

Handout 6 (Parser Combinators)

This handout explains how *parser combinators* work and how they can be implemented in Scala. Their most distinguishing feature is that they are very easy to implement (admittedly it is only easy in a functional programming language). Another good point of parser combinators is that they can deal with any kind of input as long as this input is of “sequence-kind”, for example a string or a list of tokens. The only two properties of the input we need is to be able to test when it is empty and “sequentially” take it apart. Strings and lists fit this bill. However, parser combinators also have their drawbacks. For example they require that the grammar to be parsed is *not* left-recursive and they are efficient only when the grammar is unambiguous. It is the responsibility of the grammar designer to ensure these two properties.

The general idea behind parser combinators is to transform the input into sets of pairs, like so

$$\underbrace{\text{list of tokens}}_{\text{input}} \Rightarrow \underbrace{\text{set of (parsed input, unparsed input)}}_{\text{output}}$$

Given the extended effort we have spent in order to implement a lexer in order to generate list of tokens, it might be surprising that in what follows we shall often use strings as input. This is for making the explanation more lucid. It does not make our previous work on lexers obsolete (remember they transform a string into a list of tokens). Lexers will still be needed for building a somewhat realistic compiler.

But as said, parser combinators are relatively agnostic about what kind of input they process. In my Scala code I use the following polymorphic types for parser combinators:

input: I output: T

That is they take as input something of type I and return a set of pairs of type `Set[(T, I)]`. Since the input needs to be of “sequence-kind”, I actually have to often write `I <% Seq[_]` for the input type. This ensures the input is a subtype of Scala sequences. The first component of the generated pairs corresponds to what the parser combinator was able to process from the input and the second is the unprocessed part of the input (therefore the type of this unprocessed part is the same as the input). As we shall see shortly, a parser combinator might return more than one such pair; the idea is that there are potentially several ways of how to parse the input. As a concrete example, consider the string

`i f f o o _ t e s t b a r`

We might have a parser combinator which tries to interpret this string as a keyword (`if`) or as an identifier (`iffoo`). Then the output will be the set

`{(if, foo_testbar), (iffoo, _testbar)}`

where the first pair means the parser could recognise `if` from the input and leaves the rest as ‘unprocessed’ as the second component of the pair; in the other case it could recognise `iffoo` and leaves `_testbar` as unprocessed. If the parser cannot recognise anything from the input at all, then parser combinators just return the empty set `{}`. This will indicate something “went wrong”...or more precisely, nothing could be parsed.

Also important to note is that the type `T` for the processed part is different from the input type. In the example above is just happens to be the same. The reason for the difference is that in general we are interested in transforming our input into something “different”...for example into a tree; or if we implement the grammar for arithmetic expressions, we might be interested in the actual integer number the arithmetic expression, say `1 + 2 * 3`, stands for. In this way we can use parser combinators to implement relatively easily a calculator, for instance.

The main idea of parser combinators is that we can easily build parser combinators out of smaller components following very closely the structure of a grammar. In order to implement this in an object-oriented programming language, like Scala, we need to specify an abstract class for parser combinators. This abstract class states that the function `parse` takes an argument of type `I` and returns a set of type `Set[(T, I)]`.

```
abstract class Parser[I, T] {  
  def parse(ts: I): Set[(T, I)]  
  
  def parse_all(ts: I): Set[T] =  
    for ((head, tail) <- parse(ts); if (tail.isEmpty))  
      yield head  
}
```

It is the obligation in each instance (parser combinator) to supply an implementation for `parse`. From this function we can then “centrally” derive the function `parse_all`, which just filters out all pairs whose second component is not empty (that is has still some unprocessed part). The reason is that at the end of the parsing we are only interested in the results where all the input has been consumed and no unprocessed part is left over.

One of the simplest parser combinators recognises just a single character, say `c`, from the beginning of strings. Its behaviour can be described as follows:

- If the head of the input string starts with a `c`, then return the set

$$\{(c, \textit{tail of } s)\}$$

where *tail of s* is the unprocessed part of the input string.

- Otherwise return the empty set `{}`.

The input type of this simple parser combinator for characters is `String` and the output type is `Char`. This means `parse` returns `Set[(Char, String)]`. The code in Scala is as follows:

```
case class CharParser(c: Char) extends Parser[String, Char] {
  def parse(in: String) =
    if (in.head == c) Set((c, in.tail)) else Set()
}
```

You can see the `parse` tests whether the first character of the input string `in` is equal to `c`. If yes, then it splits the string into the recognised part `c` and the unprocessed part `in.tail`. In case `in` does not start with `c` then the parser returns the empty set (in Scala `Set()`). Since this parser recognises characters and just returns characters as the processed part, the output type of the parser is `Char`.

If we want to parse a list of tokens and interested in recognising a number token, we could write something like this

```
case object NumParser extends Parser[List[Token], Int] {
  def parse(ts: List[Token]) = ts match {
    case Num_token(s)::ts => Set((s.toInt, ts))
    case _ => Set ()
  }
}
```

In this parser the input is of type `List[Token]`. The function `parse` looks at the input `ts` and checks whether the first token is a `Num_token`. Let us assume our lexer generated these tokens for numbers. But this parser does not just return this token (and the rest of the list), like the `CharParser` above, rather extracts the string `s` from the token and converts it into an integer. The hope is that the lexer did its work well and this conversion always succeeds. The consequence of this is that the output type for this parser is `Int`, not `Token`. Such a conversion would be needed if we want to implement a simple calculator program.

These simple parsers that just look at the input and do a simple transformation are often called *atomic* parser combinators. More interesting are the parser combinators that build larger parsers out of smaller component parsers. There are three such parser combinators that can be implemented generically. The *alternative parser combinator* is as follows: given two parsers, say, p and q , we apply both parsers to the input (remember parsers are functions) and combine the output (remember they are sets of pairs):

$$p(\text{input}) \cup q(\text{input})$$

In Scala we can implement alternative parser combinator as follows

```

class AltParser[I, T]
  (p: => Parser[I, T],
   q: => Parser[I, T]) extends Parser[I, T] {
  def parse(in: I) = p.parse(in) ++ q.parse(in)
}

```

The types of this parser combinator are again generic (we have I for the input type, and T for the output type). The alternative parser builds a new parser out of two existing parsers p and q given as arguments. Both parsers need to be able to process input of type I and return in parse the same output type Set[(T, I)].¹ The alternative parser should run the input with the first parser p (producing a set of pairs) and then run the same input with q (producing another set of pairs). The result should be then just the union of both sets, which is the operation ++ in Scala.

The alternative parser combinator allows us to construct a parser that parses either a character a or b using the CharParser shown above. For this we can write

```

new AltParser(CharParser('a'), CharParser('b'))

```

Later on we will use Scala mechanism for introducing some more readable shorthand notation for this, like "a" || "b". Let us look in detail at what this parser combinator produces with some sample strings

input strings	output
a c d e	→ {(a, c d e)}
b c d e	→ {(b, c d e)}
c c d e	→ {}

We receive in the first two cases a successful output (that is a non-empty set). In each case, either a or b is in the processed part, and cde in the unprocessed part. Clearly this parser cannot parse anything with ccde, therefore the empty set is returned.

A bit more interesting is the *sequence parser combinator*. Given two parsers, say again, p and q, we want to apply first the input to p producing a set of pairs; then apply q to all the unparsed parts in the pairs; and then combine the results. Mathematically we would write something like this for the expected set of pairs:

$$\{((output_1, output_2), u_2) \mid (output_1, u_1) \in p(input) \wedge (output_2, u_2) \in q(u_1)\}$$

¹There is an interesting detail of Scala, namely the => in front of the types of p and q. They will prevent the evaluation of the arguments before they are used. This is often called *lazy evaluation* of the arguments. We will explain this later.

Notice that the p will first be run on the input, producing pairs of the form $(output_1, u_1)$ where the u_1 stands for the unprocessed, or left-over, parts. We want that q runs on all these unprocessed parts u_1 . This again will produce some processed part, p and q , we apply both parsers to the input (remember parsers are functions) and combine the output (remember they are sets of pairs)

$$p(\text{input}) \cup q(\text{input})$$

In Scala we would implement alternative parser combinator as follows

```
class AltParser[I, T]
  (p: => Parser[I, T],
   q: => Parser[I, T]) extends Parser[I, T] {
  def parse(in: I) = p.parse(in) ++ q.parse(in)
}
```

The types of this parser combinator are again generic (we just have I for the input type, and T for the output type). The alternative parser builds a new parser out of two existing parsers p and q . Both need to be able to process input of type I and return the same output type $\text{Set}[(T, I)]$.² Therefore the output type of this parser is T . The alternative parser should run the input with the first parser p (producing a set of pairs) and then run the same input with q (producing another set of pairs). The result should be then just the union of both sets, which is the operation `++` in Scala.

The alternative parser combinator already allows us to construct a parser that parses either a character a or b , as

```
new AltParser(CharParser('a'), CharParser('b'))
```

Later on we will use again Scala mechanism for introducing some more readable shorthand notation for this, like `"a" | "b"`. Let us look in detail at what this parser combinator produces with some sample strings

input strings	output
a c d e	→ {(a, c d e)}
b c d e	→ {(b, c d e)}
c c d e	→ {}

We receive in the first two cases a successful output (that is a non-empty set). In each case, either a or b is in the processed part, and cde in the unprocessed part. Clearly this parser cannot parse anything in the string $ccde$, therefore the empty set.

²There is an interesting detail of Scala, namely the `=>` in front of the types of p and q . They will prevent the evaluation of the arguments before they are used. This is often called *lazy evaluation* of the arguments. We will explain this later.

A bit more interesting is the *sequence parser combinator*. Given two parsers, say again, p and q , we want to apply first the input to p producing a set of pairs; then apply q to all the unparsed parts in the pairs; and then combine the results like

$$\{(output_1, output_2), u_2 \mid (output_1, u_1) \in p(input) \wedge (output_2, u_2) \in q(u_1)\}$$

Notice that the p will first be run on the input, producing pairs of the form $output_1$ and unprocessed part u_1 . This unprocessed part is fed into the second parser q . The overall result of the sequence parser combinator is pairs of the form $((output_1, output_2), u_2)$. This means the unprocessed part of both parsers is the unprocessed part the second parser q produces leaves as left-over. The processed parts of both parsers is a pair consisting of the outputs of p and q , namely $(output_1, output_2)$. This behaviour can be implemented in Scala as follows:

```
class SeqParser[I, T, S]
  (p: => Parser[I, T],
   q: => Parser[I, S]) extends Parser[I, (T, S)] {
  def parse(in: I) =
    for ((output1, u1) <- p.parse(in);
         (output2, u2) <- q.parse(u1))
      yield ((output1, output2), u2)
}
```

This parser takes again as arguments two parsers, p and q . It implements `parse` as follows: let first run the parser p on the input producing a set of pairs $(output_1, u_1)$. The u_1 stands for the unprocessed parts left over by p . Let then q run on these unprocessed parts producing again a set of pairs. The output of the sequence parser combinator is then a set containing pairs where the first components are again pairs, namely what the first parser could parse together with what the second parser could parse; the second component is the unprocessed part left over after running the second parser q . Therefore the input type of the sequence parser combinator is as usual I , but the output type is

$$\text{Set}(((T, S), I))$$

If any of the runs of p and q fail, that is produce the empty set, then `parse` will also produce the empty set. Notice that we have now two output types for the sequence parser combinator, because in general p and q might produce different things (for example first we recognise a number and then a string corresponding to an operator).

With the shorthand notation we shall introduce later for the sequence parser combinator, we can write for example `"a" ~ "b"`, which is the parser combinator that first recognises the character `a` from a string and then `b`. Let us look again at three examples of how this parser combinator processes some strings:

input strings	output
a b c d e	→ {((a, b), c d e)}
b a c d e	→ {}
c c c d e	→ {}

In the first line we have a successful parse, because the string starts with `ab`, which is the prefix we are looking for. But since the parsing combinator is constructed as sequence of the two simple (atomic) parsers for `a` and `b`, the result is a nested pair of the form `((a, b), cde)`. It is *not* a simple pair `(ab, cde)` as one might erroneously expect. The parser returns the empty set in the other examples, because they do not fit with what the parser is supposed to parse.

A slightly more complicated parser is `("a" || "b") ~ "c"` which parses as first character either an `a` or `b`, followed by a `c`. This parser produces the following outputs.

input strings	output
a c d e	→ {((a, c), d e)}
b c d e	→ {((b, c), d e)}
a b d e	→ {}

Now consider the parser `("a" ~ "b") ~ "c"` which parses `a`, `b`, `c` in sequence. This parser produces the following outputs.

input strings	output
a b c d e	→ {(((a, b), c), d e)}
a b d e	→ {}
b c d e	→ {}

The second and third example fail, because something is “missing” in the sequence we are looking for. Also notice how the results nest with sequences: the parsed part is a nested pair of the form `((a, b), c)`. Two more examples: first consider the parser `("a" ~ "a") ~ "a"` and the input `aaaa`:

input string	output
a a a a	→ {(((a, a), a), a)}

Notice how again the results nest deeper and deeper as pairs (the last `a` is in the unprocessed part). To consume everything of this string we can use the parser `((("a" ~ "a") ~ "a") ~ "a"`. Then the output is as follows:

input string	output
a a a a	→ {((((a, a), a), a), "")}

This is an instance where the parser consumed completely the input, meaning the unprocessed part is just the empty string. So if we called `parse_all`, instead of `parse`, we would get back the result

```
{(((a, a), a), a)}
```

where the unprocessed (empty) parts have been stripped away from the pairs; everything where the second part was not empty has been thrown away as well, because they represent ultimately-unsuccessful-parses.

Note carefully that constructing a parser such `"a" || ("a" ~ "b")` will result in a typing error. The intention is that we want to parse an `a`, or an `a` followed by a `b`. However, the first parser has as output type a single character (recall the type of `CharParser`), but the second parser produces a pair of characters as output. The alternative parser is required to have both component parsers to have the same type—we need to be able to build the union of two sets, which means in Scala they need to be of the same type. We will see later how we can build this parser without the typing error.

The next parser combinator, called *semantic action*, does not actually combine smaller parsers, but applies a function to the result of a parser. It is implemented in Scala as follows

```
class FunParser[I, T, S]
  (p: => Parser[I, T],
   f: T => S) extends Parser[I, S] {
  def parse(in: I) =
    for ((head, tail) <- p.parse(in)) yield (f(head), tail)
}
```

This parser combinator takes a parser `p` with output type `T` as one argument as well as a function `f` with type `T => S`. The parser `p` produces sets of type `(T, I)`. The semantic action combinator then applies the function `f` to all the parser outputs. Since this function is of type `T => S`, we obtain a parser with output type `S`. Again Scala lets us introduce some shorthand notation for this parser combinator. Therefore we will write `p ==> f` for it.

What are semantic actions good for? Well, they allow us to transform the parsed input into a datastructure we can use for further processing. A simple example would be to transform parsed characters into ASCII numbers. Suppose we define a function `f` (from characters to ints) and use a `CharParser` for the character `c`.

```
val f = (c: Char) => c.toInt
val c = new CharParser('c')
```

Then we can run the following parsers on the input `cbd`:

```
c.parse("cbd")
(c ==> f).parse("cbd")
```

The first line we obtain the result `Set(('c', "bd"))`, whereas the second produces `Set((99, "bd"))`—the character has been transformed into an ASCII number.

A slightly less contrived example is about parsing numbers (recall `NumParser` above). However, we want to do this here for strings. For this assume we have the following `RegexParser`.

```
import scala.util.matching.Regex

case class RegexParser(reg: Regex) extends Parser[String, String] {
  def parse(in: String) = reg.findPrefixMatchOf(in) match {
    case None => Set()
    case Some(m) => Set((m.matched, m.after.toString))
  }
}
```

This parser takes a regex as argument and splits up a string into a prefix and the rest according to this regex (`reg.findPrefixMatchOf` generates a match—in the successful case—and the corresponding strings can be extracted with `matched` and `after`). We can now define a `NumParser` for strings as follows:

```
val NumParser = RegexParser("[0-9]+".r)
```

This parser will recognise a number at the beginning of a string, for example

```
NumParser.parse("123abc")
```

produces `Set((123, abc))`. The problem is that `123` is still a string. We need to convert it into the corresponding `Int`. We can do this as follows

```
(NumParser ==> (s => s.toInt)).parse("123abc")
```

The semantic action in form of a function converts a string into an `Int`. Let us come back to semantic actions when we are going to implement a simple calculator.

Shorthand notation for parser combinators

Before we proceed, let us just explain the shorthand notation for parser combinators. Like for regular expressions, the shorthand notation will make our life much easier when writing actual parsers.

How to build parsers using parser combinators?

The beauty of parser combinators is the ease with which they can be implemented and how easy it is to translate context-free grammars into code (though the grammars need to be non-left-recursive). To demonstrate this recall our context-free grammar for palindromes:

$$P ::= a \cdot P \cdot a \mid b \cdot P \cdot b \mid a \mid b \mid \epsilon$$

Given the parser combinators for alternatives and sequences, this grammar should be straightforward to implement. The first idea would be

```
lazy val Pal : Parser[String, String] =
  (("a" ~ Pal ~ "a") | ("b" ~ Pal ~ "b") | "a" | "b" | "")
```

Unfortunately, this does not work as it produces a typing error. The reason is that the parsers "a", "b" and "" all produce strings and therefore can be put into an alternative ... | "a" | "b" | "". But both "a" ~ Pal ~ "a" and "b" ~ Pal ~ "b" produce pairs of the form $((_,_),_)$ —that is how the sequence parser combinator nests results when ~ is used between two components. The solution is to use a semantic action that “flattens” these pairs and appends the corresponding strings, like

```
lazy val Pal : Parser[String, String] =
  (("a" ~ Pal ~ "a") ==> { case ((x, y), z) => x + y + z } |
  ("b" ~ Pal ~ "b") ==> { case ((x, y), z) => x + y + z } |
  "a" | "b" | "")
```

Implementing an Interpreter