Automata and Formal Languages (7)

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CFGs

A context-free grammar (CFG) G consists of:

- a finite set of nonterminal symbols (upper case)
- a finite terminal symbols or tokens (lower case)
- a start symbol (which must be a nonterminal)
- a set of rules

$$A \rightarrow \text{rhs}$$

where rhs are sequences involving terminals and nonterminals (can also be empty).

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We can also allow rules

$$A \rightarrow \text{rhs}_1 | \text{rhs}_2 | \ldots$$

A CFG Derivation

1. Begin with a string with only the start symbol S
2. Replace any non-terminal X in the string by the right-hand side of some production $X \rightarrow \text{rhs}$
3. Repeat 2 until there are no non-terminals

$$S \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots$$

Language of a CFG

Let G be a context-free grammar with start symbol S. Then the language $L(G)$ is:

$$\{c_1 \ldots c_n \mid \forall i.\ c_i \in T \wedge S \rightarrow^* c_1 \ldots c_n\}$$

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- Terminals are so-called because there are no rules for replacing them
- Once generated, terminals are "permanent"
- Terminals ought to be tokens of the language (at least in this course)

Arithmetic Expressions

$$\begin{array}{lcl}
E & \rightarrow & num_token \\
E & \rightarrow & E \cdot + \cdot E \\
E & \rightarrow & E \cdot - \cdot E \\
E & \rightarrow & E \cdot * \cdot E \\
E & \rightarrow & (\cdot E \cdot)
\end{array}$$

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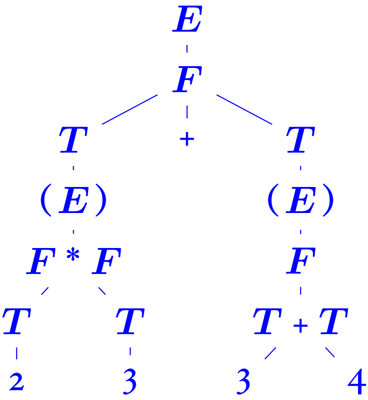
A CFG is left-recursive if it has a nonterminal E such that $E \rightarrow^+ E \cdot \ldots$

Parse Trees

$$
\begin{array}{lcl}
E & \rightarrow & F \mid F \cdot * \cdot F \\
F & \rightarrow & T \mid T \cdot + \cdot T \mid T \cdot - \cdot T \\
T & \rightarrow & num\_token \mid (\cdot E \cdot)
\end{array}
$$

(2*3)+(3+4)



Ambiguous Grammars

A CFG is ambiguous if there is a string that has at least parse trees.

$$
\begin{array}{lcl}
E & \rightarrow & num\_token \\
E & \rightarrow & E \cdot + \cdot E \\
E & \rightarrow & E \cdot - \cdot E \\
E & \rightarrow & E \cdot * \cdot E \\
E & \rightarrow & (\cdot E \cdot)
\end{array}
$$

$1 + 2 * 3 + 4$

Dangling Else

Another ambiguous grammar:

$$
\begin{array}{rcl}
E & \rightarrow & \text{if } E \text{ then } E \\
 & | & \text{if } E \text{ then } E \text{ else } E \\
 & | & \text{id}
\end{array}
$$

$\text{if a then if x then y else c}$