Compilers and Formal Languages (5)

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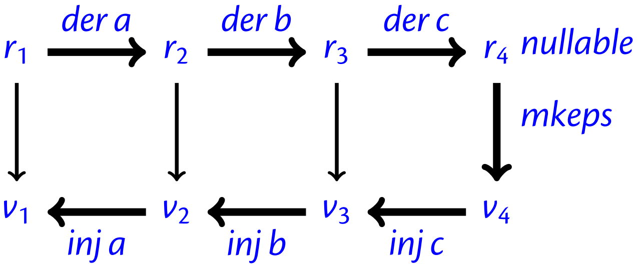
Slides & Progs:	KEATS (also homework is there)

Last Week
Regexes and Values

Regular expressions and their corresponding values:

$$
\begin{array}{lcl}
r & ::= & \mathbf{0} \\
& | & \mathbf{1} \\
& | & c \\
& | & r_1 \cdot r_2 \\
& | & r_1 + r_2 \\
& & \\
& | & r^*
\end{array}
\qquad
\begin{array}{lcl}
v & ::= & \\
& & \mathit{Empty} \\
& | & \mathit{Char}(c) \\
& | & \mathit{Seq}(v_1, v_2) \\
& | & \mathit{Left}(v) \\
& | & \mathit{Right}(v) \\
& | & \mathit{Stars}[v_1, \ldots v_n]
\end{array}
$$

r_1: $a \cdot (b \cdot c)$
r_2: $\mathbf{1} \cdot (b \cdot c)$
r_3: $(\mathbf{0} \cdot (b \cdot c)) + (\mathbf{1} \cdot c)$
r_4: $(\mathbf{0} \cdot (b \cdot c)) + ((\mathbf{0} \cdot c) + \mathbf{1})$

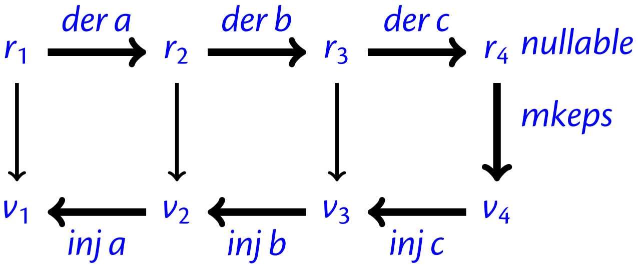


v_1: $Seq(Char(a), Seq(Char(b), Char(c)))$
v_2: $Seq(Empty, Seq(Char(b), Char(c)))$
v_3: $Right(Seq(Empty, Char(c)))$
v_4: $Right(Right(Empty))$

$|v_1|$: abc
$|v_2|$: bc
$|v_3|$: c
$|v_4|$: $[]$

Simplification

- If we simplify after the derivative, then we are builing the value for the simplified regular expression, but *not* for the original regular expression.



$$(b \cdot c) + (\mathbf{0} + \mathbf{1}) \mapsto (b \cdot c) + \mathbf{1}$$

Records

- new regex: $(x : r)$ new value: $Rec(x, v)$

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- $nullable(x : r) \stackrel{\text{def}}{=} nullable(r)$
- $der\, c\, (x : r) \stackrel{\text{def}}{=} (x : der\, c\, r)$
- $mkeps(x : r) \stackrel{\text{def}}{=} Rec(x, mkeps(r))$
- $inj\, (x : r)\, c\, Rec(x, v) \stackrel{\text{def}}{=} Rec(x, inj\, r\, c\, v)$

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- $inj\, (x : r)\, c\, Rec(x, v) \stackrel{\text{def}}{=} Rec(x, inj\, r\, c\, v)$

for extracting subpatterns $(z : ((x : ab) + (y : ba))$

Environments

Obtaining the "recorded" parts of a value:

$$
\begin{aligned}
env(Empty) &\stackrel{\text{def}}{=} [] \\
env(Char(c)) &\stackrel{\text{def}}{=} [] \\
env(Left(v)) &\stackrel{\text{def}}{=} env(v) \\
env(Right(v)) &\stackrel{\text{def}}{=} env(v) \\
env(Seq(v_1, v_2)) &\stackrel{\text{def}}{=} env(v_1) \,@\, env(v_2) \\
env(Stars[v_1, \ldots, v_n]) &\stackrel{\text{def}}{=} env(v_1) \,@ \ldots @\, env(v_n) \\
env(Rec(x : v)) &\stackrel{\text{def}}{=} (x : |v|) :: env(v)
\end{aligned}
$$

While Tokens

$$\texttt{WHILE\_REGS} \stackrel{\text{def}}{=} \big((\texttt{"k"} : \texttt{KEYWORD}) + (\texttt{"i"} : \texttt{ID}) + (\texttt{"o"} : \texttt{OP}) + (\texttt{"n"} : \texttt{NUM}) + (\texttt{"s"} : \texttt{SEMI}) + (\texttt{"p"} : (\texttt{LPAREN} + \texttt{RPAREN})) + (\texttt{"b"} : (\texttt{BEGIN} + \texttt{END})) + (\texttt{"w"} : \texttt{WHITESPACE})\big)^*$$

```
"if true then then 42 else +"
```

```
KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)
```

```
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KEYWORD(if),
IDENT(true),
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KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)
```
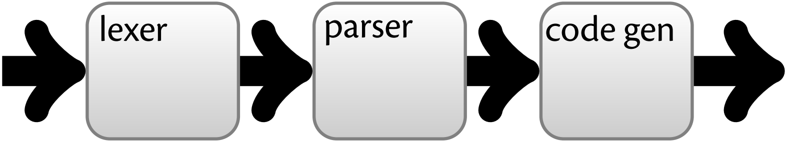

Coursework: Submissions

- Scala (29)
- Haskell (1)
- Kotlin (1)
- Rust (1)

Please get in contact if you intend to do CW Strand 2.

Lexer, Parser



Today a parser.

What Parsing is Not

Usually parsing does not check semantic correctness, e.g.

- whether a function is not used before it is defined
- whether a function has the correct number of arguments or are of correct type
- whether a variable can be declared twice in a scope

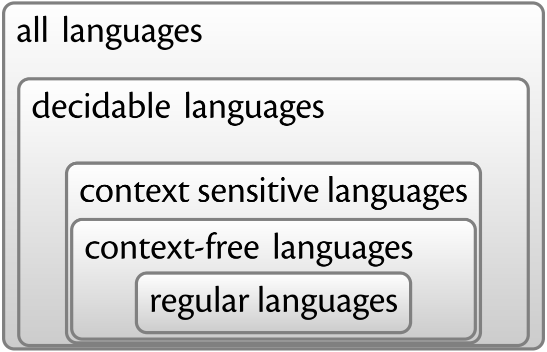
Regular Languages

While regular expressions are very useful for lexing, there is no regular expression that can recognise the language $a^n b^n$.

$$(((()()))()) \quad \text{vs.} \quad (((()()))()))$$

So we cannot find out with regular expressions whether parentheses are matched or unmatched. Also regular expressions are not recursive, e.g. $(1 + 2) + 3$.

Hierarchy of Languages



CF Grammars

A **context-free grammar** G consists of

- a finite set of nonterminal symbols (e.g. A upper case)
- a finite set terminal symbols or tokens (lower case)
- a start symbol (which must be a nonterminal)
- a set of rules

$$A ::= rhs$$

where rhs are sequences involving terminals and nonterminals, including the empty sequence ϵ.

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- a set of rules

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where rhs are sequences involving terminals and nonterminals, including the empty sequence ϵ.

We also allow rules

$$A ::= rhs_1 | rhs_2 | \ldots$$

Palindromes

A grammar for palindromes over the alphabet $\{a, b\}$:

$$S ::= a \cdot S \cdot a$$

$$S ::= b \cdot S \cdot b$$

$$S ::= a$$

$$S ::= b$$

$$S ::= \epsilon$$

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or

$$S ::= a \cdot S \cdot a \mid b \cdot S \cdot b \mid a \mid b \mid \epsilon$$

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or

$$S ::= a \cdot S \cdot a \mid b \cdot S \cdot b \mid a \mid b \mid \epsilon$$

Can you find the grammar rules for matched parentheses?

Arithmetic Expressions

$$
\begin{aligned}
E ::= &\ num\_token \\
|\ & E \cdot + \cdot E \\
|\ & E \cdot - \cdot E \\
|\ & E \cdot * \cdot E \\
|\ & (\cdot E \cdot)
\end{aligned}
$$

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\end{aligned}
$$

```
1 + 2 * 3 + 4
```

A CFG Derivation

1. Begin with a string containing only the start symbol, say S
2. Replace any nonterminal X in the string by the right-hand side of some production $X ::= rhs$
3. Repeat 2 until there are no nonterminals left

$$S \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots$$

Example Derivation

$$S ::= \epsilon \mid a \cdot S \cdot a \mid b \cdot S \cdot b$$

$$\begin{array}{lcl} S & \rightarrow & aSa \\ & \rightarrow & abSba \\ & \rightarrow & abaSaba \\ & \rightarrow & abaaba \end{array}$$

Example Derivation

$$
\begin{aligned}
E ::= &\ num\_token \\
&\mid E \cdot + \cdot E \\
&\mid E \cdot - \cdot E \\
&\mid E \cdot * \cdot E \\
&\mid (\cdot E \cdot)
\end{aligned}
$$

$$
\begin{aligned}
E \rightarrow\ & E * E \\
\rightarrow\ & E + E * E \\
\rightarrow\ & E + E * E + E \\
\rightarrow^{+}\ & 1 + 2 * 3 + 4
\end{aligned}
$$

Example Derivation

$$
\begin{aligned}
E ::= &\ num\_token \\
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E \rightarrow &\quad E * E \\
\rightarrow &\quad E + E * E \\
\rightarrow &\quad E + E * E + E \\
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\qquad
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Context Sensitive Grammars

It is much harder to find out whether a string is parsed by a context sensitive grammar:

$$\begin{aligned} S &::= bSAA \mid \epsilon \\ A &::= a \\ bA &::= Ab \end{aligned}$$

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\begin{aligned}
S &::= bSAA \mid \epsilon \\
A &::= a \\
bA &::= Ab
\end{aligned}
$$

$$S \rightarrow \ldots \rightarrow^? ababaa$$

```
Time flies like an arrow;
fruit flies like bananas.
```

Language of a CFG

Let G be a context-free grammar with start symbol $\mathbf{S}$. Then the language $L(G)$ is:

$$\{c_1 \ldots c_n \mid \forall i.\ c_i \in T \wedge \mathbf{S} \rightarrow^* c_1 \ldots c_n\}$$

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Let G be a context-free grammar with start symbol S. Then the language $L(G)$ is:

$$\{c_1 \ldots c_n \mid \forall i.\ c_i \in T \wedge S \rightarrow^* c_1 \ldots c_n\}$$

- Terminals, because there are no rules for replacing them.
- Once generated, terminals are "permanent".
- Terminals ought to be tokens of the language (but can also be strings).

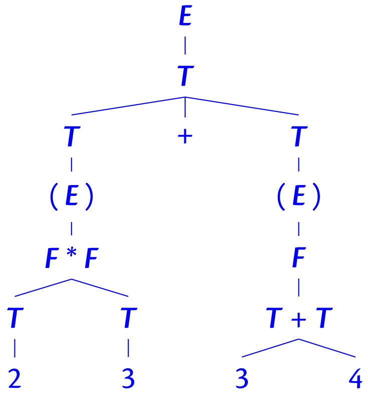
Parse Trees

$$E ::= T \mid T \cdot + \cdot E \mid T \cdot - \cdot E$$

$$T ::= F \mid F \cdot * \cdot T$$

$$F ::= num\_token \mid (\cdot E \cdot)$$

(2*3)+(3+4)



Arithmetic Expressions

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$$\begin{aligned} E ::= &\ \textit{num\_token} \\ &|\ E \cdot + \cdot E \\ &|\ E \cdot - \cdot E \\ &|\ E \cdot * \cdot E \\ &|\ (\cdot E \cdot) \end{aligned}$$

A CFG is **left-recursive** if it has a nonterminal E such that $E \rightarrow^{+} E \cdot \ldots$

Ambiguous Grammars

A grammar is **ambiguous** if there is a string that has at least two different parse trees.

$$
\begin{aligned}
E ::= &\ num\_token \\
| &\ E \cdot + \cdot E \\
| &\ E \cdot - \cdot E \\
| &\ E \cdot * \cdot E \\
| &\ (\cdot E \cdot)
\end{aligned}
$$

`1 + 2 * 3 + 4`

'Dangling' Else

Another ambiguous grammar:

$$
\begin{array}{rcl}
E & \rightarrow & \text{if } E \text{ then } E \\
 & | & \text{if } E \text{ then } E \text{ else } E \\
 & | & \ldots
\end{array}
$$

```
if a then if x then y else c
```

Parser Combinators

One of the simplest ways to implement a parser, see

`https://vimeo.com/142341803`

Parser combinators:

$$\underbrace{\text{list of tokens}}_{\text{input}} \Rightarrow \underbrace{\text{set of (parsed input, unparsed input)}}_{\text{output}}$$

- atomic parsers
- sequencing
- alternative
- semantic action

Atomic parsers, for example, number tokens

$$\texttt{Num(123)} :: \textit{rest} \;\Rightarrow\; \{(\texttt{Num(123)}, \textit{rest})\}$$

- you consume one or more token from the input (stream)
- also works for characters and strings

Alternative parser (code $p \,||\, q$)

- apply p and also q; then combine the outputs

$$p(\text{input}) \cup q(\text{input})$$

Sequence parser (code $p \sim q$)

- apply first p producing a set of pairs
- then apply q to the unparsed part
- then combine the results:

((output$_1$, output$_2$), unparsed part)

$$\{((o_1, o_2), u_2) \mid$$
$$(o_1, u_1) \in p(\text{input}) \wedge$$
$$(o_2, u_2) \in q(u_1)\}$$

Function parser (code $p \Rightarrow f$)

- apply p producing a set of pairs
- then apply the function f to each first component

$$\{(f(o_1), u_1) \mid (o_1, u_1) \in p(\text{input})\}$$

Function parser (code $p \Rightarrow f$)

- apply p producing a set of pairs
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$$\{(f(o_1), u_1) \mid (o_1, u_1) \in p(\text{input})\}$$

f is the semantic action ("what to do with the parsed input")

Semantic Actions

Addition

$$T \sim + \sim E \Rightarrow \underbrace{f\big((x,y),z\big) \Rightarrow x + z}_{\text{semantic action}}$$

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Multiplication

$$F \sim * \sim T \Rightarrow f\big((x,y),z\big) \Rightarrow x * z$$

Parenthesis

$$(\sim E \sim) \Rightarrow f\big((x,y),z\big) \Rightarrow y$$

Types of Parsers

- **Sequencing**: if p returns results of type T, and q results of type S, then $p \sim q$ returns results of type

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$$T$$

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- **Sequencing**: if p returns results of type T, and q results of type S, then $p \sim q$ returns results of type
$$T \times S$$
- **Alternative**: if p returns results of type T then q must also have results of type T, and $p \mid\mid q$ returns results of type
$$T$$
- **Semantic Action**: if p returns results of type T and f is a function from T to S, then $p \Rightarrow f$ returns results of type
$$S$$

Input Types of Parsers

- input: token list
- output: set of (output_type, token list)

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actually it can be any input type as long as it is a kind of sequence (for example a string)

Scannerless Parsers

- input: string
- output: set of (output_type, string)

but using lexers is better because whitespaces or comments can be filtered out; then input is a sequence of tokens

Successful Parses

- input: string
- output: set of (output_type, string)

a parse is successful whenever the input has been fully "consumed" (that is the second component is empty)

Abstract Parser Class

```
abstract class Parser[I, T] {
  def parse(ts: I): Set[(T, I)]

  def parse_all(ts: I) : Set[T] =
    for ((head, tail) <- parse(ts);
         if (tail.isEmpty)) yield head
}
```

```scala
class AltParser[I, T](p: => Parser[I, T],
                      q: => Parser[I, T])
                          extends Parser[I, T] {
  def parse(sb: I) = p.parse(sb) ++ q.parse(sb)
}

class SeqParser[I, T, S](p: => Parser[I, T],
                         q: => Parser[I, S])
                             extends Parser[I, (T, S)] {
  def parse(sb: I) =
    for ((head1, tail1) <- p.parse(sb);
         (head2, tail2) <- q.parse(tail1))
            yield ((head1, head2), tail2)
}

class FunParser[I, T, S](p: => Parser[I, T], f: T => S)
                                  extends Parser[I, S] {
  def parse(sb: I) =
    for ((head, tail) <- p.parse(sb))
      yield (f(head), tail)
}
```

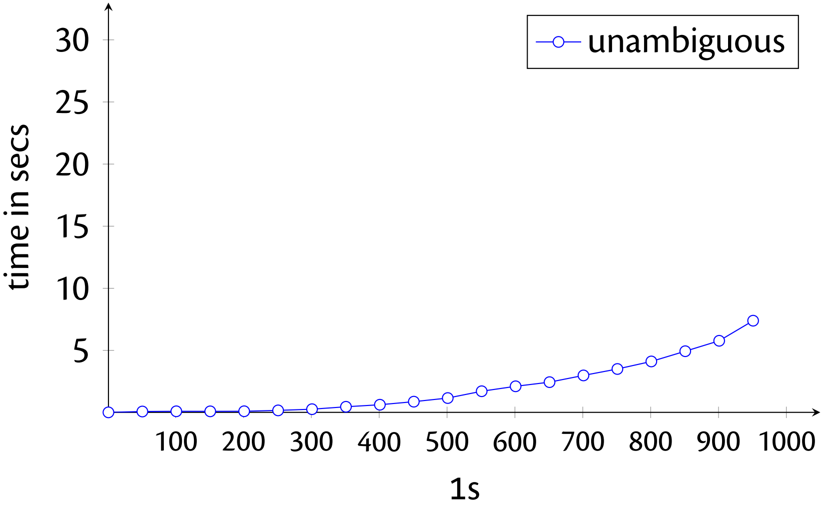
Two Grammars

Which languages are recognised by the following two grammars?

$$
\begin{array}{lcl}
S & \rightarrow & 1 \cdot S \cdot S \\
  & \mid & \epsilon
\end{array}
$$

$$
\begin{array}{lcl}
U & \rightarrow & 1 \cdot U \\
  & \mid & \epsilon
\end{array}
$$

Ambiguous Grammars



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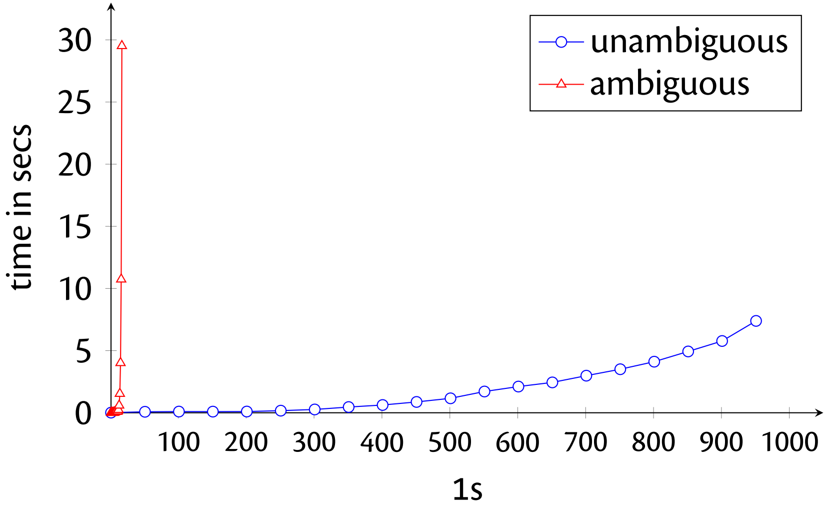


While-Language

$$
\begin{array}{rcl}
\textit{Stmt} & ::= & \text{skip} \\
& | & \textit{Id} := \textit{AExp} \\
& | & \text{if } \textit{BExp} \text{ then } \textit{Block} \text{ else } \textit{Block} \\
& | & \text{while } \textit{BExp} \text{ do } \textit{Block} \\
\textit{Stmts} & ::= & \textit{Stmt} \;;\; \textit{Stmts} \\
& | & \textit{Stmt} \\
\textit{Block} & ::= & \{ \textit{Stmts} \} \\
& | & \textit{Stmt} \\
\textit{AExp} & ::= & \ldots \\
\textit{BExp} & ::= & \ldots
\end{array}
$$

An Interpreter

$$\{$$
$$x := 5;$$
$$y := x * 3;$$
$$y := x * 4;$$
$$x := u * 3$$
$$\}$$

- the interpreter has to record the value of x before assigning a value to y

An Interpreter

$$\begin{array}{l} \{ \\ \quad x := 5; \\ \quad y := x * 3; \\ \quad y := x * 4; \\ \quad x := u * 3 \\ \} \end{array}$$

- the interpreter has to record the value of x before assigning a value to y
- `eval(stmt, env)`

Interpreter

$$
\begin{aligned}
\text{eval}(n, E) &\stackrel{\text{def}}{=} n \\
\text{eval}(x, E) &\stackrel{\text{def}}{=} E(x) \quad \text{lookup } x \text{ in } E \\
\text{eval}(a_1 + a_2, E) &\stackrel{\text{def}}{=} \text{eval}(a_1, E) + \text{eval}(a_2, E) \\
\text{eval}(a_1 - a_2, E) &\stackrel{\text{def}}{=} \text{eval}(a_1, E) - \text{eval}(a_2, E) \\
\text{eval}(a_1 * a_2, E) &\stackrel{\text{def}}{=} \text{eval}(a_1, E) * \text{eval}(a_2, E)
\end{aligned}
$$

$$
\begin{aligned}
\text{eval}(a_1 = a_2, E) &\stackrel{\text{def}}{=} \text{eval}(a_1, E) = \text{eval}(a_2, E) \\
\text{eval}(a_1 \,!= a_2, E) &\stackrel{\text{def}}{=} \neg(\text{eval}(a_1, E) = \text{eval}(a_2, E)) \\
\text{eval}(a_1 < a_2, E) &\stackrel{\text{def}}{=} \text{eval}(a_1, E) < \text{eval}(a_2, E)
\end{aligned}
$$

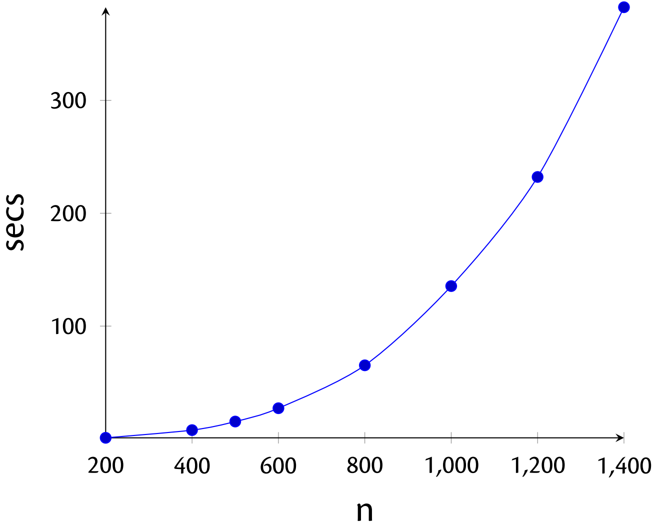
Interpreter (2)

$$\begin{array}{lcl}
\text{eval}(\text{skip}, E) & \stackrel{\text{def}}{=} & E \\
\text{eval}(x := a, E) & \stackrel{\text{def}}{=} & E(x \mapsto \text{eval}(a, E)) \\
\multicolumn{3}{l}{\text{eval}(\text{if } b \text{ then } cs_1 \text{ else } cs_2, E) \stackrel{\text{def}}{=}} \\
 & & \text{if eval}(b, E) \text{ then eval}(cs_1, E) \\
 & & \qquad\qquad\quad \text{else eval}(cs_2, E) \\
\multicolumn{3}{l}{\text{eval}(\text{while } b \text{ do } cs, E) \stackrel{\text{def}}{=}} \\
 & & \text{if eval}(b, E) \\
 & & \text{then eval}(\text{while } b \text{ do } cs, \text{eval}(cs, E)) \\
 & & \text{else } E \\
\text{eval}(\text{write } x, E) & \stackrel{\text{def}}{=} & \{\ \text{println}(E(x))\ ;\ E\ \}
\end{array}$$

Test Program

```
start := 1000;
x := start;
y := start;
z := start;
while 0 < x do {
 while 0 < y do {
  while 0 < z do { z := z - 1 };
  z := start;
  y := y - 1
 };
 y := start;
 x := x - 1
}
```

Interpreted Code



Java Virtual Machine

- introduced in 1995
- is a stack-based VM (like Postscript, CLR of .Net)
- contains a JIT compiler
- many languages take advantage of JVM's infrastructure (JRE)
- is garbage collected $\Rightarrow$ no buffer overflows
- some languages compile to the JVM: Scala, Clojure...