

Automata and Formal Languages (I)



Antikythera automaton, 100 BC (Archimedes?)

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Slides: KEATS

The Goal of this Course

A Compiler



The Goal of this Course

lexer input: a string

```
"read(n);"
```

lexer output: a sequence of tokens

```
key(read); lpar; id(n); rpar; semi
```



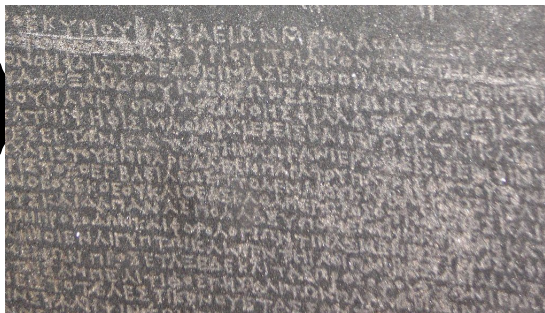
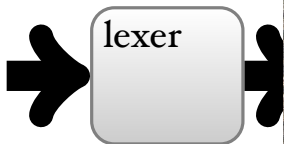
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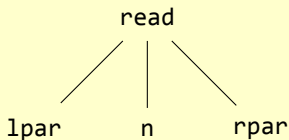


lexing \Rightarrow recognising words (Stone of Rosetta)

The Goal of this Course

parser input: a sequence of token

parser output: an abstract syntax tree



The Goal of this Course

code generator:

```
istore 2  
iload 2  
ldc 10  
isub  
ifeq Label2  
iload 2  
...
```

Compiler



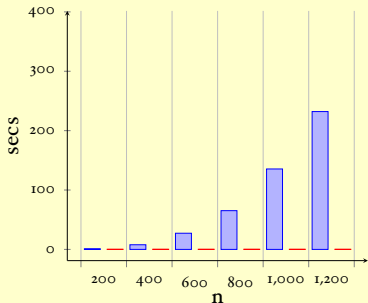
The Goal of this Course

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...
```

Compiler

parse



The subject is quite old

- Turing Machines, 1936
- Regular Expressions, 1956
- The first compiler for COBOL, 1957
(Grace Hopper)
- But surprisingly research papers are still published nowadays



Grace Hopper

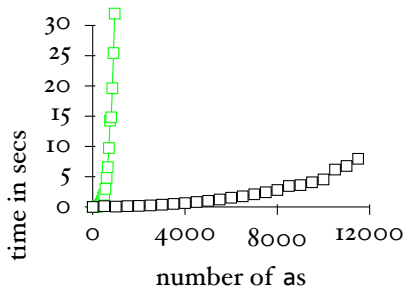
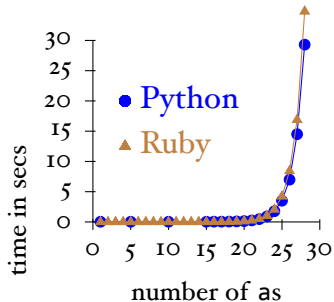
(she made it to David Letterman's Tonight Show,

<http://www.youtube.com/watch?v=aZ0xtURhfEU>)

Why Bother?

Ruby, Python
and Others

Us (after next lecture)



matching $[a?]\{n\}[a]\{n\}$ against $\underbrace{a\dots a}_n$

Lectures 1 - 5

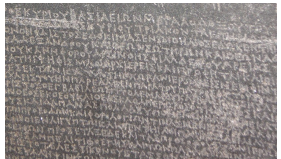
transforming strings into structured data

Lexing

(recognising “words”)

Parsing

(recognising “sentences”)



Stone of Rosetta

Lectures 1 - 5

transforming strings into structured data

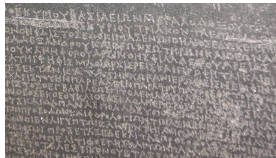
Lexing

based on regular expressions

(recognising “words”)

Parsing

(recognising “sentences”)



Stone of Rosetta

Familiar Regular Expr.

`[a-z0-9_.-]+ @ [a-z0-9_.-]+ . [a-z.]{2,6}`

<code>re*</code>	matches 0 or more times
<code>re+</code>	matches 1 or more times
<code>re?</code>	matches 0 or 1 times
<code>re{n}</code>	matches exactly n number of times
<code>re{n,m}</code>	matches at least n and at most m times
<code>[...]</code>	matches any single character inside the brackets
<code>[^...]</code>	matches any single character not inside the brackets
<code>a-zA-Z</code>	character ranges
<code>\d</code>	matches digits; equivalent to <code>[0-9]</code>
<code>.</code>	matches every character except newline
<code>(re)</code>	groups regular expressions and remembers the matched text

Today

- While the ultimate goal is to implement a small compiler (a really small one for the JVM)...

Let's start with:

- a web-crawler
- an email harvester
- (a web-scraper)

A Web-Crawler

- 1 given an URL, read the corresponding webpage
- 2 extract all links from it
- 3 call the web-crawler again for all these links

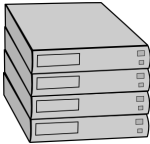
A Web-Crawler

- ➊ given an URL, read the corresponding webpage
- ➋ if not possible print, out a problem
- ➌ if possible, extract all links from it
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A Web-Crawler

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(we need a bound for the number of recursive calls)
(the purpose is to check all links on my own webpage)



Server

GET request



webpage



POST data



Browser

Scala

A simple Scala function for reading webpages:

```
import io.Source

def get_page(url: String) : String = {
  Source.fromURL(url).take(10000).mkString
}
```

Scala

A simple Scala function for reading webpages:

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import io.Source

def get_page(url: String) : String = {
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}

get_page("""http://www.inf.kcl.ac.uk/staff/urbanc/""")
```

Scala

A simple Scala function for reading webpages:

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import io.Source

def get_page(url: String) : String = {
  Source.fromURL(url).take(10000).mkString
}

get_page("""http://www.inf.kcl.ac.uk/staff/urbanc/""")
```

A slightly more complicated version for handling errors properly:

```
def get_page(url: String) : String = {
  Try(Source.fromURL(url).take(10000).mkString) getOrElse
    { println(s" Problem with: $url"); ""}
}
```

Why Scala?

twitter 

Linked 

theguardian

Morgan Stanley

CREDIT SUISSE 



edf
ENERGY

Novell.

foursquare™

HSBC 

...

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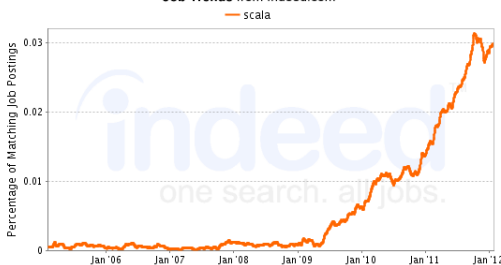
Novell.

foursquare™

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...

Job Trends from Indeed.com



5 yrs {
2013: 1%
2014: 3%
2015: 9%
2016: 27%
2017: 81%
2018: 243% 😊

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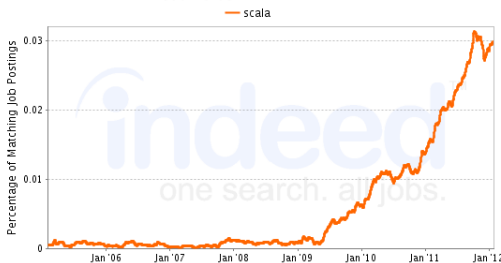
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in London today: 1 Scala job for every 30 Java jobs;
Scala programmers seem to get up to 20% better salary

Why Scala?

twitter 

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CREDIT S


edf
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Novel

foursquare

HSBC 

...

Job Trends from Indeed.com



Scala is a functional and object-oriented programming language; compiles to the JVM; does not need null-pointer exceptions; a course on Coursera

<http://www.scala-lang.org>

2016: 27%

2017: 81%

2018: 243% 😊

in London today: 1 Scala job for every 30 Java jobs;
Scala programmers seem to get up to 20% better salary

A Regular Expression

- ... is a pattern or template for specifying strings

```
"https?://[^\"]*"
```

matches for example

```
"http://www.foobar.com"
```

```
"https://www.tls.org"
```

A Regular Expression

- ... is a pattern or template for specifying strings

```
""""https?://[^\"]*"""".r
```

matches for example

```
"http://www.foobar.com"
```

```
"https://www.tls.org"
```

Finding Operations

rexp.findAllIn(string)

returns a list of all (sub)strings that match the regular expression

rexp.findFirstIn(string)

returns either

- None if no (sub)string matches or
- Some(s) with the first (sub)string

```
val http_pattern = """https?://[^\"]*""".r

def unquote(s: String) = s.drop(1).dropRight(1)

def get_all_URLs(page: String) : Set[String] =
  http_pattern.findAllIn(page).map(unquote).toSet

def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else {
    println(s"Visiting: $n $url")
    for (u <- get_all_URLs(get_page(url))) crawl(u, n - 1)
  }
}

crawl(some_start_URL, 2)
```

A version that only crawls links in “my” domain:

```
val my_urls = """urbanc""".r

def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else if (my_urls.findFirstIn(url) == None) {
    println(s"Visiting: $n $url")
    get_page(url); ()
  }
  else {
    println(s"Visiting: $n $url")
    for (u <- get_all_URLs(get_page(url))) crawl(u, n - 1)
  }
}
```

A little email harvester:

```
val http_pattern = """https?://[^\"]*""".r
val email_pattern =
  """([a-z0-9_\.-]+)@([\da-z\.-]+)\.([a-z\.-]{2,6})""".r

def print_str(s: String) =
  if (s == "") () else println(s)

def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else {
    println(s"Visiting: $n $url")
    val page = get_page(url)
    print_str(email_pattern.findAllIn(page).mkString("\n"))
    for (u <- get_all_URLs(page).par) crawl(u, n - 1)
  }
}
```

<http://net.tutsplus.com/tutorials/other/8-regular-expressions-you-should-know/>

Regular Expressions

Their inductive definition:

$r ::=$	\emptyset	null
	ϵ	empty string / "" / []
	c	character
	$r_1 + r_2$	alternative / choice
	$r_1 \cdot r_2$	sequence
	r^*	star (zero or more)

Th

```
abstract class Rexp
case object NULL extends Rexp
case object EMPTY extends Rexp
case class CHAR(c: Char) extends Rexp
case class ALT(r1: Rexp, r2: Rexp) extends Rexp
case class SEQ(r1: Rexp, r2: Rexp) extends Rexp
case class STAR(r: Rexp) extends Rexp
```

$r ::= \emptyset$	null
ϵ	empty string / "" / []
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Regular Expressions

In Scala:

```
def OPT(r: Rexp) = ALT(r, EMPTY)
```

```
def NTIMES(r: Rexp, n: Int) : Rexp = n match {  
  case 0 => EMPTY  
  case 1 => r  
  case n => SEQ(r, NTIMES(r, n - 1))  
}
```

Strings

...are lists of characters. For example "hello"

$[h, e, l, l, o]$ or just *hello*

the empty string: $[]$ or ""

the concatenation of two strings:

$s_1 @ s_2$

$foo @ bar = foobar$, $baz @ [] = baz$

Languages, Strings

- **Strings** are lists of characters, for example

$[], abc$ (Pattern match: $c::s$)

- A **language** is a set of strings, for example

$\{[], bello, foobar, a, abc\}$

- **Concatenation** of strings and languages

$foo @ bar = foobar$

$A @ B \stackrel{\text{def}}{=} \{s_1 @ s_2 \mid s_1 \in A \wedge s_2 \in B\}$

The Meaning of a Regular Expression

$$L(\emptyset) \stackrel{\text{def}}{=} \emptyset$$

$$L(\epsilon) \stackrel{\text{def}}{=} \{\epsilon\}$$

$$L(c) \stackrel{\text{def}}{=} \{[c]\}$$

$$L(r_1 + r_2) \stackrel{\text{def}}{=} L(r_1) \cup L(r_2)$$

$$L(r_1 \cdot r_2) \stackrel{\text{def}}{=} \{s_1 @ s_2 \mid s_1 \in L(r_1) \wedge s_2 \in L(r_2)\}$$

$$L(r^*) \stackrel{\text{def}}{=}$$

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$$\{s_1 @ s_2 \mid s_1 \in L(r) \wedge s_2 \in L(r)^n\}$$

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$$L(r^*) \stackrel{\text{def}}{=} \bigcup_{n \geq 0} L(r)^n$$

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The Meaning of Matching

A regular expression r matches a string s provided

$$s \in L(r)$$

...and the point of the next lecture is to decide this problem as fast as possible (unlike Python, Ruby)

Written Exam

- Accounts for 75%.
- You will understand the question “*Is this relevant for the exam?*” is very demotivating for the lecturer!
- Deal: Whatever is in the homework (and is not marked “*optional*”) is relevant for the exam.
- Each lecture has also a handout. There are also handouts about notation and Scala.

Coursework

- Accounts for 25%. Two strands. Choose **one!**

Strand 1

- four programming subtasks:
 - matcher (5%, 16.10.)
 - lexer (5%, 06.11.)
 - parser (5%, 27.11.)
 - compiler (10%, 11.12.)

Strand 2

- one task: prove the correctness of a regular expression matcher in the Isabelle theorem prover
- 25%, submission 11.12.

- Solving more than one strand will **not** give you more marks.
- The exam will contain in much, much smaller form elements from both (but will also be in lectures and HW).

Questions?