Homework 5

1. Consider the basic regular expressions

$$r ::= \varnothing \mid \epsilon \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

   and suppose you want to show a property $P(r)$ for all regular expressions r by structural induction. Write down which cases do you need to analyse. State clearly the induction hypotheses if applicable in a case.

2. Define a regular expression, written ALL, that can match every string. This definition should be in terms of the following extended regular expressions:

$$r ::= \varnothing \mid \epsilon \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^* \mid \; \sim r$$

3. Define the following regular expressions

r^+	(one or more matches)
$r^?$	(zero or one match)
$r^{\{n\}}$	(exactly n matches)
$r^{\{m,n\}}$	(at least m and maximal n matches, with the assumption $m \leq n$)

   in terms of the usual basic regular expressions

$$r ::= \varnothing \mid \epsilon \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

4. Give the regular expressions for lexing a language consisting of identifiers, left-parenthesis `(`, right-parenthesis `)`, numbers that can be either positive or negative, and the operations `+`, `-` and `*`.

   Decide whether the following strings can be lexed in this language?

   (a) `"(a3+3)*b"`
   (b) `")()++-33"`
   (c) `"(b42/3)*3"`

   In case they can, give the corresponding token sequences. (Hint: Observe the maximal munch rule and the priorities of your regular expressions that make the process of lexing unambiguous.)

5. (Optional) Recall the definitions for *Der* and *der* from the lectures. Prove by induction on r the property that

$$L(der\, c\, r) = Der\, c\, (L(r))$$

   holds.