

# Automata and Formal Languages (I)



Antikythera automaton, 100 BC (Archimedes?)

Email: christian.urban at kcl.ac.uk

Office: SI.27 (1st floor Strand Building)

Slides: KEATS

# The Goal of this Course

## A Compiler



# The Goal of this Course

lexer input: a string

```
"read(n);"
```

lexer output: a sequence of tokens

```
key(read); lpar; id(n); rpar; semi
```



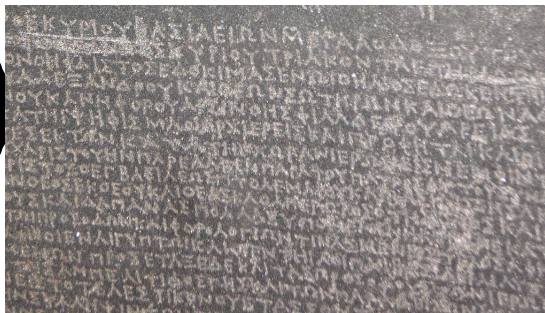
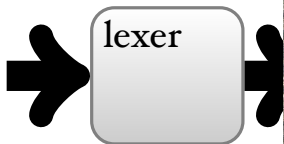
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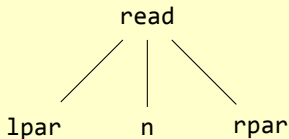


lexing  $\Rightarrow$  recognising words (Stone of Rosetta)

# The Goal of this Course

parser input: a sequence of token

parser output: an abstract syntax tree



# The Goal of this Course

code generator:

```
istore 2  
iload 2  
ldc 10  
isub  
ifeq Label2  
iload 2  
...
```

## Compiler



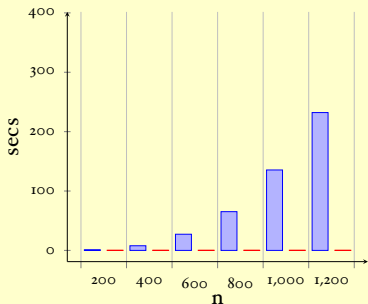
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## Compiler

parse



# The subject is quite old

- Turing Machines, 1936
- Regular Expressions, 1956
- The first compiler for COBOL, 1957  
(Grace Hopper)
- But surprisingly research papers are still published nowadays



Grace Hopper

(she made it to David Letterman's Tonight Show,

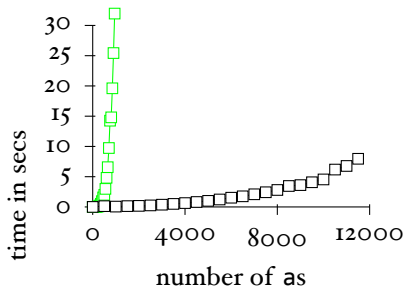
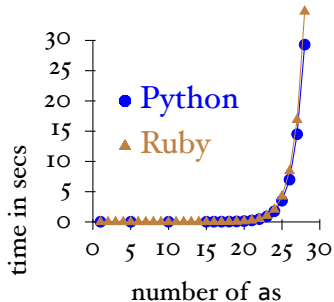
<http://www.youtube.com/watch?v=aZ0xtURhfEU>)



# Why Bother?

Ruby, Python  
and Others

Us (after next lecture)



matching  $[a?]\{n\}[a]\{n\}$  against  $\underbrace{a\dots a}_n$

# Lectures 1 - 5

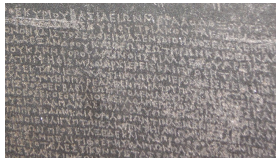
transforming strings into structured data

## Lexing

(recognising “words”)

## Parsing

(recognising “sentences”)



Stone of Rosetta

# Lectures 1 - 5

transforming strings into structured data

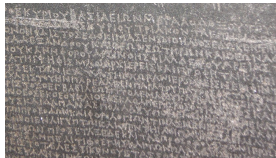
## Lexing

based on regular expressions

(recognising “words”)

## Parsing

(recognising “sentences”)



Stone of Rosetta

# Familiar Regular Expr.

`[a-z0-9_.-]+ @ [a-z0-9_.-]+ . [a-z.]{2,6}`

<code>re*</code>	matches 0 or more times
<code>re+</code>	matches 1 or more times
<code>re?</code>	matches 0 or 1 times
<code>re{n}</code>	matches exactly n number of times
<code>re{n,m}</code>	matches at least n and at most m times
<code>[...]</code>	matches any single character inside the brackets
<code>[^...]</code>	matches any single character not inside the brackets
<code>a-zA-Z</code>	character ranges
<code>\d</code>	matches digits; equivalent to <code>[0-9]</code>
<code>.</code>	matches every character except newline
<code>(re)</code>	groups regular expressions and remembers the matched text

# Today

- the ultimate goal is to implement a small compiler (a really small one for the JVM)

Let's start with:

- a web-crawler
- an email harvester
- a web-scraper

# A Web-Crawler

- 1 given an URL, read the corresponding webpage
- 2 extract all links from it
- 3 call the web-crawler again for all these links

# A Web-Crawler

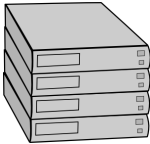
- ➊ given an URL, read the corresponding webpage
- ➋ if not possible print, out a problem
- ➌ if possible, extract all links from it
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- ➋ if not possible print, out a problem
- ➌ if possible, extract all links from it
- ➍ call the web-crawler again for all these links

(we need a bound for the number of recursive calls)  
(the purpose is to check all links on my own webpage)





Server

GET request



webpage



POST data



Browser

# Scala

A simple Scala function for reading webpages:

```
import io.Source

def get_page(url: String) : String = {
  Source.fromURL(url).take(10000).mkString
}
```

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get_page("""http://www.inf.kcl.ac.uk/staff/urbanc/""")
```

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}

get_page("""http://www.inf.kcl.ac.uk/staff/urbanc/""")
```

A slightly more complicated version for handling errors properly:

```
def get_page(url: String) : String = {
  Try(Source.fromURL(url).take(10000).mkString) getOrElse
    { println(s" Problem with: $url"); ""}
}
```

# Why Scala?

twitter 

Linked 

theguardian

Morgan Stanley

CREDIT SUISSE 



edf  
ENERGY

Novell.

foursquare™

HSBC 

...

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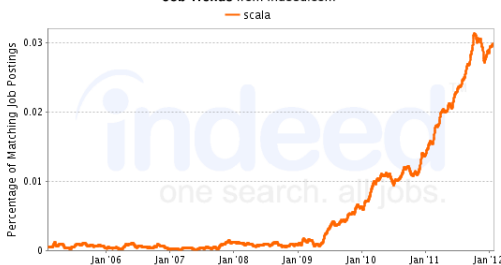
Novell

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Job Trends from Indeed.com



5 yrs {

- 2013: 1%
- 2014: 3%
- 2015: 9%
- 2016: 27%
- 2017: 81%
- 2018: 243% 😊

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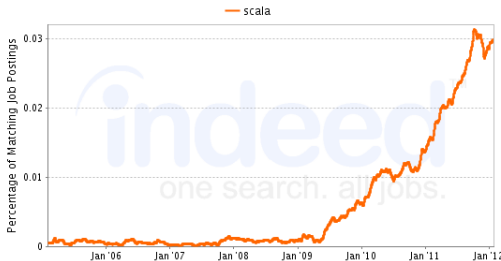
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**in London today:** 1 Scala job for every 30 Java jobs;  
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Job Trends from Indeed.com



Scala is a functional and object-oriented programming language; compiles to the JVM; does not need null-pointer exceptions; a course on Coursera

<http://www.scala-lang.org>

2016: 27%

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2018: 243% 😊

**in London today:** 1 Scala job for every 30 Java jobs;  
Scala programmers seem to get up to 20% better salary

# A Regular Expression

- ... is a pattern or template for specifying strings

```
"https?://[^\"]*"
```

matches for example

```
"http://www.foobar.com"
```

```
"https://www.tls.org"
```

# A Regular Expression

- ... is a pattern or template for specifying strings

```
""""https?://[^\"]*"""".r
```

matches for example

```
"http://www.foobar.com"
```

```
"https://www.tls.org"
```

# Finding Operations

**rexp.findAllIn(string)**

returns a list of all (sub)strings that match the regular expression

**rexp.findFirstIn(string)**

returns either

- None if no (sub)string matches or
- Some(s) with the first (sub)string

```

val http_pattern = """https?://[^\"]*""".r

def unquote(s: String) = s.drop(1).dropRight(1)

def get_all_URLs(page: String) : Set[String] =
  http_pattern.findAllIn(page).map(unquote).toSet

def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else {
    println(s"Visiting: $n $url")
    for (u <- get_all_URLs(get_page(url))) crawl(u, n - 1)
  }
}

crawl(some_start_URL, 2)

```

A version that only crawls links in “my” domain:

```
val my_urls = """urbanc""".r

def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else if (my_urls.findFirstIn(url) == None) {
    println(s"Visiting: $n $url")
    get_page(url); ()
  }
  else {
    println(s"Visiting: $n $url")
    for (u <- get_all_URLs(get_page(url))) crawl(u, n - 1)
  }
}
```

## A little email harvester:

```
val http_pattern = """https?://[^\"]*""".r
val email_pattern =
  """([a-z0-9_\. -]+)@([\da-z\.-]+)\.([a-z\.]{2,6})""".r

def print_str(s: String) =
  if (s == "") () else println(s)

def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else {
    println(s"Visiting: $n $url")
    val page = get_page(url)
    print_str(email_pattern.findAllIn(page).mkString("\n"))
    for (u <- get_all_URLs(page).par) crawl(u, n - 1)
  }
}
```

<http://net.tutsplus.com/tutorials/other/8-regular-expressions-you-should-know/>

# Regular Expressions

Their inductive definition:

$r ::=$	$\emptyset$	null
	$\epsilon$	empty string / "" / []
	$c$	character
	$r_1 + r_2$	alternative / choice
	$r_1 \cdot r_2$	sequence
	$r^*$	star (zero or more)



Th

```
abstract class Rexp
case object NULL extends Rexp
case object EMPTY extends Rexp
case class CHAR(c: Char) extends Rexp
case class ALT(r1: Rexp, r2: Rexp) extends Rexp
case class SEQ(r1: Rexp, r2: Rexp) extends Rexp
case class STAR(r: Rexp) extends Rexp
```

$r ::= \emptyset$	null
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$r^*$	star (zero or more)

# Regular Expressions

In Scala:

```
def OPT(r: Rexp) = ALT(r, EMPTY)
```

```
def NTIMES(r: Rexp, n: Int) : Rexp = n match {  
  case 0 => EMPTY  
  case 1 => r  
  case n => SEQ(r, NTIMES(r, n - 1))  
}
```

# Strings

...are lists of characters. For example "hello"

$[h, e, l, l, o]$  or just *hello*

the empty string:  $[]$  or ""

the concatenation of two strings:

$s_1 @ s_2$

$foo @ bar = foobar$ ,  $baz @ [] = baz$

# Languages, Strings

- **Strings** are lists of characters, for example

$[], abc$  (Pattern match:  $c::s$ )

- A **language** is a set of strings, for example

$\{[], bello, foobar, a, abc\}$

- **Concatenation** of strings and sets

$foo @ bar = foobar$

$A @ B \stackrel{\text{def}}{=} \{s_1 @ s_2 \mid s_1 \in A \wedge s_2 \in B\}$

# The Meaning of a Regular Expression

$$L(\emptyset) \stackrel{\text{def}}{=} \emptyset$$

$$L(\epsilon) \stackrel{\text{def}}{=} \{\epsilon\}$$

$$L(c) \stackrel{\text{def}}{=} \{[c]\}$$

$$L(r_1 + r_2) \stackrel{\text{def}}{=} L(r_1) \cup L(r_2)$$

$$L(r_1 \cdot r_2) \stackrel{\text{def}}{=} \{s_1 @ s_2 \mid s_1 \in L(r_1) \wedge s_2 \in L(r_2)\}$$

$$L(r^*) \stackrel{\text{def}}{=}$$

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$$L(r^*) \stackrel{\text{def}}{=} \{ \}$$

$$L(r)^\circ \stackrel{\text{def}}{=} \{ \}$$

$$L(r)^{n+1} \stackrel{\text{def}}{=} L(r) @ L(r)^n$$

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$$L(r^*) \stackrel{\text{def}}{=} \bigcup_{n \geq 0} L(r)^n$$

$$L(r)^\circ \stackrel{\text{def}}{=} \{\emptyset\}$$

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# The Meaning of Matching

A regular expression  $r$  matches a string  $s$  provided

$$s \in L(r)$$

...and the point of the next lecture is to decide this problem as fast as possible (unlike Python, Ruby)

# Written Exam

- Accounts for 75%.
- You will understand the question “*Is this relevant for the exam?*” is very demotivating for the lecturer!
- Deal: Whatever is in the homework (and is not marked “*optional*”) is relevant for the exam.
- Each lecture has also a handout. There are also handouts about notation and Scala.

# Coursework

- Accounts for 25%. Two strands. Choose **one!**

## Strand 1

- four programming subtasks:
  - matcher (5%, 16.10.)
  - lexer (5%, 06.11.)
  - parser (5%, 27.11.)
  - compiler (10%, 11.12.)

## Strand 2

- one task: prove the correctness of a regular expression matcher in the Isabelle theorem prover
- 25%, submission 11.12.

- Solving more than one strand will **not** give you more marks.
- The exam will contain in much, much smaller form elements from both (but will also be in lectures and HW).

**Questions?**