Compilers and Formal Languages

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Slides & Progs: KEATS (also homework is there)

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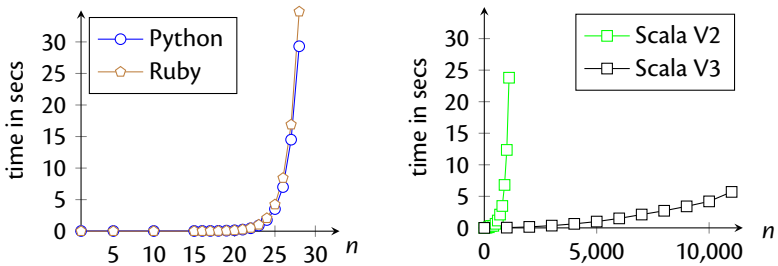
Let's Implement an Efficient Regular Expression Matcher

Graphs: $a^{?\{n\}} \cdot a^{\{n\}}$ and strings $\underbrace{a \ldots a}_{n}$



In the handouts is a similar graph for $(a^*)^* \cdot b$ and Java 8, JavaScript and Python.

(Basic) Regular Expressions

Their inductive definition:

r	$::=$	$\mathbf{0}$	nothing
	$\mid$	$\mathbf{1}$	empty string / "" / $[]$
	$\mid$	c	character
	$\mid$	$r_1 + r_2$	alternative / choice
	$\mid$	$r_1 \cdot r_2$	sequence
	$\mid$	r^*	star (zero or more)

When Are Two Regular Expressions Equivalent?

Two regular expressions r_1 and r_2 are **equivalent** provided:

$$r_1 \equiv r_2 \stackrel{\text{def}}{=} L(r_1) = L(r_2)$$

Some Concrete Equivalences

$$
\begin{array}{rcl}
(a + b) + c & \equiv & a + (b + c) \\
a + a & \equiv & a \\
a + b & \equiv & b + a \\
(a \cdot b) \cdot c & \equiv & a \cdot (b \cdot c) \\
c \cdot (a + b) & \equiv & (c \cdot a) + (c \cdot b)
\end{array}
$$

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c \cdot (a + b) & \equiv & (c \cdot a) + (c \cdot b)
\end{array}
$$

$$
\begin{array}{rcl}
a \cdot a & \not\equiv & a \\
a + (b \cdot c) & \not\equiv & (a + b) \cdot (a + c)
\end{array}
$$

Some Corner Cases

$$
\begin{array}{rcl}
a \cdot \mathbf{0} & \not\equiv & a \\
a + \mathbf{1} & \not\equiv & a \\
\mathbf{1} & \equiv & \mathbf{0}^* \\
\mathbf{1}^* & \equiv & \mathbf{1} \\
\mathbf{0}^* & \not\equiv & \mathbf{0}
\end{array}
$$

Some Simplification Rules

$$
\begin{array}{rcl}
r + \mathbf{0} & \equiv & r \\
\mathbf{0} + r & \equiv & r \\
r \cdot \mathbf{1} & \equiv & r \\
\mathbf{1} \cdot r & \equiv & r \\
r \cdot \mathbf{0} & \equiv & \mathbf{0} \\
\mathbf{0} \cdot r & \equiv & \mathbf{0} \\
r + r & \equiv & r
\end{array}
$$

Simplification Example

$$
\begin{array}{lcl}
((\mathbf{1} \cdot b) + \mathbf{0}) \cdot r & \Rightarrow & (\underline{(\mathbf{1} \cdot b)} + \mathbf{0}) \cdot r \\
& = & (\underline{b + \mathbf{0}}) \cdot r \\
& = & b \cdot r
\end{array}
$$

Simplification Example

$$
\begin{array}{lcl}
((\mathbf{0} \cdot b) + \mathbf{0}) \cdot r & \Rightarrow & (\underline{(\mathbf{0} \cdot b)} + \mathbf{0}) \cdot r \\
& = & (\underline{\mathbf{0} + \mathbf{0}}) \cdot r \\
& = & \mathbf{0} \cdot r \\
& = & \mathbf{0}
\end{array}
$$

Semantic Derivative

The **Semantic Derivative** of a language w.r.t. to a character c:

$$Der\, c\, A \stackrel{\text{def}}{=} \{s \mid c :: s \in A\}$$

For $A = \{foo, bar, frak\}$ then

$$\begin{array}{lcl} Der\, f\, A & = & \{oo, rak\} \\ Der\, b\, A & = & \{ar\} \\ Der\, a\, A & = & \{\} \end{array}$$

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$$\begin{array}{lcl} Der\, f\, A & = & \{oo, rak\} \\ Der\, b\, A & = & \{ar\} \\ Der\, a\, A & = & \{\} \end{array}$$

We can extend this definition to strings

$$Ders\, s\, A = \{s' \mid s @ s' \in A\}$$

The Specification for Matching

A regular expression r matches a string s provided:

$$s \in L(r)$$

...and the point of the this lecture is to decide this problem as fast as possible (unlike Python, Ruby, Java etc)

Brzozowski's Algorithm (1)

...whether a regular expression can match the empty string:

$$
\begin{array}{lcl}
nullable(\mathbf{0}) & \stackrel{\text{def}}{=} & false \\
nullable(\mathbf{1}) & \stackrel{\text{def}}{=} & true \\
nullable(c) & \stackrel{\text{def}}{=} & false \\
nullable(r_1 + r_2) & \stackrel{\text{def}}{=} & nullable(r_1) \vee nullable(r_2) \\
nullable(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & nullable(r_1) \wedge nullable(r_2) \\
nullable(r^*) & \stackrel{\text{def}}{=} & true
\end{array}
$$

The Derivative of a Rexp

If r matches the string $c :: s$, what is a regular expression that matches just s?

der c r gives the answer, Brzozowski 1964

The Derivative of a Rexp

$$
\begin{array}{lcl}
der\,c\,(\mathbf{0}) & \stackrel{\text{def}}{=} & \mathbf{0} \\
der\,c\,(\mathbf{1}) & \stackrel{\text{def}}{=} & \mathbf{0} \\
der\,c\,(d) & \stackrel{\text{def}}{=} & \text{if } c = d \text{ then } \mathbf{1} \text{ else } \mathbf{0} \\
der\,c\,(r_1 + r_2) & \stackrel{\text{def}}{=} & der\,c\,r_1 + der\,c\,r_2 \\
der\,c\,(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & \text{if } nullable(r_1) \\
 & & \text{then } (der\,c\,r_1) \cdot r_2 + der\,c\,r_2 \\
 & & \text{else } (der\,c\,r_1) \cdot r_2 \\
der\,c\,(r^*) & \stackrel{\text{def}}{=} & (der\,c\,r) \cdot (r^*)
\end{array}
$$

The Derivative of a Rexp

$$
\begin{array}{lcl}
der\, c\, (\mathbf{0}) & \stackrel{\text{def}}{=} & \mathbf{0} \\
der\, c\, (\mathbf{1}) & \stackrel{\text{def}}{=} & \mathbf{0} \\
der\, c\, (d) & \stackrel{\text{def}}{=} & \text{if } c = d \text{ then } \mathbf{1} \text{ else } \mathbf{0} \\
der\, c\, (r_1 + r_2) & \stackrel{\text{def}}{=} & der\, c\, r_1 + der\, c\, r_2 \\
der\, c\, (r_1 \cdot r_2) & \stackrel{\text{def}}{=} & \text{if } nullable(r_1) \\
 & & \text{then } (der\, c\, r_1) \cdot r_2 + der\, c\, r_2 \\
 & & \text{else } (der\, c\, r_1) \cdot r_2 \\
der\, c\, (r^*) & \stackrel{\text{def}}{=} & (der\, c\, r) \cdot (r^*)
\end{array}
$$

$$
\begin{array}{lcl}
ders\, []\, r & \stackrel{\text{def}}{=} & r \\
ders\, (c :: s)\, r & \stackrel{\text{def}}{=} & ders\, s\, (der\, c\, r)
\end{array}
$$

Examples

Given $r \stackrel{\text{def}}{=} ((a \cdot b) + b)^*$ what is

$$der\, a\, r = ?$$
$$der\, b\, r = ?$$
$$der\, c\, r = ?$$

Derivative Example

Given $r \stackrel{\text{def}}{=} ((a \cdot b) + b)^*$ what is

$$
\begin{array}{lcl}
der\, a\, ((a \cdot b) + b)^* & \Rightarrow & der\, a\, \underline{((a \cdot b) + b)^*} \\
& = & (der\, a\, (\underline{(a \cdot b) + b})) \cdot r \\
& = & ((der\, a\, (\underline{a \cdot b})) + (der\, a\, b)) \cdot r \\
& = & (((der\, a\, \underline{a}) \cdot b) + (der\, a\, b)) \cdot r \\
& = & ((\mathbf{1} \cdot b) + (der\, a\, \underline{b})) \cdot r \\
& = & ((\mathbf{1} \cdot b) + \mathbf{0}) \cdot r
\end{array}
$$

The Brzozowski Algorithm

$$matcher\ r\ s \stackrel{\text{def}}{=} nullable(ders\ s\ r)$$

Brzozowski: An Example

Does r_1 match abc?

Step 1:	build derivative of a and r_1	$(r_2 = der\, a\, r_1)$
Step 2:	build derivative of b and r_2	$(r_3 = der\, b\, r_2)$
Step 3:	build derivative of c and r_3	$(r_4 = der\, c\, r_3)$
Step 4:	the string is exhausted: test whether r_4 can recognise the empty string	$(nullable(r_4))$
Output:	result of the test $\Rightarrow$ *true* or *false*	

The Idea of the Algorithm

If we want to recognise the string abc with regular expression r_1 then

1. $Der\, a\, (L(r_1))$

The Idea of the Algorithm

If we want to recognise the string abc with regular expression r_1 then

1. $Der\, a\, (L(r_1))$
2. $Der\, b\, (Der\, a\, (L(r_1)))$

The Idea of the Algorithm

If we want to recognise the string abc with regular expression r_1 then

1. $Der\, a\, (L(r_1))$
2. $Der\, b\, (Der\, a\, (L(r_1)))$
3. $Der\, c\, (Der\, b\, (Der\, a\, (L(r_1))))$

4. finally we test whether the empty string is in this set; same for $Ders\, abc\, (L(r_1))$.

The matching algorithm works similarly, just over regular expressions instead of sets.

The Idea with Derivatives

Input: string abc and regular expression r

1. $der\, a\, r$
2. $der\, b\, (der\, a\, r)$
3. $der\, c\, (der\, b\, (der\, a\, r))$

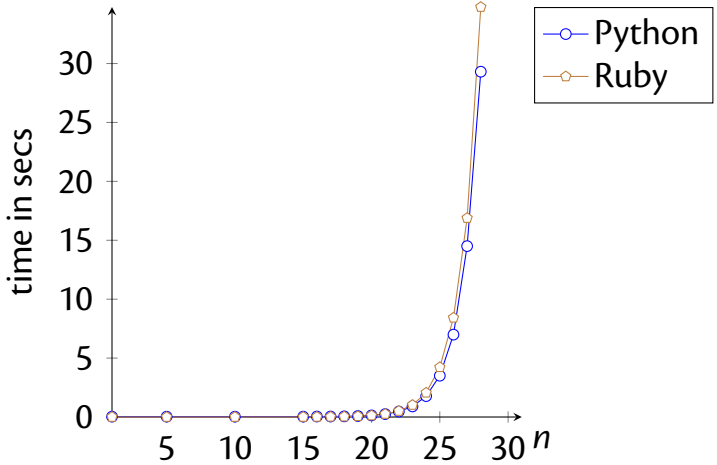
The Idea with Derivatives

Input: string abc and regular expression r

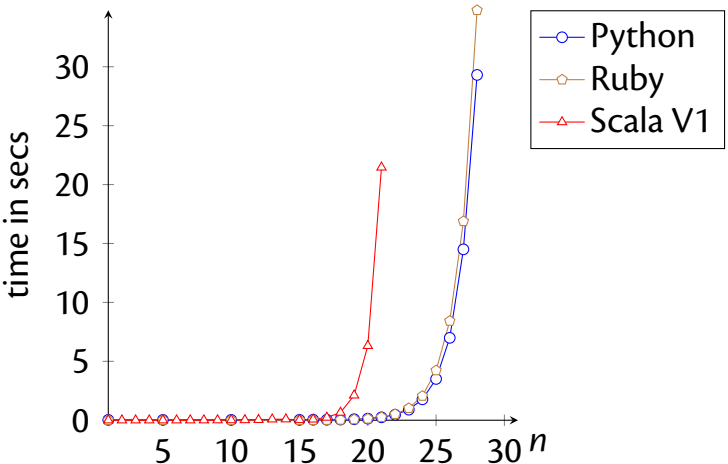
1. $der\, a\, r$
2. $der\, b\, (der\, a\, r)$
3. $der\, c\, (der\, b\, (der\, a\, r))$

4. finally check whether the last regular expression can match the empty string

$a?\{n\} \cdot a\{n\}$



Oops... $a^?\{n\} \cdot a\{n\}$



A Problem

We represented the "n-times" $a^{\{n\}}$ as a sequence regular expression:

0: $\mathbf{1}$
1: a
2: $a \cdot a$
3: $a \cdot a \cdot a$
…
13: $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$
…
20:

This problem is aggravated with $a^?$ being represented as $a + \mathbf{1}$.

Solving the Problem

What happens if we extend our regular expressions with explicit constructors

$$\begin{array}{rcl} r & ::= & \ldots \\ & | & r^{\{n\}} \\ & | & r^{?} \end{array}$$

What is their meaning?
What are the cases for *nullable* and *der*?

der for *n*-times

Case $n = 2$ and $r \cdot r$:

$$der\, c\, (r \cdot r) \stackrel{\text{def}}{=} \begin{array}{l} \text{if } \; nullable(r) \\ \text{then } \; (der\, c\, r) \cdot r + der\, c\, r \\ \text{else } \; (der\, c\, r) \cdot r \end{array}$$

der for *n*-times

Case $n = 2$ and $r \cdot r$:

$$der\, c\, (r \cdot r) \stackrel{\text{def}}{=} \begin{array}{l} \text{if } nullable(r) \\ \text{then } (der\, c\, r) \cdot r + der\, c\, r \\ \text{else } (der\, c\, r) \cdot r \end{array}$$

my claim
(in this case)

$$\equiv (der\, c\, r) \cdot r$$

der for *n*-times

Case $n = 2$ and $r \cdot r$:

$$
\begin{array}{lcl}
der\, c\, (r \cdot r) & \stackrel{\text{def}}{=} & \text{if } nullable(r) \\
& & \text{then } (der\, c\, r) \cdot r + der\, c\, r \\
& & \text{else } (der\, c\, r) \cdot r \\
\\
\text{my claim} & \equiv & (der\, c\, r) \cdot r \\
\text{(in this case)} & &
\end{array}
$$

We know $nullable(r)$ holds!

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$$(der\, c\, r) \cdot r + der\, c\, r$$

We know $nullable(r)$ holds!

$$(der\, c\, r) \cdot r + der\, c\, r \quad \equiv \quad (der\, c\, r) \cdot r + (der\, c\, r) \cdot \mathbf{1}$$

We know $nullable(r)$ holds!

$$\begin{array}{lcl} (der\,c\,r) \cdot r + der\,c\,r & \equiv & (der\,c\,r) \cdot r + (der\,c\,r) \cdot \mathbf{1} \\ & \equiv & (der\,c\,r) \cdot (r + \mathbf{1}) \end{array}$$

We know $nullable(r)$ holds!

$$
\begin{array}{lcl}
(der\,c\,r) \cdot r + der\,c\,r & \equiv & (der\,c\,r) \cdot r + (der\,c\,r) \cdot \mathbf{1} \\
& \equiv & (der\,c\,r) \cdot (r + \mathbf{1}) \\
& \equiv & (der\,c\,r) \cdot r \\
& & \quad \text{(remember } r \text{ is nullable)}
\end{array}
$$

We know $nullable(r)$ holds!

$$\begin{array}{lcl}(der\,c\,r)\cdot r + der\,c\,r & \equiv & (der\,c\,r)\cdot r + (der\,c\,r)\cdot \mathbf{1}\\ & \equiv & (der\,c\,r)\cdot (r + \mathbf{1})\\ & \equiv & (der\,c\,r)\cdot r\end{array}$$

(remember r is nullable)

$$der\,c\,(r\cdot r) \stackrel{\text{def}}{=} \begin{array}{l}\text{if } nullable(r)\\ \text{then } (der\,c\,r)\cdot r + der\,c\,r\\ \text{else } (der\,c\,r)\cdot r\end{array}$$

We know $nullable(r)$ holds!

$$\begin{array}{lcl}(der\,c\,r)\cdot r + der\,c\,r & \equiv & (der\,c\,r)\cdot r + (der\,c\,r)\cdot \mathbf{1}\\ & \equiv & (der\,c\,r)\cdot (r + \mathbf{1})\\ & \equiv & (der\,c\,r)\cdot r\end{array}$$

(remember r is nullable)

$$der\,c\,(r\cdot r) \stackrel{\text{def}}{=} \begin{array}{l}\text{if } nullable(r)\\ \text{then } (der\,c\,r)\cdot r\\ \text{else } (der\,c\,r)\cdot r\end{array}$$

We know $nullable(r)$ holds!

$$\begin{aligned}(der\,c\,r) \cdot r + der\,c\,r &\equiv (der\,c\,r) \cdot r + (der\,c\,r) \cdot \mathbf{1} \\ &\equiv (der\,c\,r) \cdot (r + \mathbf{1}) \\ &\equiv (der\,c\,r) \cdot r \\ &\qquad \text{(remember } r \text{ is nullable)}\end{aligned}$$

$$der\,c\,(r \cdot r) \stackrel{\text{def}}{=} (der\,c\,r) \cdot r$$

	$r\{n\}$	der
$n = 0$:	$\mathbf{1}$	$\mathbf{0}$
$n = 1$:	r	$(der\, c\, r)$
$n = 2$:	$r \cdot r$	$(der\, c\, r) \cdot r$
$n = 3$:	$r \cdot r \cdot r$	???
$\vdots$		

	$r\{n\}$	der
$n = 0$:	$\mathbf{1}$	$\mathbf{0}$
$n = 1$:	r	$(der\, c\, r)$
$n = 2$:	$r \cdot r$	$(der\, c\, r) \cdot r$
$n = 3$:	$r \cdot r \cdot r$	$(der\, c\, r) \cdot r \cdot r$
$\vdots$		

	$r\{n\}$	der
$n = 0$:	$\mathbf{1}$	$\mathbf{0}$
$n = 1$:	r	$(der\, c\, r)$
$n = 2$:	$r \cdot r$	$(der\, c\, r) \cdot r$
$n = 3$:	$r \cdot r \cdot r$	$(der\, c\, r) \cdot r \cdot r$
$\vdots$		

$$nullable(r\{n\}) \stackrel{\text{def}}{=} \text{if } n = 0 \text{ then } true \text{ else } nullable(r)$$

$$der\, c\, (r\{n\}) \stackrel{\text{def}}{=} \text{if } n = 0 \text{ then } \mathbf{0} \text{ else } (der\, c\, r) \cdot r\{n - 1\}$$

$$
\begin{array}{lcl}
der\,c\,(r \cdot r \cdot r) & \stackrel{\text{def}}{=} & \text{if } nullable(r) \\
 & & \text{then } (der\,c\,r) \cdot r \cdot r + der\,c\,(r \cdot r) \\
 & & \text{else } (der\,c\,r) \cdot r \cdot r
\end{array}
$$

$$
\begin{array}{lcl}
der\,c\,(r \cdot r \cdot r) & \stackrel{\text{def}}{=} & \text{if } nullable(r) \\
 & & \text{then } (der\,c\,r) \cdot r \cdot r + (der\,c\,r) \cdot r \\
 & & \text{else } (der\,c\,r) \cdot r \cdot r
\end{array}
$$

$$
\begin{array}{lcl}
der\,c\,(r \cdot r \cdot r) & \stackrel{\text{def}}{=} & \text{if } nullable(r) \\
 & & \text{then } (der\,c\,r) \cdot (r \cdot r + r) \\
 & & \text{else } (der\,c\,r) \cdot r \cdot r
\end{array}
$$

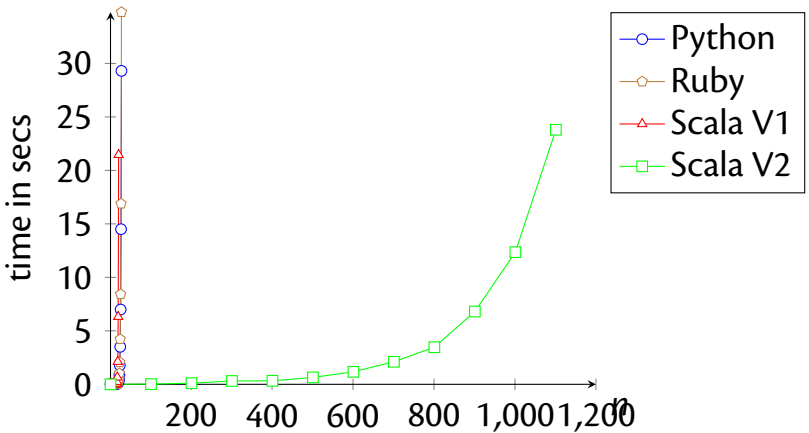
$$
\begin{array}{lcl}
der\,c\,(r \cdot r \cdot r) & \stackrel{\text{def}}{=} & \text{if } nullable(r) \\
 & & \text{then } (der\,c\,r) \cdot (r \cdot (r + \mathbf{1})) \\
 & & \text{else } (der\,c\,r) \cdot r \cdot r
\end{array}
$$

$$
\begin{array}{lcl}
der\,c\,(r \cdot r \cdot r) & \stackrel{\text{def}}{=} & \text{if } nullable(r) \\
 & & \text{then } (der\,c\,r) \cdot (r \cdot r) \\
 & & \text{else } (der\,c\,r) \cdot r \cdot r
\end{array}
$$

$$
\begin{array}{lcl}
der\,c\,(r \cdot r \cdot r) & \stackrel{\text{def}}{=} & \text{if } nullable(r) \\
 & & \text{then } (der\,c\,r) \cdot r \cdot r \\
 & & \text{else } (der\,c\,r) \cdot r \cdot r
\end{array}
$$

$$der\,c\,(r \cdot r \cdot r) \stackrel{\text{def}}{=} (der\,c\,r) \cdot r \cdot r$$

Brzozowski: $a^?\{n\} \cdot a\{n\}$



Examples

Recall the example of $r \stackrel{\text{def}}{=} ((a \cdot b) + b)^*$ with

$$\begin{array}{l} der\, a\, r = ((\mathbf{1} \cdot b) + \mathbf{0}) \cdot r \\ der\, b\, r = ((\mathbf{0} \cdot b) + \mathbf{1}) \cdot r \\ der\, c\, r = ((\mathbf{0} \cdot b) + \mathbf{0}) \cdot r \end{array}$$

What are these regular expressions equivalent to?

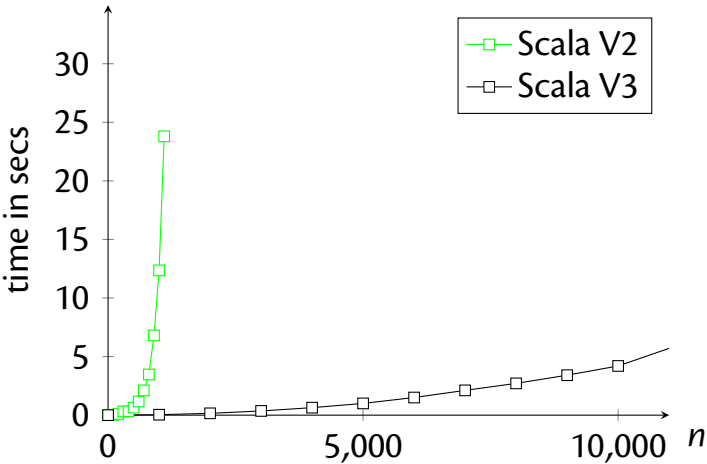
Simplification Rules

$$
\begin{array}{rcl}
r + \mathbf{0} & \Rightarrow & r \\
\mathbf{0} + r & \Rightarrow & r \\
r \cdot \mathbf{1} & \Rightarrow & r \\
\mathbf{1} \cdot r & \Rightarrow & r \\
r \cdot \mathbf{0} & \Rightarrow & \mathbf{0} \\
\mathbf{0} \cdot r & \Rightarrow & \mathbf{0} \\
r + r & \Rightarrow & r
\end{array}
$$

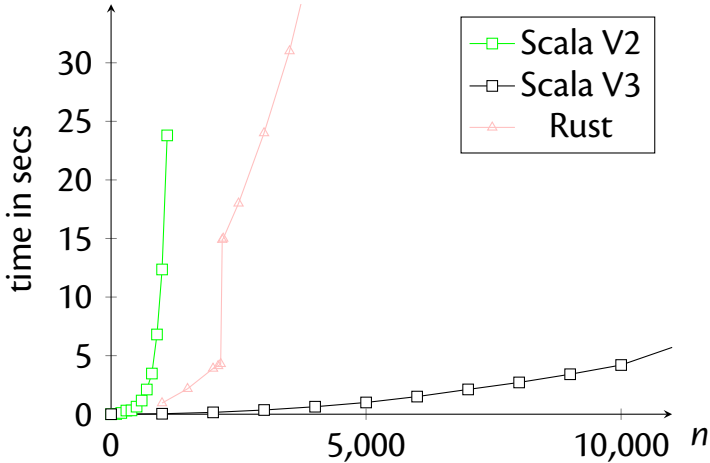
```
def ders(s: List[Char], r: Rexp) : Rexp = s match {
  case Nil => r
  case c::s => ders(s, simp(der(c, r)))
}
```

```
def simp(r: Rexp) : Rexp = r match {
  case ALT(r1, r2) => {
    (simp(r1), simp(r2)) match {
      case (ZERO, r2s) => r2s
      case (r1s, ZERO) => r1s
      case (r1s, r2s) =>
        if (r1s == r2s) r1s else ALT(r1s, r2s)
    }
  }
  case SEQ(r1, r2) => {
    (simp(r1), simp(r2)) match {
      case (ZERO, _) => ZERO
      case (_, ZERO) => ZERO
      case (ONE, r2s) => r2s
      case (r1s, ONE) => r1s
      case (r1s, r2s) => SEQ(r1s, r2s)
    }
  }
  case r => r
}
```

Brzozowski: $a^?\{n\} \cdot a\{n\}$

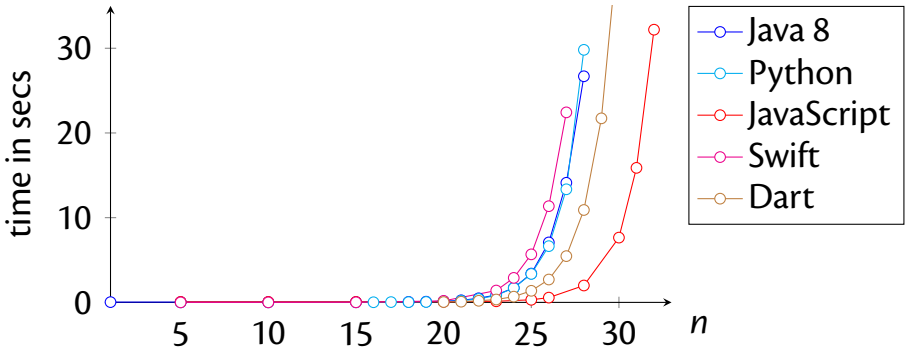


Brzozowski: $a^?\{n\} \cdot a\{n\}$



code by Archie Collard

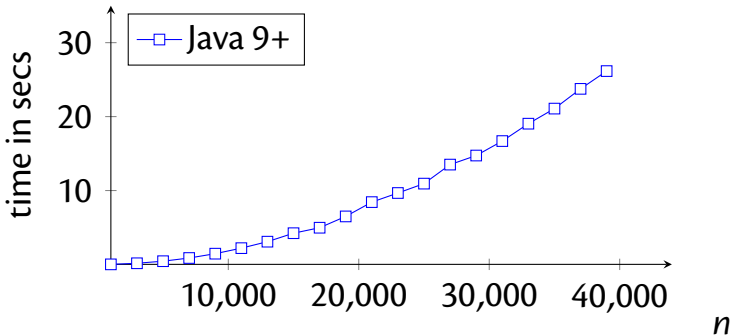
Another Example $(a^*)^* \cdot b$



Regex: $(a^*)^* \cdot b$

Strings of the form $\underbrace{a \ldots a}_{n}$

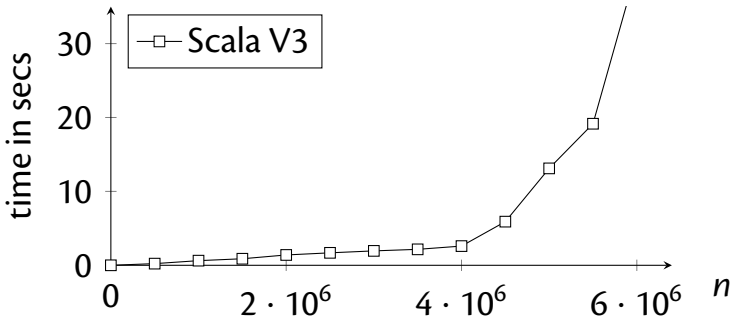
Same Example in Java 9+



Regex: $(a^*)^* \cdot b$

Strings of the form $\underbrace{a \ldots a}_{n}$

...and with Brzozowski



Regex: $(a^*)^* \cdot b$

Strings of the form $\underbrace{a \ldots a}_{n}$

What is good about this Alg.

extends to most regular expressions, for example $\sim r$ (next slide)

is easy to implement in a functional language

the algorithm is already quite old; there is still work to be done to use it as a tokenizer (that is relatively new work)

we can prove its correctness...(another video)

Negation of Regular Expr's

$\sim r$ (everything that r cannot recognise)

$$L(\sim r) \stackrel{\text{def}}{=} UNIV - L(r)$$

$$nullable(\sim r) \stackrel{\text{def}}{=} not\,(nullable(r))$$

$$der\,c\,(\sim r) \stackrel{\text{def}}{=} \sim (der\,c\,r)$$

Negation of Regular Expr's

$\sim r$ (everything that r cannot recognise)

$$L(\sim r) \stackrel{\text{def}}{=} UNIV - L(r)$$

$$nullable(\sim r) \stackrel{\text{def}}{=} not\,(nullable(r))$$

$$der\,c\,(\sim r) \stackrel{\text{def}}{=} \sim (der\,c\,r)$$

Used often for recognising comments:

$$/ \cdot * \cdot (\sim ([a\text{-}z]^* \cdot * \cdot / \cdot [a\text{-}z]^*)) \cdot * \cdot /$$

The Specification for Matching

A regular expression r matches a string s provided:

$$s \in L(r)$$

matcher $s\ r$ if and only if $s \in L(r)$

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nullable and *der*

The central properties:

$$nullable(r) \quad \textit{if and only if} \quad [] \in L(r)$$

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$$L(der\, c\, r) \;=\; Der\, c\, (L(r))$$

nullable and *der*

The central properties:

$$\forall r.\quad nullable(r)\ \textit{ if and only if }\ [] \in L(r)$$

$$\forall r\, c.\ L(der\, c\, r)\ =\ Der\, c\, (L(r))$$

Proofs about Rexps

Remember their inductive definition:

$$
\begin{array}{rcl}
r & ::= & \mathbf{0} \\
 & | & \mathbf{1} \\
 & | & c \\
 & | & r_1 \cdot r_2 \\
 & | & r_1 + r_2 \\
 & | & r^*
\end{array}
$$

If we want to prove something, say a property $P(r)$, for all regular expressions r then ...

Proofs about Rexp (2)

P holds for $\mathbf{0}$, $\mathbf{1}$ and c

P holds for $r_1 + r_2$ under the assumption that P already holds for r_1 and r_2.

P holds for $r_1 \cdot r_2$ under the assumption that P already holds for r_1 and r_2.

P holds for r^* under the assumption that P already holds for r.

Proofs about Rexp

Assume $P(r)$ is the property:

$$nullable(r) \text{ if and only if } [] \in L(r)$$

$\text{der c } (r^*) \text{ def} = (\text{der c r}) \cdot (r^*)$

Why in the example (slide 19) the first step is:

$\text{der a } ((a \cdot b) + b)^* = (\text{der a } ((a \cdot b) + b)) \cdot r$

and not

$\text{der a } ((a \cdot b) + b) = (\text{der a } ((a \cdot b) + b)) \cdot (r^*)$

Would it be possible to find and go over a few examples from the Brzozowski Algorithm, as it doesn't seem to be as simple as it sounds?

Is it possible to make a visual example of how using simp() function on a (a*)*.b regular expression reduces its runtime? If not it's fine. I am just very surprised that it is so efficient.

Do you think the algorithm can be still improved (made faster)?

Do the regular expression matchers in Python and Java 8 have more features than the one implemented in this module? Or is there another reason for their inefficiency?

Will we discuss the broader Chomsky hierarchy of languages at some point?