

# Automata and Formal Languages (7)

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# CFGs

A **context-free** grammar (CFG)  $G$  consists of:

- a finite set of nonterminal symbols (upper case)
- a finite terminal symbols or tokens (lower case)
- a start symbol (which must be a nonterminal)
- a set of rules

$$A \rightarrow \text{rhs}_1 | \text{rhs}_2 | \dots$$

where **rhs** are sequences involving terminals and nonterminals (can also be empty).

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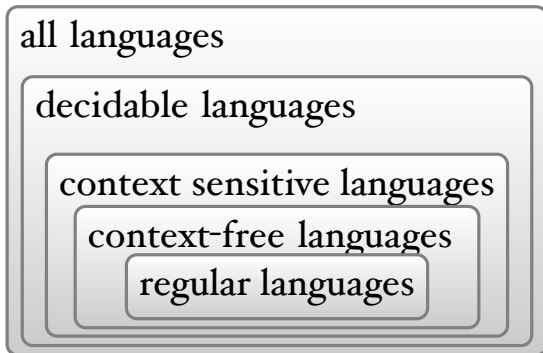
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# Hierarchy of Languages

Recall that languages are sets of strings.



# Arithmetic Expressions

A grammar for arithmetic expressions and numbers:

$$\begin{aligned} E &\rightarrow E \cdot + \cdot E \mid E \cdot * \cdot E \mid (\cdot E \cdot) \mid N \\ N &\rightarrow N \cdot N \mid 0 \mid 1 \mid \dots \mid 9 \end{aligned}$$

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# Numbers

$$N \rightarrow N \cdot N \mid 0 \mid 1 \mid \dots \mid 9$$

A non-left-recursive, non-ambiguous grammar for numbers:

$$N \rightarrow 0 \cdot N \mid 1 \cdot N \mid \dots \mid 0 \mid 1 \mid \dots \mid 9$$

# Operator Precedences

To disambiguate

$$E \rightarrow E \cdot + \cdot E \mid E \cdot * \cdot E \mid (\cdot E \cdot) \mid N$$

Decide on how many precedence levels, say  
highest for  $()$ , medium for  $*$ , lowest for  $+$

$$\begin{aligned} E_{low} &\rightarrow E_{med} \cdot + \cdot E_{low} \mid E_{med} \\ E_{med} &\rightarrow E_{hi} \cdot * \cdot E_{med} \mid E_{hi} \\ E_{hi} &\rightarrow (\cdot E_{low} \cdot) \mid N \end{aligned}$$



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What happens with  $1 + 3 + 4$ ?

# Removing Left-Recursion

The rule for numbers is directly left-recursive:

$$N \rightarrow N \cdot N \mid 0 \mid 1 \quad (\dots)$$

Translate

$$\begin{array}{l} N \rightarrow N \cdot \alpha \\ \quad \mid \beta \end{array} \quad \Rightarrow \quad \begin{array}{l} N \rightarrow \beta \cdot N' \\ N' \rightarrow \alpha \cdot N' \\ \quad \mid \epsilon \end{array}$$

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Which means

$$\begin{array}{l} N \rightarrow 0 \cdot N' \mid 1 \cdot N' \\ N' \rightarrow N \cdot N' \mid \epsilon \end{array}$$

# Chomsky Normal Form

All rules must be of the form

$$A \rightarrow a$$

or

$$A \rightarrow B \cdot C$$

No rule can contain  $\epsilon$ .

# $\epsilon$ -Removal

- 1 If  $A \rightarrow \alpha \cdot B \cdot \beta$  and  $B \rightarrow \epsilon$  are in the grammar, then add  $A \rightarrow \alpha \cdot \beta$  (iterate if necessary).
- 2 Throw out all  $B \rightarrow \epsilon$ .

$$\begin{aligned} N &\rightarrow o \cdot N' \mid I \cdot N' \\ N' &\rightarrow N \cdot N' \mid \epsilon \end{aligned}$$

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# CYK Algorithm

If grammar is in Chomsky normalform ...

$S \rightarrow N \cdot P$

$P \rightarrow V \cdot N$

$N \rightarrow N \cdot N$

$N \rightarrow \text{students} \mid \text{Jeff} \mid \text{geometry} \mid \text{trains}$

$V \rightarrow \text{trains}$

Jeff trains geometry students

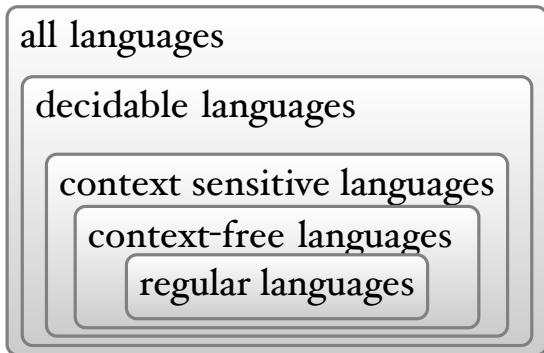
# CYK Algorithm

- fastest possible algorithm for recognition problem
- runtime is  $O(n^3)$
- grammars need to be transferred into CNF



# Hierarchy of Languages

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# Context Sensitive Grms

$$S \Rightarrow bSAA \mid \epsilon$$

$$A \Rightarrow a$$

$$bA \Rightarrow Ab$$

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$$S \Rightarrow \dots \Rightarrow? "ababaa"$$

*Stmt* → skip  
| *Id* := *AExp*  
| if *BExp* then *Block* else *Block*  
| while *BExp* do *Block*  
| read *Id*  
| write *Id*  
| write *String*

*Stmts* → *Stmt* ; *Stmts*  
| *Stmt*

*Block* → { *Stmts* }  
| *Stmt*

*AExp* → ...

*BExp* → ...

```
1 write "Fib";
2 read n;
3 minus1 := 0;
4 minus2 := 1;
5 while n > 0 do {
6     temp := minus2;
7     minus2 := minus1 + minus2;
8     minus1 := temp;
9     n := n - 1
10 };
11 write "Result";
12 write minus2
```

# An Interpreter

```
{  
   $x := 5;$   
   $y := x * 3;$   
   $y := x * 4;$   
   $x := u * 3$   
}
```

- the interpreter has to record the value of  $x$  before assigning a value to  $y$

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- `eval(stmt, env)`

# Interpreter

$\text{eval}(n, E)$	$\stackrel{\text{def}}{=} n$
$\text{eval}(x, E)$	$\stackrel{\text{def}}{=} E(x) \quad \text{lookup } x \text{ in } E$
$\text{eval}(a_1 + a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) + \text{eval}(a_2, E)$
$\text{eval}(a_1 - a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) - \text{eval}(a_2, E)$
$\text{eval}(a_1 * a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) * \text{eval}(a_2, E)$
$\text{eval}(a_1 = a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) = \text{eval}(a_2, E)$
$\text{eval}(a_1 \neq a_2, E)$	$\stackrel{\text{def}}{=} \neg(\text{eval}(a_1, E) = \text{eval}(a_2, E))$
$\text{eval}(a_1 < a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) < \text{eval}(a_2, E)$



# Interpreter (2)

$$\text{eval}(\text{skip}, E) \stackrel{\text{def}}{=} E$$

$$\text{eval}(x := a, E) \stackrel{\text{def}}{=} E(x \mapsto \text{eval}(a, E))$$

$$\begin{aligned} \text{eval}(\text{if } b \text{ then } cs_1 \text{ else } cs_2, E) &\stackrel{\text{def}}{=} \\ &\text{if } \text{eval}(b, E) \text{ then } \text{eval}(cs_1, E) \\ &\text{else } \text{eval}(cs_2, E) \end{aligned}$$

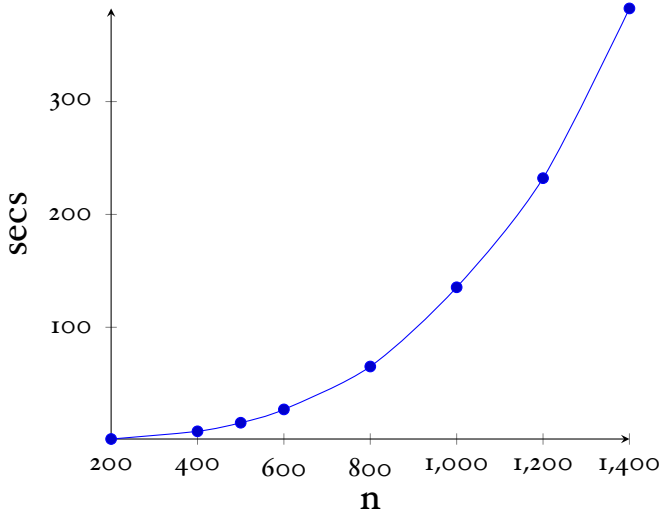
$$\begin{aligned} \text{eval}(\text{while } b \text{ do } cs, E) &\stackrel{\text{def}}{=} \\ &\text{if } \text{eval}(b, E) \\ &\text{then } \text{eval}(\text{while } b \text{ do } cs, \text{eval}(cs, E)) \\ &\text{else } E \end{aligned}$$

$$\text{eval}(\text{write } x, E) \stackrel{\text{def}}{=} \{ \text{println}(E(x)) ; E \}$$

# Test Program

```
1  start := 1000;
2  x := start;
3  y := start;
4  z := start;
5  while 0 < x do {
6    while 0 < y do {
7      while 0 < z do { z := z - 1 };
8      z := start;
9      y := y - 1
10   };
11  y := start;
12  x := x - 1
13 }
```

# Interpreted Code



# Java Virtual Machine

- introduced in 1995
- is a stack-based VM (like Postscript, CLR of .Net)
- contains a JIT compiler
- many languages take advantage of JVM's infrastructure (JRE)
- is garbage collected  $\Rightarrow$  no buffer overflows
- some languages compile to the JVM: Scala, Clojure...