Automata and Formal Languages (4)

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Slides: KEATS (also home work is there)

Last Week

Last week I showed you

- a tokenizer taking a list of regular expressions
- tokenization identifies lexeme in an input stream of characters (or string) and cathegorizes them into tokens

Two Rules

- Longest match rule (maximal munch rule): The longest initial substring matched by any regular expression is taken as next token.
- Rule priority: For a particular longest initial substring, the first regular expression that can match determines the token.

"if true then then 42 else +"

KEYWORD:

"if","then","else",

WHITESPACE:

" ","\n",

IDENT:

LETTER · (LETTER + DIGIT + "_")*

NUM:

(NONZERODIGIT · DIGIT*) + "0"

OP:

"+"

COMMENT:

"/*" · (ALL* · "*/" · ALL*) · "*/"

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"if true then then 42 else +"
```

KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)

"if true then then 42 else +"

KEYWORD(if),
IDENT(true),
KEYWORD(then),
KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)

There is one small problem with the tokenizer.
How should we tokenize:

"x - 3"

OP:

"+", "-"

NUM:

(NONZERODIGIT · DIGIT*) + "0"

NUMBER:

NUM + ("-" · NUM)

Deterministic Finite Automata

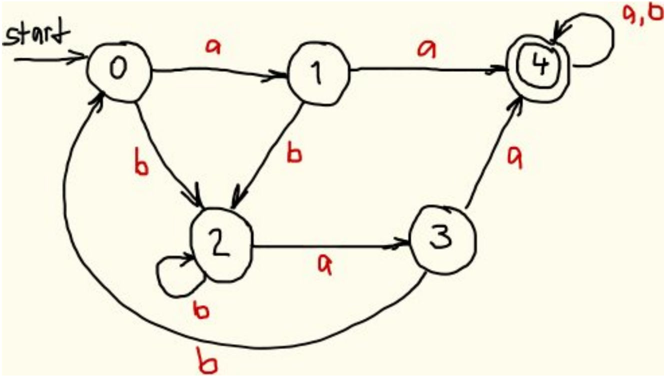
A deterministic finite automaton consists of:

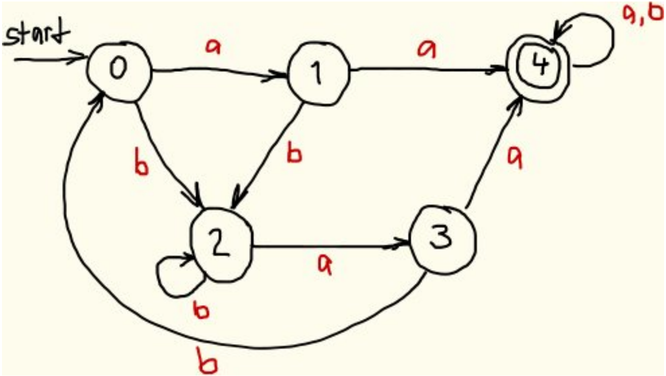
- a finite set of states
- one of these states is the start state
- some states are accepting states, and
- there is transition function

  which takes a state and a character as arguments and produces a new state

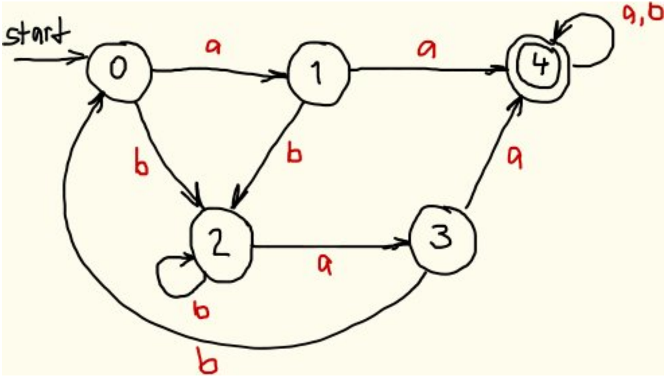
  this function might not always be defined everywhere

$$A(Q, q_0, F, \delta)$$


start
0
1
2
3
4
a
a
a
a
b
b
b
b
a,b



- start can be an accepting state
- there is no accepting state
- all states are accepting



for this automaton δ is the function

$$\begin{array}{lll} (q_0, a) \rightarrow q_1 & (q_1, a) \rightarrow q_4 & (q_4, a) \rightarrow q_4 \\ (q_0, b) \rightarrow q_2 & (q_1, b) \rightarrow q_2 & (q_4, b) \rightarrow q_4 \end{array} \quad \ldots$$

Accepting a String

Given

$$A(Q, q_0, F, \delta)$$

you can define

$$\hat{\delta}(q, "\,") = q$$
$$\hat{\delta}(q, c :: s) = \hat{\delta}(\delta(q, c), s)$$

Accepting a String

Given

$$A(Q, q_0, F, \delta)$$

you can define

$$\hat{\delta}(q, "") = q$$
$$\hat{\delta}(q, c :: s) = \hat{\delta}(\delta(q, c), s)$$

Whether a string s is accepted by A?

$$\hat{\delta}(q_0, s) \in F$$

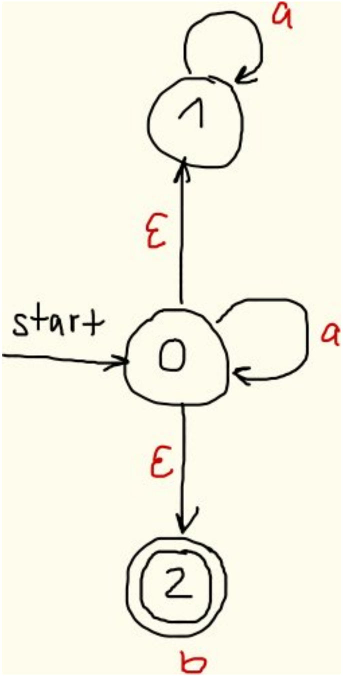
Non-Deterministic Finite Automata

A non-deterministic finite automaton consists again of:

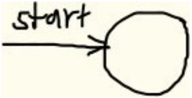
- a finite set of states
- one of these states is the start state
- some states are accepting states, and
- there is transition relation

$$(q_1, a) \rightarrow q_2$$
$$(q_1, a) \rightarrow q_3$$

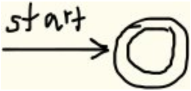
$$(q_1, \epsilon) \rightarrow q_2$$


a
1
ε
start
0
a
ε
2
b

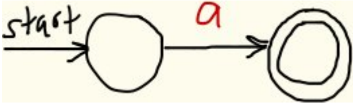
$\varnothing$



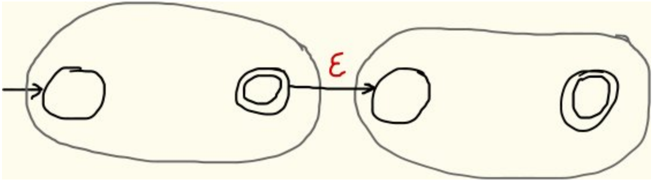
ϵ

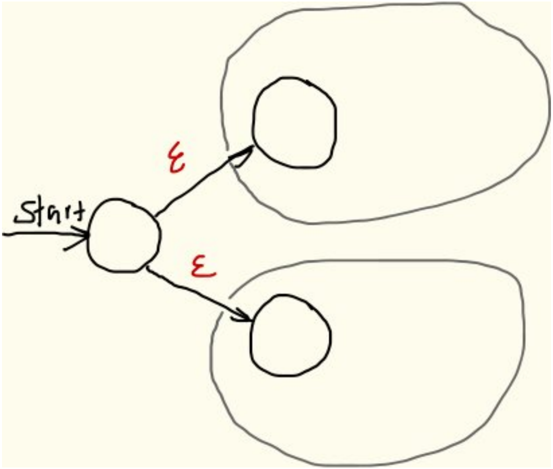


c



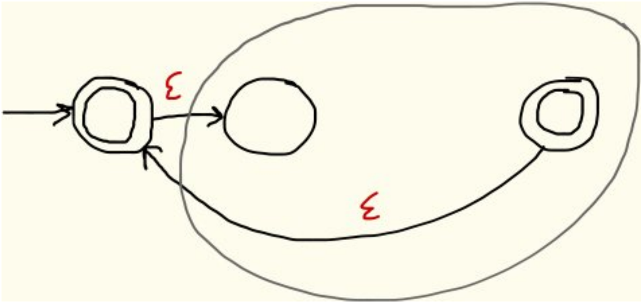
$r_1 \cdot r_2$

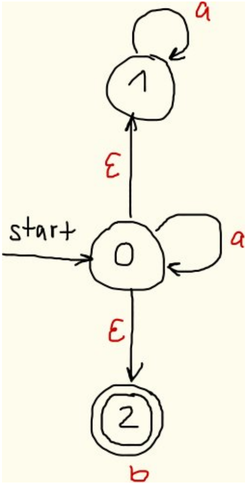




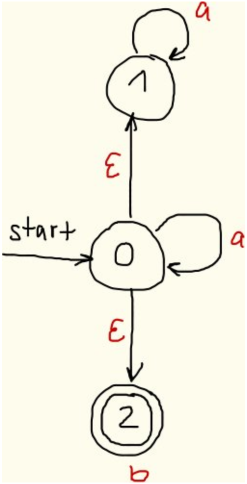
$r_1 + r_2$

r^*

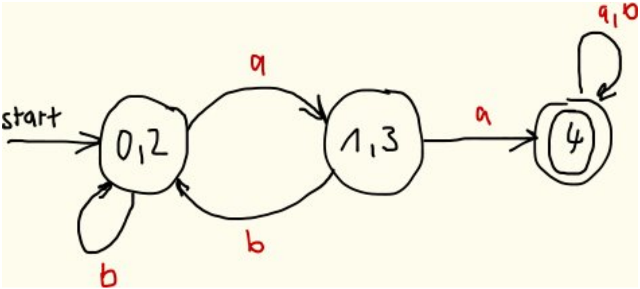




	a	b
$\varnothing$	$\varnothing$	$\varnothing$
$\{0\}$	$\{0,1,2\}$	$\{2\}$
$\{1\}$	$\{1\}$	$\varnothing$
$\{2\}$	$\varnothing$	$\{2\}$
$\{0,1\}$	$\{0,1,2\}$	$\{2\}$
$\{0,2\}$	$\{0,1,2\}$	$\{2\}$
$\{1,2\}$	$\{1\}$	$\{2\}$
$\{0,1,2\}$	$\{0,1,2\}$	$\{2\}$



	a	b
$\varnothing$	$\varnothing$	$\varnothing$
$\{0\}$	$\{0, 1, 2\}$	$\{2\}$
$\{1\}$	$\{1\}$	$\varnothing$
$\{2\}$ *	$\varnothing$	$\{2\}$
$\{0, 1\}$	$\{0, 1, 2\}$	$\{2\}$
$\{0, 2\}$ *	$\{0, 1, 2\}$	$\{2\}$
$\{1, 2\}$ *	$\{1\}$	$\{2\}$
s: $\{0, 1, 2\}$ *	$\{0, 1, 2\}$	$\{2\}$


start
0,2
1,3
4
a
a
b
b
a,b

Languages

A language is regular iff there exists a regular expression that recognises all its strings.

Languages

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not all languages are regular, e.g. $\mathsf{a}^n\mathsf{b}^n$.

- Assuming you have the alphabet {a, b, c}
- Give a regular expression that can recognise all strings that have at least one b.

- The star-case in our proof needs the following lemma

$$\mathrm{Der}\,c\,A^* = (\mathrm{Der}\,c\,A) \mathbin{@} A^*$$

- If "" $\in A$, then $\mathrm{Der}\,c\,(A \mathbin{@} B) = (\mathrm{Der}\,c\,A) \mathbin{@} B \cup (\mathrm{Der}\,c\,B)$
- If "" $\notin A$, then $\mathrm{Der}\,c\,(A \mathbin{@} B) = (\mathrm{Der}\,c\,A) \mathbin{@} B$