recognises the same language:



10. Given the following deterministic finite automaton over the alphabet $\{0, 1\}$, find the corresponding minimal automaton. In case states can be merged, state clearly which states can be merged.



11. Given the following finite deterministic automaton over the alphabet $\{a, b\}$:



Give a regular expression that can recognise the same language as this automaton. (Hint: If you use Brzozwski's method, you can assume Arden's lemma which states that an equation of the form $q = q \cdot r + s$ has the unique solution $q = s \cdot r^*$.)

12. If a non-deterministic finite automaton (NFA) has n states. How many states does a deterministic automaton (DFA) that can recognise the same language as the NFA maximal need?

13. **(Optional)** This question is for you to provide regular feedback to me: for example what were the most interesting, least interesting, or confusing parts in this lecture? Any problems with my Scala code? Please feel free to share any other questions or concerns.