Automata and Formal Languages (5)

Email: christian.urban at kcl.ac.uk
Office: S1.27 (1st floor Strand Building)
Slides: KEATS (also home work is there)

Deterministic Finite Automata

A DFA $A(Q, q_0, F, \delta)$ consists of:

- a finite set of states Q
- one of these states is the start state q_0
- some states are accepting states F
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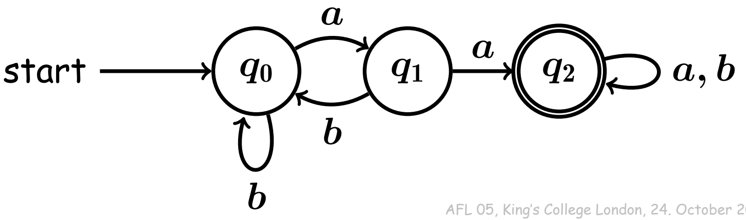

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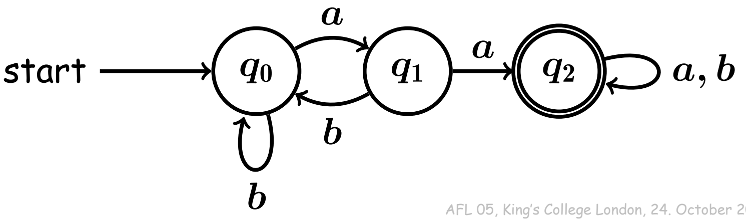

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$$L(A) \stackrel{\text{def}}{=} \{s \mid \hat{\delta}(q_0, s) \in F\}$$

Non-Deterministic Finite Automata

An NFA $A(Q, q_0, F, \delta)$ consists again of:

- a finite set of states
- one of these states is the start state
- some states are accepting states
- a transition relation

$$(q_1, a) \rightarrow q_2 \qquad (q_1, \epsilon) \rightarrow q_2$$
$$(q_1, a) \rightarrow q_3$$

Non-Deterministic Finite Automata

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$$(q_1, a) \rightarrow q_3$$

A string s is accepted by an NFA, if there is a "lucky" sequence to an accepting state.

Last Week

Last week I showed you

- an algorithm for automata minimisation
- an algorithm for transforming a regular expression into an NFA
- an algorithm for transforming an NFA into a DFA (subset construction)

This Week

Go over the algorithms again, but with two new things and . . .

- with the example: what is the regular expression that accepts every string, except those ending in aa?
- Go over the proof for $L(rev(r)) = Rev(L(r))$.
- Anything else so far.

Proofs By Induction

- P holds for $\varnothing$, ϵ and c
- P holds for $r_1 + r_2$ under the assumption that P already holds for r_1 and r_2.
- P holds for $r_1 \cdot r_2$ under the assumption that P already holds for r_1 and r_2.
- P holds for r^* under the assumption that P already holds for r.

$$P(r): \quad L(rev(r)) = Rev(L(r))$$

What is the regular expression that accepts every string, except those ending in aa?

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$$(a + b)^*ba$$
$$(a + b)^*ab$$
$$(a + b)^*bb$$

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$(a + b)^*ba$
$(a + b)^*ab$
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a
" "

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a
""

What are the strings to be avoided?

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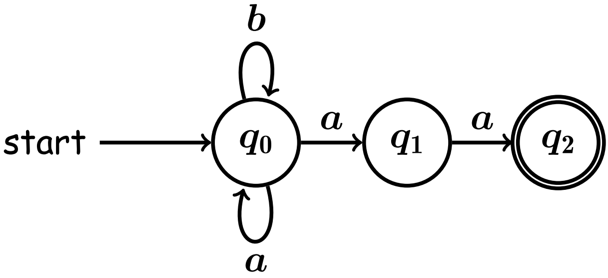
a
" "

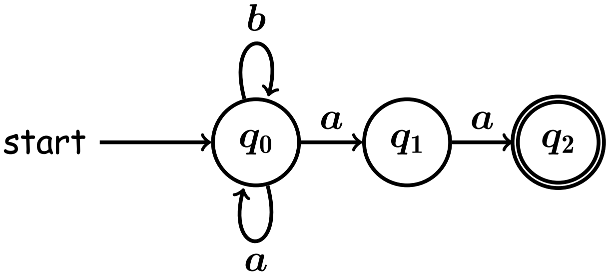
What are the strings to be avoided?

$(a + b)^*aa$

An NFA for $(a + b)^*aa$



An NFA for (a + b)*aa

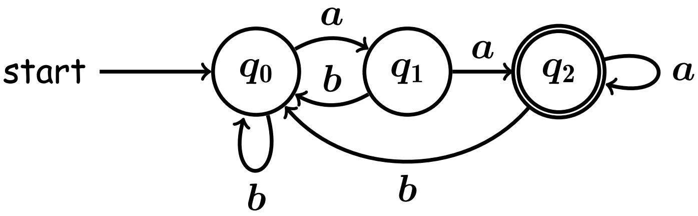


Minimisation for DFAs
Subset Construction for NFAs

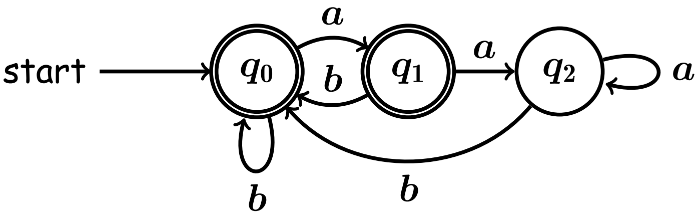
DFA Minimisation

1. Take all pairs (q, p) with $q \neq p$
2. Mark all pairs that accepting and non-accepting states
3. For all unmarked pairs (q, p) and all characters c tests wether
$$(\delta(q,c), \delta(p,c))$$
are marked. If yes, then also mark (q, p).
4. Repeat last step until nothing changed.
5. All unmarked pairs can be merged.

Minimal DFA $(a + b)^*aa$



Minimal DFA not (a + b)*aa



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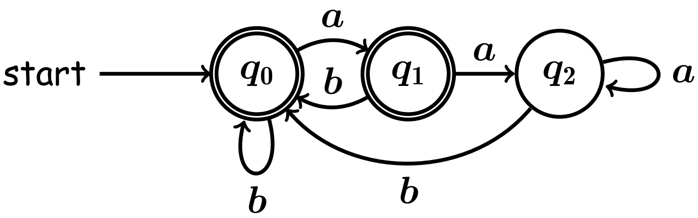
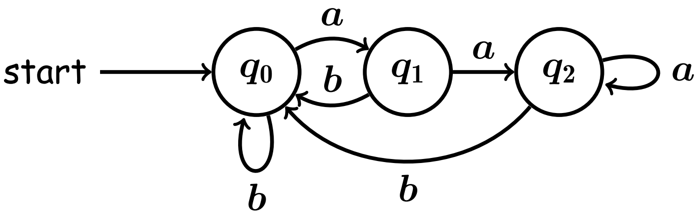
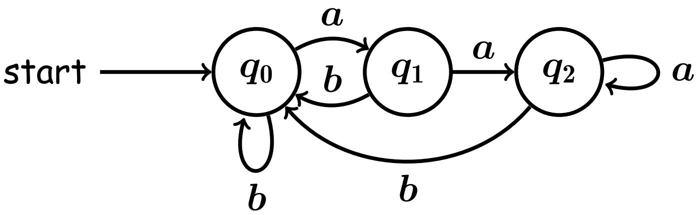
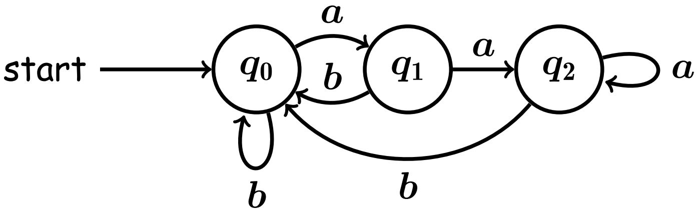


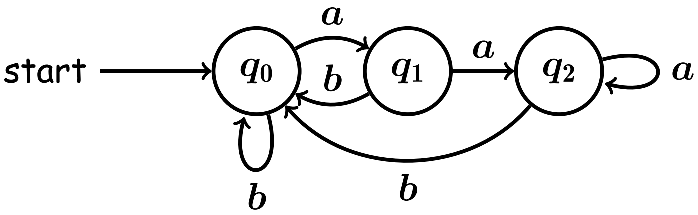
How to get from a DFA to a regular expression?


start
a
b
a
a
b
b
q_0
q_1
q_2

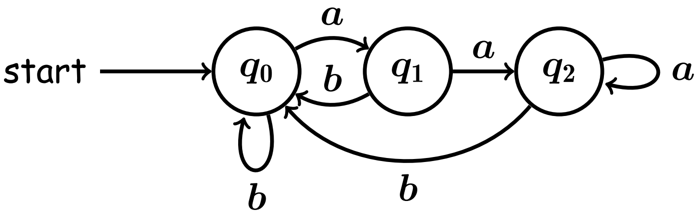


$$
\begin{aligned}
q_0 &= 2\,q_0 + 3\,q_1 + 4\,q_2 \\
q_1 &= 2\,q_0 + 3\,q_1 + 1\,q_2 \\
q_2 &= 1\,q_0 + 5\,q_1 + 2\,q_2
\end{aligned}
$$


start
q_0
q_1
q_2
a
a
a
b
b
b



$$q_0 = \epsilon + q_0\, b + q_1\, b + q_2\, b$$
$$q_1 = q_0\, a$$
$$q_2 = q_1\, a + q_2\, a$$



$$
\begin{aligned}
q_0 &= \epsilon + q_0\, b + q_1\, b + q_2\, b \\
q_1 &= q_0\, a \\
q_2 &= q_1\, a + q_2\, a
\end{aligned}
$$

Arden's Lemma:

If $q = q\, r + s$ then $q = s\, r^*$

Algorithms on Automata

- Reg $\rightarrow$ NFA: Thompson-McNaughton-Yamada method
- NFA $\rightarrow$ DFA: Subset Construction
- DFA $\rightarrow$ Reg: Brzozowski's Algebraic Method
- DFA minimisation: Hopcrofts Algorithm
- complement DFA

Grammars

$$
\begin{array}{lcl}
E & \rightarrow & F + (F \cdot \texttt{"}*\texttt{"} \cdot F) + (F \cdot \texttt{"}\backslash\texttt{"} \cdot F) \\
F & \rightarrow & T + (T \cdot \texttt{"+"} \cdot T) + (T \cdot \texttt{"}-\texttt{"} \cdot T) \\
T & \rightarrow & num + (\texttt{"(\"} \cdot E \cdot \texttt{")"})
\end{array}
$$

E, F and T are non-terminals
E is start symbol
num, (,), + ... are terminals

`(2*3)+(3+4)`

$$
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T & \rightarrow & num + (\texttt{"}(\texttt{"} \cdot E \cdot \texttt{"})\texttt{"})
\end{array}
$$

`(2*3)+(3+4)`