Compilers and Formal Languages

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Slides & Progs: KEATS (also homework is there)

1 Introduction, Languages	6 While-Language
2 Regular Expressions, Derivatives	7 Compilation, JVM
3 Automata, Regular Languages	8 Compiling Functional Languages
4 Lexing, Tokenising	9 Optimisations
5 Grammars, Parsing	10 LLVM

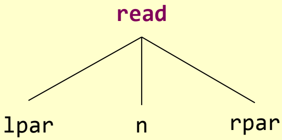
Parser

Parser

parser input: a sequence of tokens

```
key(read) lpar id(n) rpar semi
```

parser output: an abstract syntax tree



What Parsing is Not

Usually parsing does not check semantic correctness, e.g.

- whether a function is not used before it is defined
- whether a function has the correct number of arguments or are of correct type
- whether a variable can be declared twice in a scope

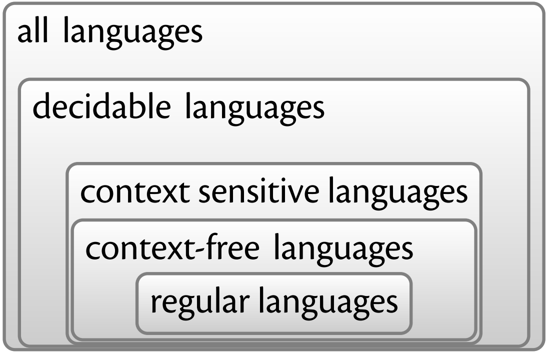
Regular Languages

While regular expressions are very useful for lexing, there is no regular expression that can recognise the language $a^n b^n$.

$$(((()()))()) \quad \text{vs.} \quad (((()()))()))$$

So we cannot find out with regular expressions whether parentheses are matched or unmatched. Also regular expressions are not recursive, e.g. $(1 + 2) + 3$.

Hierarchy of Languages



Time flies like an arrow.
Fruit flies like bananas.

CFGs

A **context-free grammar** G consists of

- a finite set of nonterminal symbols (e.g. A upper case)
- a finite set terminal symbols or tokens (lower case)
- a start symbol (which must be a nonterminal)
- a set of rules

$$A ::= rhs$$

where *rhs* are sequences involving terminals and nonterminals, including the empty sequence ϵ.

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where rhs are sequences involving terminals and nonterminals, including the empty sequence ϵ.

We also allow rules

$$A ::= rhs_1 | rhs_2 | \ldots$$

Palindromes

A grammar for palindromes over the alphabet $\{a, b\}$:

$$S ::= a \cdot S \cdot a$$

$$S ::= b \cdot S \cdot b$$

$$S ::= a$$

$$S ::= b$$

$$S ::= \epsilon$$

Palindromes

A grammar for palindromes over the alphabet $\{a, b\}$:

$$\mathbf{S} ::= a \cdot \mathbf{S} \cdot a \mid b \cdot \mathbf{S} \cdot b \mid a \mid b \mid \epsilon$$

Arithmetic Expressions

$$
\begin{aligned}
E ::= &\; 0 \mid 1 \mid 2 \mid \ldots \mid 9\\
&\mid E \cdot + \cdot E\\
&\mid E \cdot - \cdot E\\
&\mid E \cdot * \cdot E\\
&\mid (\cdot E \cdot)
\end{aligned}
$$

Arithmetic Expressions

$$\begin{aligned} E ::= {} & 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\ \mid {} & E \cdot + \cdot E \\ \mid {} & E \cdot - \cdot E \\ \mid {} & E \cdot * \cdot E \\ \mid {} & (\cdot E \cdot) \end{aligned}$$

1 + 2 * 3 + 4

A CFG Derivation

Begin with a string containing only the start symbol, say S

Replace any nonterminal X in the string by the right-hand side of some production $X ::= rhs$

Repeat 2 until there are no nonterminals left

$$S \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots$$

Example Derivation

$$S ::= \epsilon \mid a \cdot S \cdot a \mid b \cdot S \cdot b$$

$$\begin{array}{rcl} S & \rightarrow & aSa \\ & \rightarrow & abSba \\ & \rightarrow & abaSaba \\ & \rightarrow & abaaba \end{array}$$

Example Derivation

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\begin{aligned}
E ::= \; & 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\
& \mid E \cdot + \cdot E \\
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& \mid E \cdot * \cdot E \\
& \mid (\cdot E \cdot)
\end{aligned}
$$

$$
\begin{aligned}
E \rightarrow \quad & E * E \\
\rightarrow \quad & E + E * E \\
\rightarrow \quad & E + E * E + E \\
\rightarrow^{+} \; & 1 + 2 * 3 + 4
\end{aligned}
$$

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&\mid (\cdot E \cdot)
\end{aligned}
$$

$$
\begin{array}{llll}
E \rightarrow & E * E & E \rightarrow & E + E \\
\rightarrow & E + E * E & \rightarrow & E + E + E \\
\rightarrow & E + E * E + E & \rightarrow & E + E * E + E \\
\rightarrow^{+} & 1 + 2 * 3 + 4 & \rightarrow^{+} & 1 + 2 * 3 + 4
\end{array}
$$

Language of a CFG

Let G be a context-free grammar with start symbol $\mathbf{S}$.
Then the language $L(G)$ is:

$$\{c_1 \ldots c_n \mid \forall i.\ c_i \in T \wedge \mathbf{S} \rightarrow^* c_1 \ldots c_n\}$$

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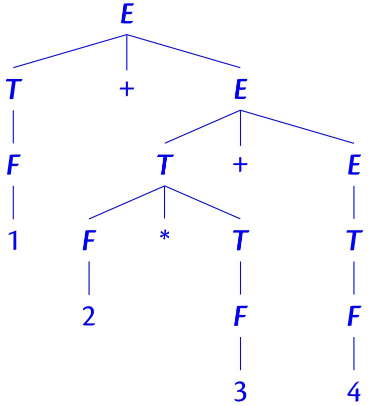
- Terminals, because there are no rules for replacing them.
- Once generated, terminals are "permanent".
- Terminals ought to be tokens of the language (but can also be strings).

Parse Trees

$$E ::= T \mid T \cdot + \cdot E \mid T \cdot - \cdot E$$

$$T ::= F \mid F \cdot * \cdot T$$

$$F ::= 0...9 \mid (\cdot E \cdot)$$



1 + 2 * 3 + 4

Arithmetic Expressions

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\end{aligned}
$$

A CFG is **left-recursive** if it has a nonterminal E such that $E \rightarrow^{+} E \cdot \ldots$

Ambiguous Grammars

A grammar is **ambiguous** if there is a string that has at least two different parse trees.

$$
\begin{aligned}
E ::= {} & 0...9 \\
\mid {} & E \cdot + \cdot E \\
\mid {} & E \cdot - \cdot E \\
\mid {} & E \cdot * \cdot E \\
\mid {} & (\cdot E \cdot)
\end{aligned}
$$

`1 + 2 * 3 + 4`

'Dangling' Else

Another ambiguous grammar:

$$
\begin{array}{rcl}
E & \rightarrow & \text{if } E \text{ then } E \\
  & | & \text{if } E \text{ then } E \text{ else } E \\
  & | & \ldots
\end{array}
$$

```
if a then if x then y else c
```

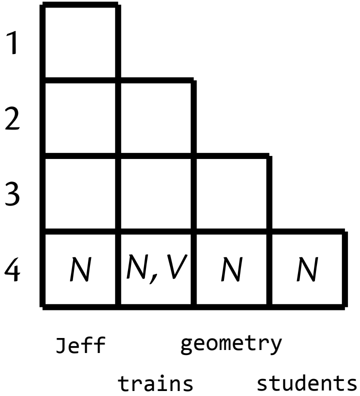
CYK Algorithm

Suppose the grammar:

$$
\begin{array}{lcl}
S & ::= & N \cdot P \\
P & ::= & V \cdot N \\
N & ::= & N \cdot N \\
N & ::= & \texttt{students} \mid \texttt{Jeff} \mid \texttt{geometry} \mid \texttt{trains} \\
V & ::= & \texttt{trains}
\end{array}
$$

`Jeff trains geometry students`

CYK Algorithm



$$
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 & & \mid \texttt{geometry} \mid \texttt{trains} \\
V & ::= & \texttt{trains}
\end{array}
$$

Chomsky Normal Form

A grammar for palindromes over the alphabet $\{a, b\}$:

$$\mathbf{S} ::= a \cdot \mathbf{S} \cdot a \mid b \cdot \mathbf{S} \cdot b \mid a \cdot a \mid b \cdot b \mid a \mid b$$

CYK Algorithm

fastest possible algorithm for recognition problem

runtime is $O(n^3)$

grammars need to be transformed into CNF

Context Sensitive Grammars

It is much harder to find out whether a string is parsed by a context sensitive grammar:

$$
\begin{aligned}
S &::= bSAA \mid \epsilon \\
A &::= a \\
bA &::= Ab
\end{aligned}
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$$S \rightarrow \ldots \rightarrow^{?} ababaa$$

```
Time flies like an arrow;
fruit flies like bananas.
```

Parser Combinators

One of the simplest ways to implement a parser, see

`https://vimeo.com/142341803`

Parser combinators:

$$\underbrace{\text{list of tokens}}_{\text{input}} \Rightarrow \underbrace{\text{set of (parsed input, unparsed input)}}_{\text{output}}$$

- atomic parsers
- sequencing
- alternative
- semantic action

Atomic parsers, for example, number tokens

$$\texttt{Num(123)} :: \textit{rest} \;\Rightarrow\; \{(\texttt{Num(123)}, \textit{rest})\}$$

- you consume one or more token from the input (stream)
- also works for characters and strings

Alternative parser (code $p \mid\mid q$)

apply p and also q; then combine the outputs

$$p(\text{input}) \cup q(\text{input})$$

Sequence parser (code $p \sim q$)

apply first p producing a set of pairs
then apply q to the unparsed part
then combine the results:

$$((\text{output}_1, \text{output}_2), \text{unparsed part})$$

$$\begin{aligned}\Big\{\big((o_1, o_2), u_2\big) \;\Big|\; & \\ & (o_1, u_1) \in p(\text{input}) \wedge \\ & (o_2, u_2) \in q(u_1)\Big\}\end{aligned}$$

Function parser (code $p \Rightarrow f$)

apply p producing a set of pairs

then apply the function f to each first component

$$\{(f(o_1), u_1) \mid (o_1, u_1) \in p(\text{input})\}$$

Function parser (code $p \Rightarrow f$)

apply p producing a set of pairs

then apply the function f to each first component

$$\{(f(o_1), u_1) \mid (o_1, u_1) \in p(\text{input})\}$$

f is the semantic action ("what to do with the parsed input")

Semantic Actions

Addition

$$T \sim + \sim E \Rightarrow \underbrace{f\big((x,y),z\big) \Rightarrow x+z}_{\text{semantic action}}$$

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Multiplication

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Addition

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Multiplication

$$F \sim * \sim T \Rightarrow f((x,y),z) \Rightarrow x*z$$

Parenthesis

$$(\sim E \sim) \Rightarrow f((x,y),z) \Rightarrow y$$

Types of Parsers

Sequencing: if p returns results of type T, and q results of type S, then $p \sim q$ returns results of type

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Alternative: if p returns results of type T then q must also have results of type T, and $p \;||\; q$ returns results of type

$$T$$

Semantic Action: if p returns results of type T and f is a function from T to S, then $p \Rightarrow f$ returns results of type

$$S$$

Input Types of Parsers

input: token list

output: set of (output_type, token list)

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actually it can be any input type as long as it is a kind of sequence (for example a string)

Scannerless Parsers

input: string

output: set of (output_type, string)

but using lexers is better because whitespaces or comments can be filtered out; then input is a sequence of tokens

Successful Parses

input: string

output: set of (output_type, string)

a parse is successful whenever the input has been fully "consumed" (that is the second component is empty)

Abstract Parser Class

```scala
abstract class Parser[I, T] {
  def parse(ts: I): Set[(T, I)]

  def parse_all(ts: I) : Set[T] =
    for ((head, tail) <- parse(ts);
          if (tail.isEmpty)) yield head
}
```

```scala
class AltParser[I, T](p: => Parser[I, T],
                      q: => Parser[I, T])
                              extends Parser[I, T] {
  def parse(sb: I) = p.parse(sb) ++ q.parse(sb)
}

class SeqParser[I, T, S](p: => Parser[I, T],
                         q: => Parser[I, S])
                                 extends Parser[I, (T, S)] {
  def parse(sb: I) =
    for ((head1, tail1) <- p.parse(sb);
         (head2, tail2) <- q.parse(tail1))
            yield ((head1, head2), tail2)
}

class FunParser[I, T, S](p: => Parser[I, T], f: T => S)
                                        extends Parser[I, S] {
  def parse(sb: I) =
    for ((head, tail) <- p.parse(sb))
      yield (f(head), tail)
}
```

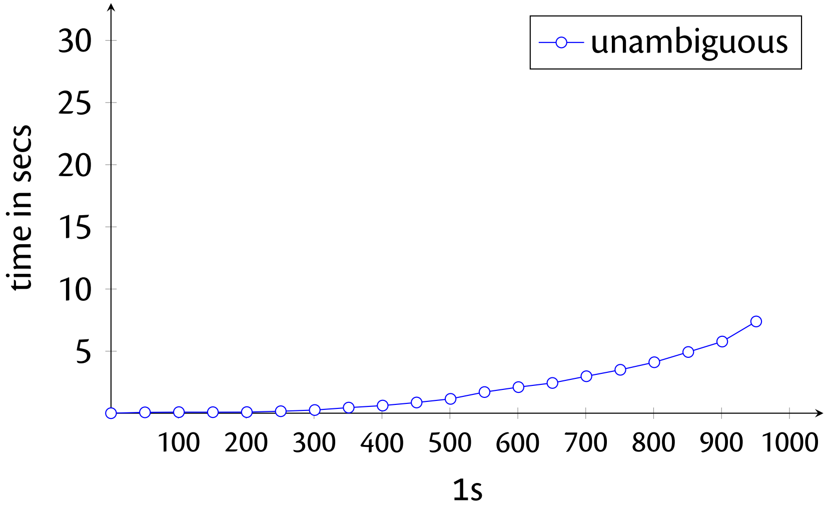
Two Grammars

Which languages are recognised by the following two grammars?

$$
\begin{array}{lcl}
S & \rightarrow & 1 \cdot S \cdot S \\
  & | & \epsilon
\end{array}
$$

$$
\begin{array}{lcl}
U & \rightarrow & 1 \cdot U \\
  & | & \epsilon
\end{array}
$$

Ambiguous Grammars



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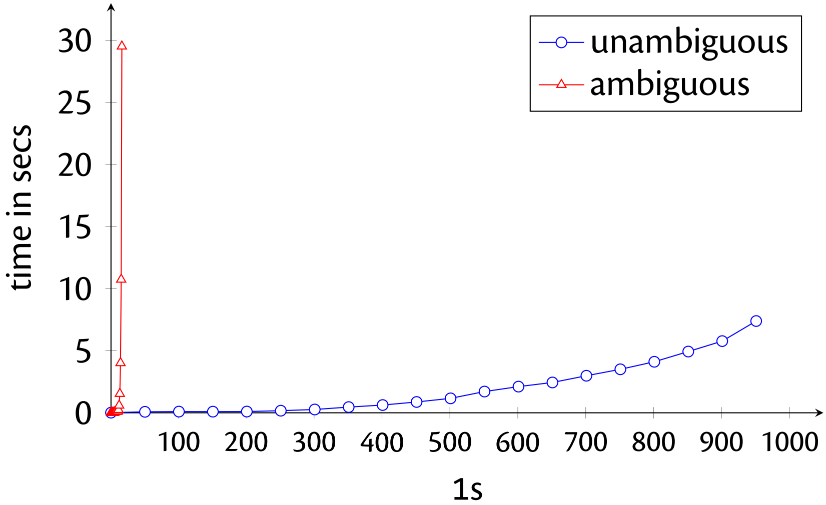

While-Language

$$
\begin{array}{rcl}
\mathit{Stmt} & ::= & \mathrm{skip} \\
& | & \mathit{Id} := \mathit{AExp} \\
& | & \mathrm{if}\ \mathit{BExp}\ \mathrm{then}\ \mathit{Block}\ \mathrm{else}\ \mathit{Block} \\
& | & \mathrm{while}\ \mathit{BExp}\ \mathrm{do}\ \mathit{Block} \\
\mathit{Stmts} & ::= & \mathit{Stmt}\ ;\ \mathit{Stmts} \\
& | & \mathit{Stmt} \\
\mathit{Block} & ::= & \{\ \mathit{Stmts}\ \} \\
& | & \mathit{Stmt} \\
\mathit{AExp} & ::= & \ldots \\
\mathit{BExp} & ::= & \ldots
\end{array}
$$

An Interpreter

$$
\begin{array}{l}
\{ \\
\quad x := 5; \\
\quad y := x * 3; \\
\quad y := x * 4; \\
\quad x := u * 3 \\
\}
\end{array}
$$

the interpreter has to record the value of x before assigning a value to y

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$$

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```
eval(stmt, env)
```

Interpreter

$$
\begin{array}{lcl}
\text{eval}(n, E) & \stackrel{\text{def}}{=} & n \\
\text{eval}(x, E) & \stackrel{\text{def}}{=} & E(x) \quad \text{lookup } x \text{ in } E \\
\text{eval}(a_1 + a_2, E) & \stackrel{\text{def}}{=} & \text{eval}(a_1, E) + \text{eval}(a_2, E) \\
\text{eval}(a_1 - a_2, E) & \stackrel{\text{def}}{=} & \text{eval}(a_1, E) - \text{eval}(a_2, E) \\
\text{eval}(a_1 * a_2, E) & \stackrel{\text{def}}{=} & \text{eval}(a_1, E) * \text{eval}(a_2, E) \\
\\
\text{eval}(a_1 = a_2, E) & \stackrel{\text{def}}{=} & \text{eval}(a_1, E) = \text{eval}(a_2, E) \\
\text{eval}(a_1 \mathrel{!=} a_2, E) & \stackrel{\text{def}}{=} & \neg(\text{eval}(a_1, E) = \text{eval}(a_2, E)) \\
\text{eval}(a_1 < a_2, E) & \stackrel{\text{def}}{=} & \text{eval}(a_1, E) < \text{eval}(a_2, E)
\end{array}
$$

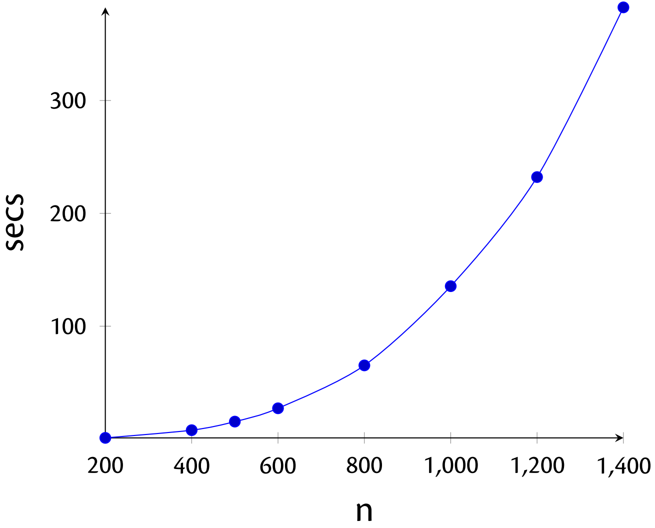
Interpreter (2)

$$
\begin{array}{lcl}
\text{eval}(\text{skip}, E) & \stackrel{\text{def}}{=} & E \\
\text{eval}(x := a, E) & \stackrel{\text{def}}{=} & E(x \mapsto \text{eval}(a, E)) \\
\text{eval}(\text{if } b \text{ then } cs_1 \text{ else } cs_2, E) & \stackrel{\text{def}}{=} & \\
\qquad \text{if eval}(b, E) \text{ then eval}(cs_1, E) & & \\
\qquad \qquad \qquad \qquad \text{else eval}(cs_2, E) & & \\
\text{eval}(\text{while } b \text{ do } cs, E) & \stackrel{\text{def}}{=} & \\
\qquad \text{if eval}(b, E) & & \\
\qquad \text{then eval}(\text{while } b \text{ do } cs, \text{eval}(cs, E)) & & \\
\qquad \text{else } E & & \\
\text{eval}(\text{write } x, E) & \stackrel{\text{def}}{=} & \{\ \text{println}(E(x))\ ;\ E\ \}
\end{array}
$$

Test Program

??

Interpreted Code



Java Virtual Machine

- introduced in 1995
- is a stack-based VM (like Postscript, CLR of .Net)
- contains a JIT compiler
- many languages take advantage of JVM's infrastructure (JRE)
- is garbage collected $\Rightarrow$ no buffer overflows
- some languages compile to the JVM: Scala, Clojure...

For CW2, please include '\' as a symbol in strings, because the collatz program contains

```
write "\n";
```

val (r1s, f1s) = simp(r1)
val (r2s, f2s) = simp(r2)
how are the first rectification functions f1s and f2s made? could you maybe show an example?