

Automata and Formal Languages (10)

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**There are more problems, than
there are programs.**

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there are programs.**

**There must be a problem for
which there is no program.**

Revision: Proofs



Subsets

$$A \subseteq B$$

$$\forall e. e \in A \Rightarrow e \in B$$

Subsets

$A \subseteq B$ and $B \subseteq A$

then $A = B$

Injective Function

f is an injective function iff

$$\forall x y. f(x) = f(y) \Rightarrow x = y$$

Cardinality

$|A| \stackrel{\text{def}}{=} \text{"how many elements"}$

$$A \subseteq B \Rightarrow |A| \leq |B|$$

Cardinality

$|A| \stackrel{\text{def}}{=} \text{"how many elements"}$

$$A \subseteq B \Rightarrow |A| \leq |B|$$

if there is an injective function
 $f : A \rightarrow B$ then $|A| \leq |B|$

Natural Numbers

$$\mathbb{N} \stackrel{\text{def}}{=} \{0, 1, 2, 3, \dots\dots\dots\}$$

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$$\mathbb{N} \stackrel{\text{def}}{=} \{0, 1, 2, 3, \dots\dots\dots\}$$

A is countable iff $|A| \leq |\mathbb{N}|$

First Question

$$|\mathbb{N} - \{0\}| \quad ? \quad |\mathbb{N}|$$

\geq or \leq or $=$

$$|\mathbb{N} - \{0, 1\}| \quad ? \quad |\mathbb{N}|$$

$$|\mathbb{N} - \{0, 1\}| \quad ? \quad |\mathbb{N}|$$

$$|\mathbb{N} - \mathbb{O}| \quad ? \quad |\mathbb{N}|$$

$\mathbb{O} \stackrel{\text{def}}{=} \text{odd numbers} \quad \{1, 3, 5, \dots\}$

$$|\mathbb{N} - \{0, 1\}| \quad ? \quad |\mathbb{N}|$$

$$|\mathbb{N} - \mathbb{O}| \quad ? \quad |\mathbb{N}|$$

$\mathbb{O} \stackrel{\text{def}}{=} \text{odd numbers} \quad \{1, 3, 5, \dots\}$

$\mathbb{E} \stackrel{\text{def}}{=} \text{even numbers} \quad \{0, 2, 4, \dots\}$

$$|\mathbb{N} \cup -\mathbb{N}| \quad ? \quad |\mathbb{N}|$$

$\mathbb{N} \stackrel{\text{def}}{=} \text{positive numbers} \quad \{0, 1, 2, 3, \dots\}$

$-\mathbb{N} \stackrel{\text{def}}{=} \text{negative numbers} \quad \{0, -1, -2, -3, \dots\}$

A is **countable** if there exists an injective $f : A \rightarrow \mathbb{N}$

A is **uncountable** if there does not exist an injective $f : A \rightarrow \mathbb{N}$

countable: $|A| \leq |\mathbb{N}|$

uncountable: $|A| > |\mathbb{N}|$

A is **countable** if there exists an injective $f : A \rightarrow \mathbb{N}$

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countable: $|A| \leq |\mathbb{N}|$

uncountable: $|A| > |\mathbb{N}|$

Does there exist such an A ?

Halting Problem

Assume a program H that decides for all programs A and all input data D whether

- $H(A, D) \stackrel{\text{def}}{=} 1$ iff $A(D)$ terminates
- $H(A, D) \stackrel{\text{def}}{=} 0$ otherwise

Halting Problem

Given such a program H define the following program C : for all programs A

- $C(A) \stackrel{\text{def}}{=} 0$ iff $H(A, A) = 0$
- $C(A) \stackrel{\text{def}}{=} 1$ otherwise

Halting Problem (2)

Given such a program H define the following program C : for all programs A

- $C(A) \stackrel{\text{def}}{=} 0$ iff $H(A, A) = 0$
- $C(A) \stackrel{\text{def}}{=} 1$ otherwise

Contradiction

$H(C, C)$ is either **0** or **1**.

- $H(C, C) = 1 \xRightarrow{\text{def } H} C(C) \downarrow \xRightarrow{\text{def } C} H(C, C) = 0$
- $H(C, C) = 0 \xRightarrow{\text{def } H} C(C) \text{ loops} \xRightarrow{\text{def } C} H(C, C) = 1$

Contradiction in both cases. So H cannot exist.