

Automata and Formal Languages (7)

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Two Weeks Ago: CFGs

A **context-free** grammar (CFG) G consists of:

- a finite set of nonterminal symbols (upper case)
- a finite terminal symbols or tokens (lower case)
- a start symbol (which must be a nonterminal)
- a set of rules

$$A \rightarrow \text{rhs}_1 | \text{rhs}_2 | \dots$$

where **rhs** are sequences involving terminals and nonterminals (can also be empty).

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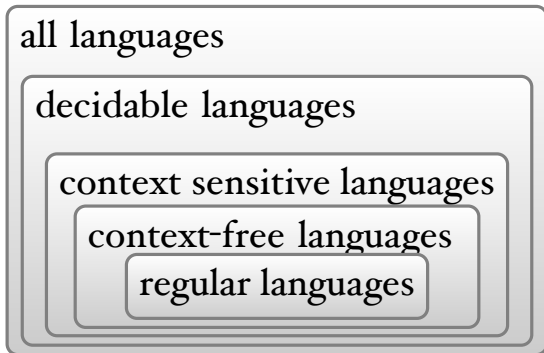
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Hierarchy of Languages

Recall that languages are sets of strings.



Arithmetic Expressions

A grammar for arithmetic expressions and numbers:

$$\begin{aligned} E &\rightarrow E \cdot + \cdot E \mid E \cdot * \cdot E \mid (\cdot E \cdot) \mid N \\ N &\rightarrow N \cdot N \mid 0 \mid 1 \mid \dots \mid 9 \end{aligned}$$

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Numbers

$$N \rightarrow N \cdot N \mid 0 \mid 1 \mid \dots \mid 9$$

A non-left-recursive, non-ambiguous grammar for numbers:

$$N \rightarrow 0 \cdot N \mid 1 \cdot N \mid \dots \mid 0 \mid 1 \mid \dots \mid 9$$

Operator Precedences

To disambiguate

$$E \rightarrow E \cdot + \cdot E \mid E \cdot * \cdot E \mid (\cdot E \cdot) \mid N$$

Decide on how many precedence levels, say
highest for $()$, medium for $*$, lowest for $+$

$$\begin{aligned} E_{low} &\rightarrow E_{med} \cdot + \cdot E_{low} \mid E_{med} \\ E_{med} &\rightarrow E_{hi} \cdot * \cdot E_{med} \mid E_{hi} \\ E_{hi} &\rightarrow (\cdot E_{low} \cdot) \mid N \end{aligned}$$

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What happens with $1 + 3 + 4$?

Removing Left-Recursion

The rule for numbers is directly left-recursive:

$$N \rightarrow N \cdot N \mid 0 \mid 1 \quad (\dots)$$

Translate

$$\begin{array}{l} N \rightarrow N \cdot \alpha \\ \quad \mid \beta \end{array} \quad \Rightarrow \quad \begin{array}{l} N \rightarrow \beta \cdot N' \\ N' \rightarrow \alpha \cdot N' \\ \quad \mid \epsilon \end{array}$$

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Which means

$$\begin{array}{l} N \rightarrow 0 \cdot N' \mid 1 \cdot N' \\ N' \rightarrow N \cdot N' \mid \epsilon \end{array}$$

Chomsky Normal Form

All rules must be of the form

$$A \rightarrow a$$

or

$$A \rightarrow B \cdot C$$

No rule can contain ϵ .

ϵ -Removal

- 1 If $A \rightarrow \alpha \cdot B \cdot \beta$ and $B \rightarrow \epsilon$ are in the grammar, then add $A \rightarrow \alpha \cdot \beta$ (iterate if necessary).
- 2 Throw out all $B \rightarrow \epsilon$.

$$N \rightarrow 0 \cdot N' \mid 1 \cdot N'$$

$$N' \rightarrow N \cdot N' \mid \epsilon$$

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CYK Algorithm

If grammar is in Chomsky normalform ...

$S \rightarrow N \cdot P$

$P \rightarrow V \cdot N$

$N \rightarrow N \cdot N$

$N \rightarrow \text{students} \mid \text{Jeff} \mid \text{geometry} \mid \text{trains}$

$V \rightarrow \text{trains}$

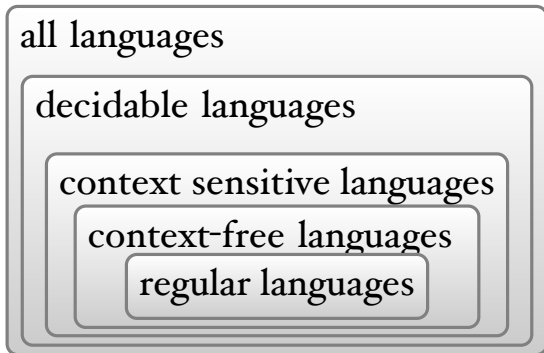
Jeff trains geometry students

CYK Algorithm

- fastest possible algorithm for recognition problem
- runtime is $O(n^3)$
- grammars need to be transferred into CNF

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Context Sensitive Grms

$$S \Rightarrow bSAA \mid \epsilon$$
$$A \Rightarrow a$$
$$bA \Rightarrow Ab$$

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$$A \Rightarrow a$$

$$bA \Rightarrow Ab$$

$$S \Rightarrow \dots \Rightarrow^? \text{ "ababaa"}$$

Stmt → skip
| *Id* := *AExp*
| if *BExp* then *Block* else *Block*
| while *BExp* do *Block*
| read *Id*
| write *Id*
| write *String*

Stmts → *Stmt* ; *Stmts*
| *Stmt*

Block → { *Stmts* }
| *Stmt*

AExp → ...

BExp → ...

```
1  /* Fibonacci numbers implemented in
2     the WHILE language */
3
4  write "Input a number ";
5  read n;
6  x := 0;    // start values
7  y := 1;
8  while n > 0 do {
9     temp := y;
10    y := x + y;
11    x := temp;
12    n := n - 1 // decrement counter
13 };
14 write "Result ";
15 write y
```

An Interpreter

```
{  
   $x := 5;$   
   $y := x * 3;$   
   $y := x * 4;$   
   $x := u * 3$   
}
```

- the interpreter has to record the value of x before assigning a value to y

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- `eval(stmt, env)`

Interpreter

$\text{eval}(n, E)$	$\stackrel{\text{def}}{=} n$
$\text{eval}(x, E)$	$\stackrel{\text{def}}{=} E(x) \quad \text{lookup } x \text{ in } E$
$\text{eval}(a_1 + a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) + \text{eval}(a_2, E)$
$\text{eval}(a_1 - a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) - \text{eval}(a_2, E)$
$\text{eval}(a_1 * a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) * \text{eval}(a_2, E)$
$\text{eval}(a_1 = a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) = \text{eval}(a_2, E)$
$\text{eval}(a_1 \neq a_2, E)$	$\stackrel{\text{def}}{=} \neg(\text{eval}(a_1, E) = \text{eval}(a_2, E))$
$\text{eval}(a_1 < a_2, E)$	$\stackrel{\text{def}}{=} \text{eval}(a_1, E) < \text{eval}(a_2, E)$

Interpreter (2)

$$\text{eval}(\text{skip}, E) \stackrel{\text{def}}{=} E$$

$$\text{eval}(x := a, E) \stackrel{\text{def}}{=} E(x \mapsto \text{eval}(a, E))$$

$$\begin{aligned} \text{eval}(\text{if } b \text{ then } cs_1 \text{ else } cs_2, E) &\stackrel{\text{def}}{=} \\ &\text{if } \text{eval}(b, E) \text{ then } \text{eval}(cs_1, E) \\ &\text{else } \text{eval}(cs_2, E) \end{aligned}$$

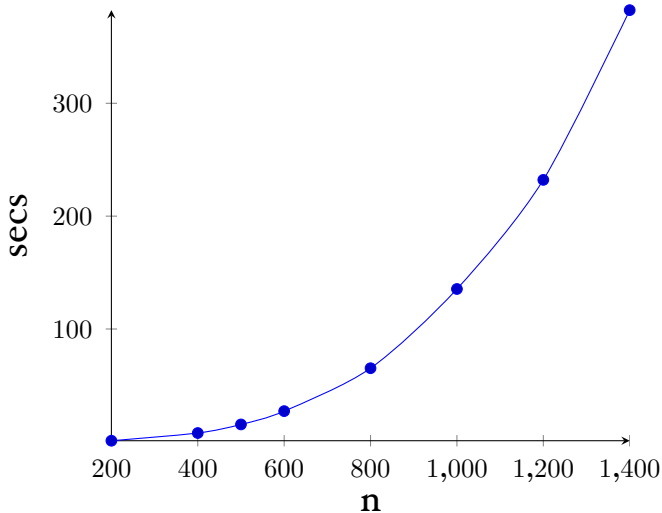
$$\begin{aligned} \text{eval}(\text{while } b \text{ do } cs, E) &\stackrel{\text{def}}{=} \\ &\text{if } \text{eval}(b, E) \\ &\text{then } \text{eval}(\text{while } b \text{ do } cs, \text{eval}(cs, E)) \\ &\text{else } E \end{aligned}$$

$$\text{eval}(\text{write } x, E) \stackrel{\text{def}}{=} \{ \text{println}(E(x)) ; E \}$$

Test Program

```
1 start := 1000; // start value
2 x := start;
3 y := start;
4 z := start;
5 while 0 < x do {
6   while 0 < y do {
7     while 0 < z do { z := z - 1 };
8     z := start;
9     y := y - 1
10  };
11  y := start;
12  x := x - 1
13  };
```

Interpreted Code



Java Virtual Machine

- introduced in 1995
- is a stack-based VM (like Postscript, CLR of .Net)
- contains a JIT compiler
- many languages take advantage of JVM's infrastructure (JRE)
- is garbage collected \Rightarrow no buffer overflows
- some languages compile to the JVM: Scala, Clojure...