Compilers and Formal Languages

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Slides & Progs: KEATS (also homework is there)

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The Goal of this Course

Write a compiler

Today a lexer.

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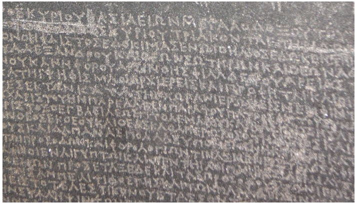

lexing $\Rightarrow$ recognising words (Stone of Rosetta)

Regular Expressions

In programming languages they are often used to recognise:

- operands, digits
- identifiers
- numbers (non-leading zeros)
- keywords
- comments

`http://www.regexper.com`

Lexing: Test Case

```
write "Fib";
read n;
minus1 := 0;
minus2 := 1;
while n > 0 do {
       temp := minus2;
       minus2 := minus1 + minus2;
       minus1 := temp;
       n := n - 1
};
write "Result";
write minus2
```

"if true then then 42 else +"

KEYWORD:

if, then, else,

WHITESPACE:

" ", \n,

IDENTIFIER:

$LETTER \cdot (LETTER + DIGIT + \_)^*$

NUM:

$(NONZERODIGIT \cdot DIGIT^*) + 0$

OP:

+, -, *, %, <, <=

COMMENT:

$/* \cdot \sim(ALL^* \cdot (*/) \cdot ALL^*) \cdot */$

```
"if true then then 42 else +"
```

```
KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)
```

```
"if true then then 42 else +"
```

```
KEYWORD(if),
IDENT(true),
KEYWORD(then),
KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)
```

There is one small problem with the tokenizer. How should we tokenize...?

"x-3"

```
ID: ...
OP:
  "+", "-"
NUM:
  (NONZERODIGIT · DIGIT*) + ''0''
NUMBER:
  NUM + ("-" · NUM)
```

The same problem with

$$(ab + a) \cdot (c + bc)$$

and the string abc.

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Or, keywords are **`if`** etc and identifiers are letters followed by "letters + numbers + _"*

`if`   `iffoo`

POSIX: Two Rules

Longest match rule (“maximal munch rule”): The longest initial substring matched by any regular expression is taken as the next token.

Rule priority: For a particular longest initial substring, the first regular expression that can match determines the token.

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traditional lexers are fast, but hairy

Sulzmann & Lu Matcher

We want to match the string abc using r_1:

$$r_1 \xrightarrow{der\ a} r_2$$

Sulzmann & Lu Matcher

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$$r_1 \xrightarrow{der\ a} r_2 \xrightarrow{der\ b} r_3$$

Sulzmann & Lu Matcher

We want to match the string *abc* using r_1:

$$r_1 \xrightarrow{der\ a} r_2 \xrightarrow{der\ b} r_3 \xrightarrow{der\ c} r_4$$

Sulzmann & Lu Matcher

We want to match the string *abc* using r_1:

$$r_1 \xrightarrow{der\ a} r_2 \xrightarrow{der\ b} r_3 \xrightarrow{der\ c} r_4 \ nullable?$$

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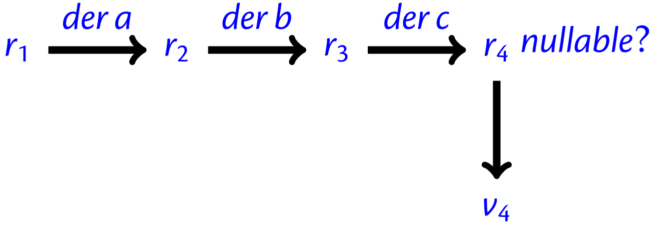
We want to match the string *abc* using r_1:



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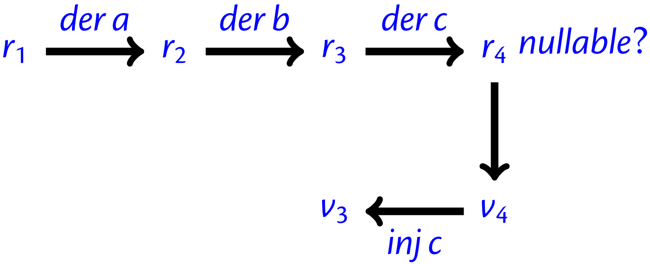

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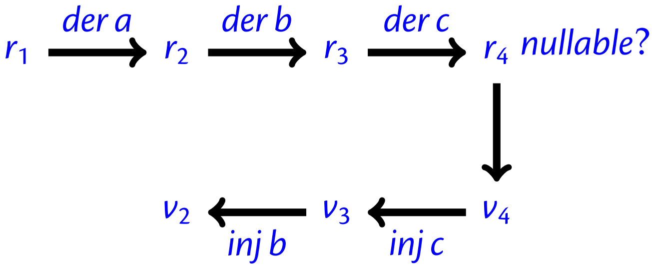

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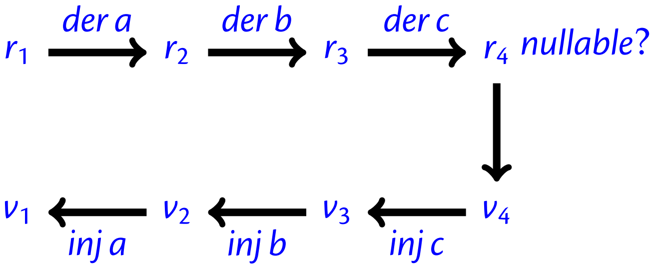

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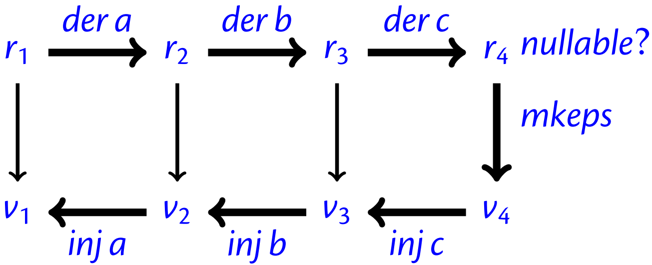


Regexes and Values

Regular expressions and their corresponding values:

$$
\begin{array}{lcl}
r & ::= & \mathbf{0} \\
 & | & \mathbf{1} \\
 & | & c \\
 & | & r_1 \cdot r_2 \\
 & | & r_1 + r_2 \\
 & & \\
 & | & r^*
\end{array}
\qquad
\begin{array}{lcl}
v & ::= & \\
 & & \mathit{Empty} \\
 & | & \mathit{Char}(c) \\
 & | & \mathit{Seq}(v_1, v_2) \\
 & | & \mathit{Left}(v) \\
 & | & \mathit{Right}(v) \\
 & | & \mathit{Stars}\,[] \\
 & | & \mathit{Stars}\,[v_1, \ldots\, v_n]
\end{array}
$$

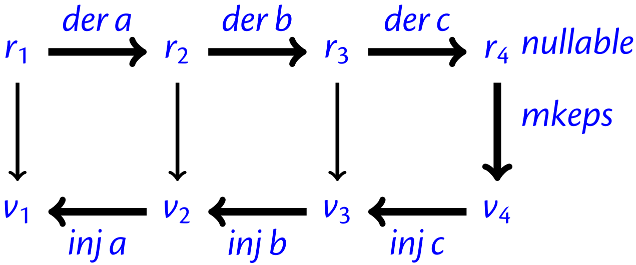
```scala
abstract class Rexp
case object ZERO extends Rexp
case object ONE extends Rexp
case class CHAR(c: Char) extends Rexp
case class ALT(r1: Rexp, r2: Rexp) extends Rexp
case class SEQ(r1: Rexp, r2: Rexp) extends Rexp
case class STAR(r: Rexp) extends Rexp

abstract class Val
case object Empty extends Val
case class Chr(c: Char) extends Val
case class Sequ(v1: Val, v2: Val) extends Val
case class Left(v: Val) extends Val
case class Right(v: Val) extends Val
case class Stars(vs: List[Val]) extends Val
```

r_1: $a \cdot (b \cdot c)$

r_2: $\mathbf{1} \cdot (b \cdot c)$

r_3: $(\mathbf{0} \cdot (b \cdot c)) + (\mathbf{1} \cdot c)$

r_4: $(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1})$



r_1: $a \cdot (b \cdot c)$
r_2: $\mathbf{1} \cdot (b \cdot c)$
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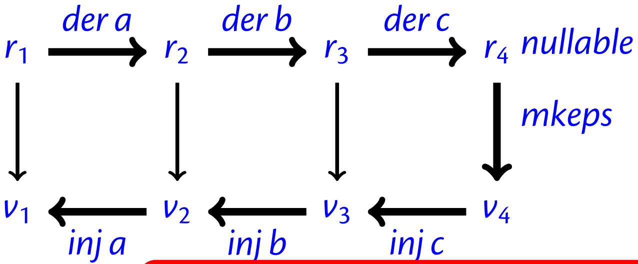


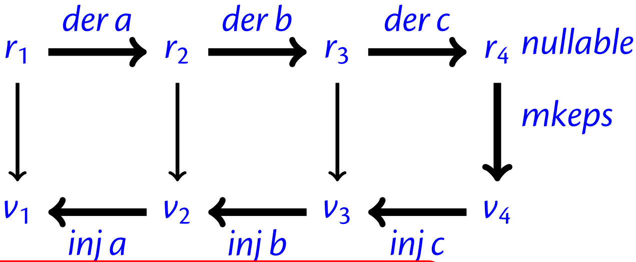
v_1: $Seq(Char(a), Seq(Char(b), Char(c)))$
v_2: $Seq(Empty, Seq(Char(b), Char(c)))$
v_3: $Right(Seq(Empty, Char(c)))$
v_4: $Right(Right(Empty))$

Flatten

Obtaining the string underlying a value:

$$
\begin{array}{lcl}
|Empty| & \stackrel{\text{def}}{=} & [] \\
|Char(c)| & \stackrel{\text{def}}{=} & [c] \\
|Left(v)| & \stackrel{\text{def}}{=} & |v| \\
|Right(v)| & \stackrel{\text{def}}{=} & |v| \\
|Seq(v_1, v_2)| & \stackrel{\text{def}}{=} & |v_1| \,@\, |v_2| \\
|Stars\,[v_1, \ldots, v_n]| & \stackrel{\text{def}}{=} & |v_1| \,@ \ldots @\, |v_n|
\end{array}
$$

r_1: $a \cdot (b \cdot c)$
r_2: $\mathbf{1} \cdot (b \cdot c)$
r_3: $(\mathbf{0} \cdot (b \cdot c)) + (\mathbf{1} \cdot c)$
r_4: $(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1})$



v_1: $Seq(Char(a), Seq(Char(b), Char(c)))$
v_2: $Seq(Empty, Seq(Char(b), Char(c)))$
v_3: $Right(Seq(Empty, Char(c)))$
v_4: $Right(Right(Empty))$

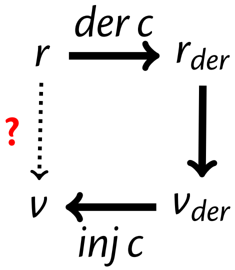
$|v_1|$: abc
$|v_2|$: bc
$|v_3|$: c
$|v_4|$: $[]$

Mkeps

Finding a (posix) value for recognising the empty string:

$$
\begin{array}{lcl}
mkeps\,(\mathbf{1}) & \stackrel{\text{def}}{=} & Empty \\
mkeps\,(r_1 + r_2) & \stackrel{\text{def}}{=} & \text{if } nullable(r_1) \\
 & & \text{then } Left(mkeps(r_1)) \\
 & & \text{else } Right(mkeps(r_2)) \\
mkeps\,(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & Seq(mkeps(r_1), mkeps(r_2)) \\
mkeps\,(r^*) & \stackrel{\text{def}}{=} & Stars\,[]
\end{array}
$$

Inject



Inject

Injecting ("Adding") a character to a value

$$
\begin{array}{lcl}
inj\,(c)\,c\,(Empty) & \stackrel{\text{def}}{=} & Char\,c\\
inj\,(r_1 + r_2)\,c\,(Left(v)) & \stackrel{\text{def}}{=} & Left(inj\,r_1\,c\,v)\\
inj\,(r_1 + r_2)\,c\,(Right(v)) & \stackrel{\text{def}}{=} & Right(inj\,r_2\,c\,v)\\
inj\,(r_1 \cdot r_2)\,c\,(Seq(v_1, v_2)) & \stackrel{\text{def}}{=} & Seq(inj\,r_1\,c\,v_1, v_2)\\
inj\,(r_1 \cdot r_2)\,c\,(Left(Seq(v_1, v_2))) & \stackrel{\text{def}}{=} & Seq(inj\,r_1\,c\,v_1, v_2)\\
inj\,(r_1 \cdot r_2)\,c\,(Right(v)) & \stackrel{\text{def}}{=} & Seq(mkeps(r_1), inj\,r_2\,c\,v)\\
inj\,(r^*)\,c\,(Seq(v, Stars\,vs)) & \stackrel{\text{def}}{=} & Stars\,(inj\,r\,c\,v\ ::\ vs)
\end{array}
$$

inj: 1st arg $\mapsto$ a rexp; 2nd arg $\mapsto$ a character; 3rd arg $\mapsto$ a value
result $\mapsto$ a value

$$inj\ (c)\ c\ (Empty) \stackrel{\text{def}}{=} Char\ c$$

$$
\begin{aligned}
inj\,(r_1 + r_2)\,c\,(Left(v)) &\stackrel{\text{def}}{=} Left(inj\,r_1\,c\,v)\\
inj\,(r_1 + r_2)\,c\,(Right(v)) &\stackrel{\text{def}}{=} Right(inj\,r_2\,c\,v)
\end{aligned}
$$

$$inj\,(r_1 \cdot r_2)\,c\,(Seq(v_1, v_2)) \stackrel{\text{def}}{=} Seq(inj\,r_1\,c\,v_1, v_2)$$
$$inj\,(r_1 \cdot r_2)\,c\,(Left(Seq(v_1, v_2))) \stackrel{\text{def}}{=} Seq(inj\,r_1\,c\,v_1, v_2)$$
$$inj\,(r_1 \cdot r_2)\,c\,(Right(v)) \stackrel{\text{def}}{=} Seq(mkeps(r_1), inj\,r_2\,c\,v)$$

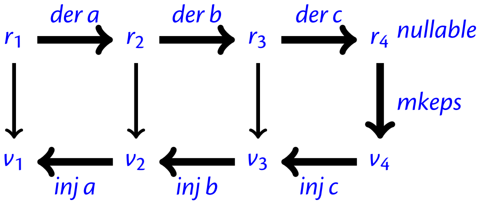
$$der\,c\,(r_1 \cdot r_2) \stackrel{\text{def}}{=} \textbf{if}\ nullable(r_1)\ \textbf{then}\ (der\,c\,r_1) \cdot r_2 + der\,c\,r_2\ \textbf{else}\ (der\,c\,r_1) \cdot r_2$$

$$inj\,(r^*)\,c\,(Seq(v, Stars\,vs)) \stackrel{\text{def}}{=} Stars\,(inj\,r\,c\,v\ ::\ vs)$$

Lexing

$$
\begin{aligned}
\mathit{lex}\; r\; [] &\stackrel{\text{def}}{=} \text{if } \mathit{nullable}(r) \text{ then } \mathit{mkeps}(r) \text{ else } \mathit{error} \\
\mathit{lex}\; r\; a :: s &\stackrel{\text{def}}{=} \mathit{inj}\; r\; a\; \mathit{lex}(\mathit{der}(a, r), s)
\end{aligned}
$$

lex: returns a value



Records

new regex: $(x : r)$ new value: $Rec(x, v)$

$(id : r_{id})$
$(key : r_{key})$

Records

new regex: $(x : r)$ new value: $Rec(x, v)$

$$nullable(x : r) \stackrel{\text{def}}{=} nullable(r)$$

$$der\,c\,(x : r) \stackrel{\text{def}}{=} der\,c\,r$$

$$mkeps(x : r) \stackrel{\text{def}}{=} Rec(x, mkeps(r))$$

$$inj\,(x : r)\,c\,v \stackrel{\text{def}}{=} Rec(x, inj\,r\,c\,v)$$

$(id : r_{id})$
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$$inj\, (x : r)\, c\, v \stackrel{\text{def}}{=} Rec(x, inj\, r\, c\, v)$$

for extracting subpatterns $(z : ((x : ab) + (y : ba))$

$(id : r_{id})$
$(key : r_{key})$

A regular expression for email addresses

$$(\text{name}: [a\text{-}z0\text{-}9\_\,.-]^{+}) \cdot @ \cdot$$
$$(\text{domain}: [a\text{-}z0\text{-}9\ -]^{+}) \cdot . \cdot$$
$$(\text{top\_level}: [a\text{-}z\ .]^{\{2,6\}})$$

`christian.urban@kcl.ac.uk`

the result environment:

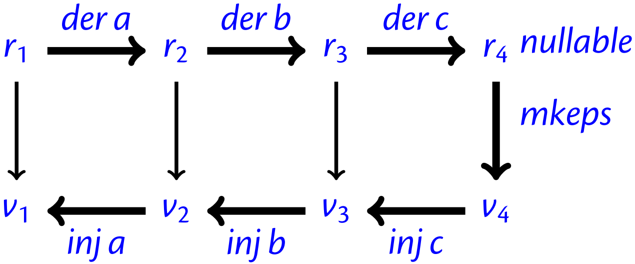
$$[(name : \texttt{christian.urban}),$$
$$(domain : \texttt{kcl}),$$
$$(top\_level : \texttt{ac.uk})]$$

While Tokens

$$\texttt{WHILE\_REGS} \stackrel{\text{def}}{=} \big((\texttt{"k"} : \texttt{KEYWORD}) + (\texttt{"i"} : \texttt{ID}) + (\texttt{"o"} : \texttt{OP}) + (\texttt{"n"} : \texttt{NUM}) + (\texttt{"s"} : \texttt{SEMI}) + (\texttt{"p"} : (\texttt{LPAREN} + \texttt{RPAREN})) + (\texttt{"b"} : (\texttt{BEGIN} + \texttt{END})) + (\texttt{"w"} : \texttt{WHITESPACE})\big)^*$$

Simplification

If we simplify after the derivative, then we are building the value for the simplified regular expression, but **not** for the original regular expression.



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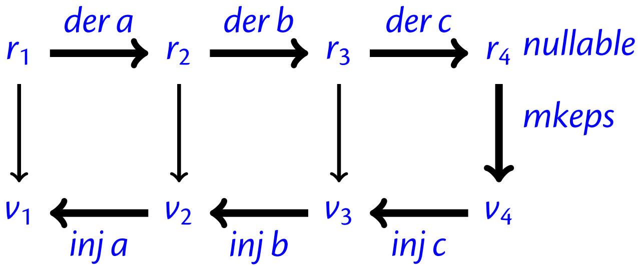

$$(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1}) \mapsto \mathbf{1}$$

Normally we would have

$$(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1})$$

and answer how this regular expression matches the empty string with the value

$$Right(Right(Empty))$$

But now we simplify this to $\mathbf{1}$ and would produce *Empty* (see *mkeps*).

Rectification

rectification functions:

$$
\begin{array}{lcll}
r \cdot \mathbf{0} & \mapsto & \mathbf{0} & \\
\mathbf{0} \cdot r & \mapsto & \mathbf{0} & \\
r \cdot \mathbf{1} & \mapsto & r & \lambda f_1\, f_2\, v.\, Seq(f_1\, v, f_2\, Empty) \\
\mathbf{1} \cdot r & \mapsto & r & \lambda f_1\, f_2\, v.\, Seq(f_1\, Empty, f_2\, v) \\
r + \mathbf{0} & \mapsto & r & \lambda f_1\, f_2\, v.\, Left(f_1\, v) \\
\mathbf{0} + r & \mapsto & r & \lambda f_1\, f_2\, v.\, Right(f_2\, v) \\
r + r & \mapsto & r & \lambda f_1\, f_2\, v.\, Left(f_1\, v)
\end{array}
$$

Rectification

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r \cdot \mathbf{0} & \mapsto & \mathbf{0} & \\
\mathbf{0} \cdot r & \mapsto & \mathbf{0} & \\
r \cdot \mathbf{1} & \mapsto & r & \lambda f_1 f_2 v.\, Seq(f_1\, v, f_2\, Empty) \\
\mathbf{1} \cdot r & \mapsto & r & \lambda f_1 f_2 v.\, Seq(f_1\, Empty, f_2\, v) \\
r + \mathbf{0} & \mapsto & r & \lambda f_1 f_2 v.\, Left(f_1\, v) \\
\mathbf{0} + r & \mapsto & r & \lambda f_1 f_2 v.\, Right(f_2\, v) \\
r + r & \mapsto & r & \lambda f_1 f_2 v.\, Left(f_1\, v)
\end{array}
$$

old *simp* returns a rexp;
new *simp* returns a rexp and a rectification function.

Rectification $\_ + \_$

$simp(r)$:

case $r = r_1 + r_2$

let $(r_{1s}, f_{1s}) = simp(r_1)$

$(r_{2s}, f_{2s}) = simp(r_2)$

case $r_{1s} = \mathbf{0}$: return $(r_{2s}, \lambda v.\, Right(f_{2s}(v)))$

case $r_{2s} = \mathbf{0}$: return $(r_{1s}, \lambda v.\, Left(f_{1s}(v)))$

case $r_{1s} = r_{2s}$: return $(r_{1s}, \lambda v.\, Left(f_{1s}(v)))$

otherwise: return $(r_{1s} + r_{2s}, f_{alt}(f_{1s}, f_{2s}))$

$$f_{alt}(f_1, f_2) \stackrel{\text{def}}{=}$$

$$\lambda v.\ \text{case } v = Left(v'):\ \text{return } Left(f_1(v'))$$

$$\text{case } v = Right(v'):\ \text{return } Right(f_2(v'))$$

```
def simp(r: Rexp): (Rexp, Val => Val) = r match {
  case ALT(r1, r2) => {
    val (r1s, f1s) = simp(r1)
    val (r2s, f2s) = simp(r2)
    (r1s, r2s) match {
      case (ZERO, _) => (r2s, F_RIGHT(f2s))
      case (_, ZERO) => (r1s, F_LEFT(f1s))
      case _ =>
         if (r1s == r2s) (r1s, F_LEFT(f1s))
         else (ALT (r1s, r2s), F_ALT(f1s, f2s))
    }
  }
  ...
}
def F_RIGHT(f: Val => Val) = (v:Val) => Right(f(v))
def F_LEFT(f: Val => Val) = (v:Val) => Left(f(v))
def F_ALT(f1: Val => Val, f2: Val => Val) =
  (v:Val) => v match {
    case Right(v) => Right(f2(v))
    case Left(v) => Left(f1(v)) }
```

Rectification $\_ \cdot \_$

$simp(r)$:...

case $r = r_1 \cdot r_2$

let $(r_{1s}, f_{1s}) = simp(r_1)$

$(r_{2s}, f_{2s}) = simp(r_2)$

case $r_{1s} = \mathbf{0}$: return $(\mathbf{0}, f_{error})$

case $r_{2s} = \mathbf{0}$: return $(\mathbf{0}, f_{error})$

case $r_{1s} = \mathbf{1}$: return $(r_{2s}, \lambda v.\, Seq(f_{1s}(Empty), f_{2s}(v)))$

case $r_{2s} = \mathbf{1}$: return $(r_{1s}, \lambda v.\, Seq(f_{1s}(v), f_{2s}(Empty)))$

otherwise: return $(r_{1s} \cdot r_{2s}, f_{seq}(f_{1s}, f_{2s}))$

$f_{seq}(f_1, f_2) \stackrel{\text{def}}{=}$

$\lambda v.$ case $v = Seq(v_1, v_2)$: return $Seq(f_1(v_1), f_2(v_2))$

```
def simp(r: Rexp): (Rexp, Val => Val) = r match {
  case SEQ(r1, r2) => {
    val (r1s, f1s) = simp(r1)
    val (r2s, f2s) = simp(r2)
    (r1s, r2s) match {
      case (ZERO, _) => (ZERO, F_ERROR)
      case (_, ZERO) => (ZERO, F_ERROR)
      case (ONE, _) => (r2s, F_SEQ_Empty1(f1s, f2s))
      case (_, ONE) => (r1s, F_SEQ_Empty2(f1s, f2s))
      case _ => (SEQ(r1s,r2s), F_SEQ(f1s, f2s))
    }
  }
  ...}
def F_SEQ_Empty1(f1: Val => Val, f2: Val => Val) =
  (v:Val) => Sequ(f1(Empty), f2(v))
def F_SEQ_Empty2(f1: Val => Val, f2: Val => Val) =
  (v:Val) => Sequ(f1(v), f2(Empty))
def F_SEQ(f1: Val => Val, f2: Val => Val) =
  (v:Val) => v match {
    case Sequ(v1, v2) => Sequ(f1(v1), f2(v2)) }
```

Rectification Example

$$(b \cdot c) + (\mathbf{0} + \mathbf{1}) \mapsto (b \cdot c) + \mathbf{1}$$

Rectification Example

$$(\underline{b \cdot c}) + (\underline{\mathbf{0} + \mathbf{1}}) \mapsto (b \cdot c) + \mathbf{1}$$

Rectification Example

$$(\underline{b \cdot c}) + (\underline{\mathbf{0} + \mathbf{1}}) \mapsto (b \cdot c) + \mathbf{1}$$

$$\begin{array}{lcl} f_{s1} & = & \lambda v.v \\ f_{s2} & = & \lambda v.Right(v) \end{array}$$

Rectification Example

$$\underline{(b \cdot c) + (\mathbf{0} + \mathbf{1})} \mapsto (b \cdot c) + \mathbf{1}$$

$$\begin{array}{rcl} f_{s1} & = & \lambda v.v \\ f_{s2} & = & \lambda v.Right(v) \end{array}$$

$$\begin{array}{l} f_{alt}(f_{s1}, f_{s2}) \stackrel{\text{def}}{=} \\ \quad \lambda v.\ \text{case } v = Left(v'):\quad \text{return } Left(f_{s1}(v')) \\ \qquad\quad \text{case } v = Right(v'): \text{return } Right(f_{s2}(v')) \end{array}$$

Rectification Example

$$\underline{(b \cdot c) + (\mathbf{0} + \mathbf{1})} \mapsto (b \cdot c) + \mathbf{1}$$

$$\begin{array}{lcl} f_{s1} & = & \lambda v.v \\ f_{s2} & = & \lambda v.Right(v) \end{array}$$

$$\lambda v. \begin{array}{l} \text{case } v = Left(v')\text{:} \quad \text{return } Left(v') \\ \text{case } v = Right(v')\text{: return } Right(Right(v')) \end{array}$$

Rectification Example

$$\underline{(b \cdot c) + (\mathbf{0} + \mathbf{1})} \mapsto (b \cdot c) + \mathbf{1}$$

$$\begin{array}{rcl} f_{s1} & = & \lambda v.v \\ f_{s2} & = & \lambda v.Right(v) \end{array}$$

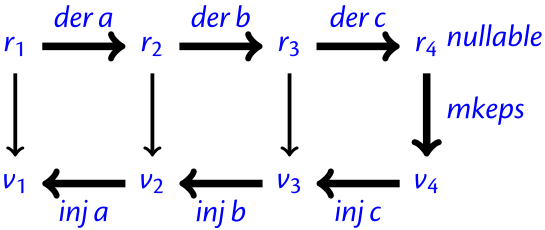
$$\begin{array}{ll} \lambda v. & \text{case } v = Left(v')\text{:} \quad \text{return } Left(v') \\ & \text{case } v = Right(v')\text{: return } Right(Right(v')) \end{array}$$

mkeps simplified case: $Right(Empty)$
rectified case: $Right(Right(Empty))$

Lexing with Simplification

$$lex\ r\ [] \stackrel{\text{def}}{=} \text{if } nullable(r) \text{ then } mkeps(r) \text{ else } error$$

$$lex\ r\ c :: s \stackrel{\text{def}}{=} \text{let } (r', frect) = simp(der(c, r))$$
$$\qquad inj\ r\ c\ (frect(lex(r', s)))$$



Environments

Obtaining the "recorded" parts of a value:

$$
\begin{array}{lcl}
env(Empty) & \stackrel{\text{def}}{=} & [] \\
env(Char(c)) & \stackrel{\text{def}}{=} & [] \\
env(Left(v)) & \stackrel{\text{def}}{=} & env(v) \\
env(Right(v)) & \stackrel{\text{def}}{=} & env(v) \\
env(Seq(v_1, v_2)) & \stackrel{\text{def}}{=} & env(v_1) \,@\, env(v_2) \\
env(Stars\,[v_1, \ldots, v_n]) & \stackrel{\text{def}}{=} & env(v_1) \,@\ldots @\, env(v_n) \\
env(Rec(x : v)) & \stackrel{\text{def}}{=} & (x : |v|) :: env(v)
\end{array}
$$

While Tokens

$$
\begin{aligned}
\texttt{WHILE\_REGS} \stackrel{\text{def}}{=} (&(\texttt{"k"} : \texttt{KEYWORD}) + \\
&(\texttt{"i"} : \texttt{ID}) + \\
&(\texttt{"o"} : \texttt{OP}) + \\
&(\texttt{"n"} : \texttt{NUM}) + \\
&(\texttt{"s"} : \texttt{SEMI}) + \\
&(\texttt{"p"} : (\texttt{LPAREN} + \texttt{RPAREN})) + \\
&(\texttt{"b"} : (\texttt{BEGIN} + \texttt{END})) + \\
&(\texttt{"w"} : \texttt{WHITESPACE}))^*
\end{aligned}
$$

```
"if true then then 42 else +"
```

```
KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)
```

```
"if true then then 42 else +"
```

```
KEYWORD(if),
IDENT(true),
KEYWORD(then),
KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)
```

Lexer: Two Rules

Longest match rule ("maximal munch rule"): The longest initial substring matched by any regular expression is taken as next token.

Rule priority: For a particular longest initial substring, the first regular expression that can match determines the token.

Environments

Obtaining the "recorded" parts of a value:

$$
\begin{array}{lcl}
env(Empty) & \stackrel{\text{def}}{=} & [] \\
env(Char(c)) & \stackrel{\text{def}}{=} & [] \\
env(Left(v)) & \stackrel{\text{def}}{=} & env(v) \\
env(Right(v)) & \stackrel{\text{def}}{=} & env(v) \\
env(Seq(v_1, v_2)) & \stackrel{\text{def}}{=} & env(v_1) \mathbin{@} env(v_2) \\
env(Stars[v_1, \ldots, v_n]) & \stackrel{\text{def}}{=} & env(v_1) \mathbin{@} \ldots \mathbin{@} env(v_n) \\
env(Rec(x : v)) & \stackrel{\text{def}}{=} & (x : |v|) :: env(v)
\end{array}
$$

While Tokens

$$
\begin{aligned}
\texttt{WHILE\_REGS} \stackrel{\text{def}}{=} \big((&\texttt{"k"} : \texttt{KEYWORD}) + \\
(&\texttt{"i"} : \texttt{ID}) + \\
(&\texttt{"o"} : \texttt{OP}) + \\
(&\texttt{"n"} : \texttt{NUM}) + \\
(&\texttt{"s"} : \texttt{SEMI}) + \\
(&\texttt{"p"} : (\texttt{LPAREN} + \texttt{RPAREN})) + \\
(&\texttt{"b"} : (\texttt{BEGIN} + \texttt{END})) + \\
(&\texttt{"w"} : \texttt{WHITESPACE})\big)^*
\end{aligned}
$$

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"if true then then 42 else +"
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KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)
```

```
"if true then then 42 else +"
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```
KEYWORD(if),
IDENT(true),
KEYWORD(then),
KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)
```

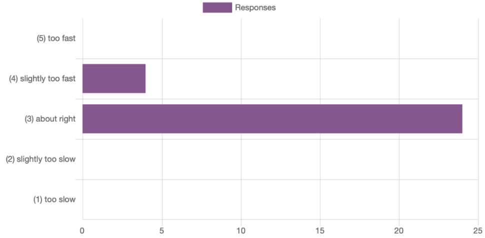
Are dfas completed by definition as in do they have a to have transitions for every char at every state?

How can you tell if a language will be regular or irregular?

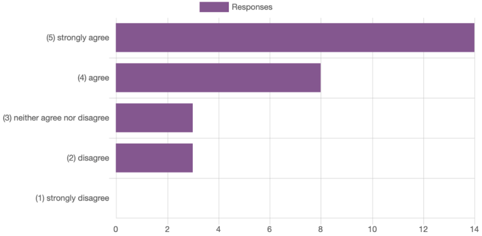
- contentwise probably the most enjoyable module I have had so far at KCL
- I personally found the coursework sheets a bit unclear. For example I couldnt see what was required from the CFUN section but once explained it actually was very easy and didnt take long to get working
- Please can tutorial sessions be recorded & linked on Keats.
- One of the best taught modules I've had, with a genuinely interested and engaging lecturer. Thanks Dr. Urban!

- Dr. Urban is honestly a great lecturer, he's incredibly helpful and responsive. He also teaches at a very good pace and explains things clearly so students who sometimes struggle with the content like myself can keep up. It's a pleasure to learn from him.
- I just wish the Coursework content was explained better, in such a way that allows students to get on with the coursework as soon as possible.
- I'm thoroughly enjoying the module so far. I find that the handouts and homework really solidify my knowledge and the feedback is extremely useful. I enjoy doing the homework and then using the feedback alongside the workshop to correct where I have gone wrong.
- I enjoy this module and think it is taught well. The homework is very useful.

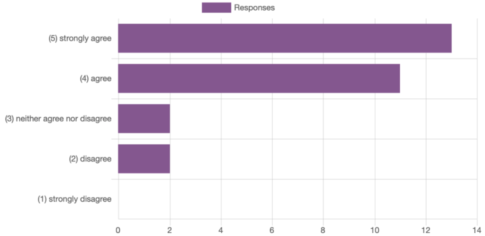
(AppropriatePace) ...teaches at a pace that is:



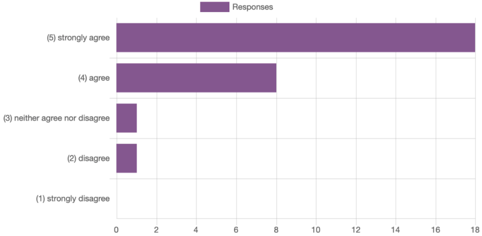
(ExplainsMaterialClearly) ...explains the material clearly



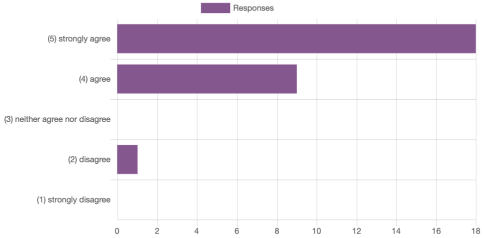
(contemporary) ..makes clear the contemporary relevance of the subject



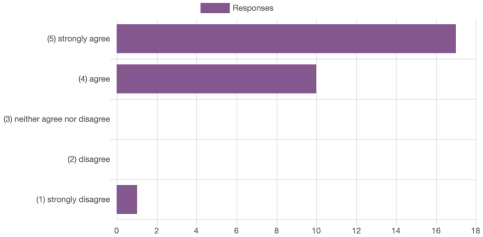
(keats) ...provides useful information on KEATS



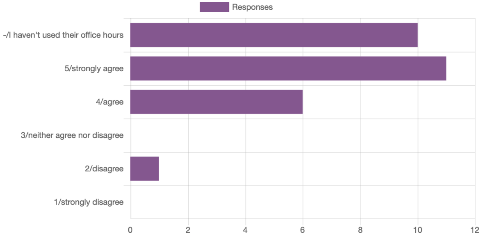
(objectives) ...has (have) made the module objectives clear



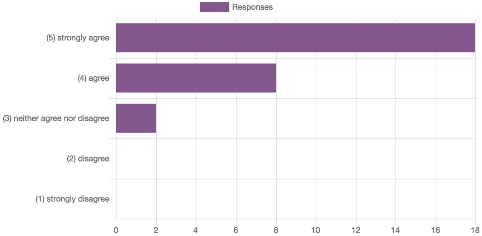
(amethods) ...has (have) made the assessment methods clear

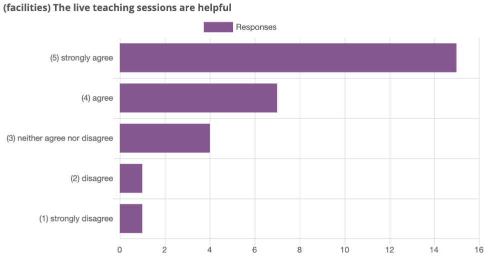


(forum) ...is available to answer questions on the discussion forum:



(Audible) The video lectures and other content on KEATS are helpful




(facilities) The live teaching sessions are helpful
Responses
(5) strongly agree
(4) agree
(3) neither agree nor disagree
(2) disagree
(1) strongly disagree
0
2
4
6
8
10
12
14
16