Homework 2

1. What is the language recognised by the regular expressions $(\varnothing^*)^*$.

2. Review the first handout about sets of strings and read the second handout. Assuming the alphabet is the set $\{a, b\}$, decide which of the following equations are true in general for arbitrary languages A, B and C:

$$
\begin{aligned}
(A \cup B) @ C &\overset{?}{=} A@C \cup B@C \\
A^* \cup B^* &\overset{?}{=} (A \cup B)^* \\
A^* @ A^* &\overset{?}{=} A^* \\
(A \cap B) @ C &\overset{?}{=} (A@C) \cap (B@C)
\end{aligned}
$$

In case an equation is true, give an explanation; otherwise give a counter-example.

3. Given the regular expressions $r_1 = \epsilon$ and $r_2 = \varnothing$ and $r_3 = a$. How many strings can the regular expressions r_1^*, r_2^* and r_3^* each match?

4. Give regular expressions for (a) decimal numbers and for (b) binary numbers. (Hint: Observe that the empty string is not a number. Also observe that leading 0s are normally not written.)

5. Decide whether the following two regular expressions are equivalent $(\epsilon + a)^* \overset{?}{\equiv} a^*$ and $(a \cdot b)^* \cdot a \overset{?}{\equiv} a \cdot (b \cdot a)^*$.

6. Given the regular expression $r = (a \cdot b + b)^*$. Compute what the derivative of r is with respect to a, b and c. Is r nullable?

7. Prove that for all regular expressions r we have

$$nullable(r) \quad \text{if and only if} \quad [] \in L(r)$$

Write down clearly in each case what you need to prove and what are the assumptions.

8. Define what is meant by the derivative of a regular expressions with respoect to a character. (Hint: The derivative is defined recursively.)

9. Assume the set Der is defined as

$$Der\ c\ A \overset{\text{def}}{=} \{s \mid c{::}s \in A\}$$

What is the relation between Der and the notion of derivative of regular expressions?

10. Give a regular expression over the alphabet $\{a, b\}$ recognising all strings that do not contain any substring bb and end in a.

11. Do $(a + b)^* \cdot b^+$ and $(a^* \cdot b^+) + (b^* \cdot b^+)$ define the same language?