Formal Languages (1)





Antikythera automaton, 100 BC (Archimedes?)

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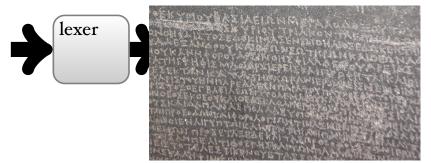
Write A Compiler



```
lexer input: a string
    "read(n);"
lexer output: a sequence of tokens
    key(read); lpar; id(n); rpar; semi
```



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```



lexing ⇒ recognising words (Stone of Rosetta)

parser input: a sequence of token parser output: an abstract syntax tree read lpar rpar code gen lexer

code generator:

istore 2
iload 2
ldc 10
isub
ifeq Label2
iload 2

A Compiler



code generator:

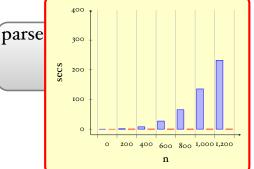
istore 2 iload 2 ldc 10

isub

ifeq Label2
iload 2

. . .

A Compiler



The subject is quite old

- Turing Machines, 1936
- Regular Expressions, 1956
- The first compiler for COBOL, 1957 (Grace Hopper)
- But surprisingly research papers are still published nowadays



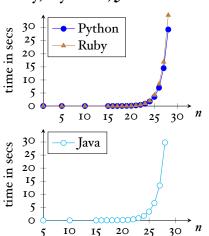
Grace Hopper

(she made it to David Letterman's Tonight Show,

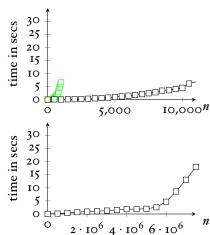
http://www.youtube.com/watch?v=aZOxtURhfEU)

Why Bother?

Ruby, Python, Java



Us (after next lecture)



matching $[a?]{n}[a]{n}$ and [a*]*b against a...a

Lectures 1 - 5

transforming strings into structured data

Lexing

(recognising "words")

Parsing

(recognising "sentences")



Stone of Rosetta

Lectures 1 - 5

transforming strings into structured data

Lexing

based on regular expressions

(recognising "words")

Parsing

(recognising "sentences")



Stone of Rosetta

Familiar Regular Expr.

$$[a-z0-9..]+ @ [a-z0-9..]+ . [a-z.]{2,6}$$

re*	matches o or more times
re+	matches I or more times
re?	matches o or 1 times
re{n}	matches exactly n number of times
re{n,m}	matches at least n and at most m times
[]	matches any single character inside the brackets
[^]	matches any single character not inside the
	brackets
a-zA-Z	character ranges
\d	matches digits; equivalent to [0-9]
•	matches every character except newline
(re)	groups regular expressions and remembers the
	matched text

Today

• While the ultimate goal is to implement a small compiler (a really small one for the JVM)...

Let's start with:

- a web-crawler
- an email harvester
- (a web-scraper)

A Web-Crawler

- o given an URL, read the corresponding webpage
- extract all links from it
- o call the web-crawler again for all these links

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(we need a bound for the number of recursive calls) (the purpose is to check all links on my own webpage)





Scala

A simple Scala function for reading webpages:

```
import io.Source

def get_page(url: String) : String = {
   Source.fromURL(url).take(10000).mkString
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get_page("""http://www.inf.kcl.ac.uk/staff/urbanc/""")
A slightly more complicated version for handling errors:
def get page(url: String) : String = {
  Try(Source.fromURL(url).take(10000).mkString).
     getOrElse { println(s" Problem with: $url"); ""}
```

A Regular Expression

• ... is a pattern or template for specifying strings

```
"https?://[^"]*"
```

matches for example

```
"http://www.foobar.com"
"https://www.tls.org"
```

A Regular Expression

• ... is a pattern or template for specifying strings

```
""""https?://[^"]*""".r
```

matches for example

```
"http://www.foobar.com"
"https://www.tls.org"
```

Finding Operations

rexp.findAllIn(string)

returns a list of all (sub)strings that match the regular expression

rexp.findFirstIn(string)

returns either

- None if no (sub)string matches or
- Some(s) with the first (sub)string

```
val http pattern = """"https?://[^"]*""".r
def unquote(s: String) = s.drop(1).dropRight(1)
def get all URLs(page: String) : Set[String] =
  http pattern.findAllIn(page).map(unquote).toSet
def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else {
    println(s"Visiting: $n $url")
    for (u <- get_all_URLs(get_page(url))) crawl(u, n - 1)</pre>
crawl(some start URL, 2)
```

A version that only crawls links in "my" domain:

```
val my urls = """urbanc"".r
def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else if (my urls.findFirstIn(url) == None) {
    println(s"Visiting: $n $url")
    get page(url); ()
  else {
    println(s"Visiting: $n $url")
    for (u <- get_all_URLs(get_page(url))) crawl(u, n - 1)</pre>
```

A little email harvester:

```
val http pattern = """"https?://[^"]*""".r
val email pattern =
  """([a-z0-9_{.-}]+)@([\da-z.-]+).([a-z.]{2,6})"".r
def print str(s: String) =
  if (s == "") () else println(s)
def crawl(url: String, n: Int) : Unit = {
  if (n == 0) ()
  else {
    println(s"Visiting: $n $url")
    val page = get page(url)
    print str(email_pattern.findAllIn(page).mkString("\n"))
    for (u <- get all URLs(page).par) crawl(u, n - 1)</pre>
```

http://net.tutsplus.com/tutorials/other/8-regular-expressions-you-should-know/

Regular Expressions

Their inductive definition:

```
abstract class Rexp
case object ZERO extends Rexp
case object ONE extends Rexp
case class CHAR(c: Char) extends Rexp
case class ALT(r1: Rexp, r2: Rexp) extends Rexp
case class SEQ(r1: Rexp, r2: Rexp) extends Rexp
case class STAR(r: Rexp) extends Rexp
```

```
r ::= 0 null
\begin{vmatrix} \mathbf{I} & \text{empty string } / \text{"" } / [] \\ c & \text{character} \\ r_1 + r_2 & \text{alternative } / \text{choice} \\ r_1 \cdot r_2 & \text{sequence} \\ r^* & \text{star (zero or more)} \end{vmatrix}
```

Regular Expressions

In Scala:

```
def OPT(r: Rexp) = ALT(r, ONE)

def NTIMES(r: Rexp, n: Int) : Rexp = n match {
  case 0 => ONE
  case 1 => r
  case n => SEQ(r, NTIMES(r, n - 1))
}
```

Strings

...are lists of characters. For example "hello"

$$[h, e, l, l, o]$$
 or just *hello*

the empty string: [] or ""

the concatenation of two strings:

$$s_1 @ s_2$$

Languages, Strings

• **Strings** are lists of characters, for example [], *abc* (Pattern match: *c*::*s*)

• A language is a set of strings, for example

• Concatenation of strings and languages

$$foo @ bar = foobar$$

$$A @ B \stackrel{\text{def}}{=} \{s_1 @ s_2 \mid s_1 \in A \land s_2 \in B\}$$

$$egin{array}{lll} L(\mathbf{o}) &\stackrel{ ext{def}}{=} & \{\} \ L(\mathbf{I}) &\stackrel{ ext{def}}{=} & \{[]\} \ L(c) &\stackrel{ ext{def}}{=} & \{[c]\} \ L(r_{\scriptscriptstyle \mathrm{I}} + r_{\scriptscriptstyle 2}) &\stackrel{ ext{def}}{=} & L(r_{\scriptscriptstyle \mathrm{I}}) \cup L(r_{\scriptscriptstyle 2}) \ L(r_{\scriptscriptstyle \mathrm{I}} \cdot r_{\scriptscriptstyle 2}) &\stackrel{ ext{def}}{=} & \{s_{\scriptscriptstyle \mathrm{I}} @ s_{\scriptscriptstyle 2} \mid s_{\scriptscriptstyle \mathrm{I}} \in L(r_{\scriptscriptstyle \mathrm{I}}) \land s_{\scriptscriptstyle 2} \in L(r_{\scriptscriptstyle 2})\} \ L(r^*) &\stackrel{ ext{def}}{=} & \end{array}$$

$$egin{aligned} L(\mathbf{o}) & \stackrel{ ext{def}}{=} & \{\} \ L(\mathbf{I}) & \stackrel{ ext{def}}{=} & \{[]\} \ L(c) & \stackrel{ ext{def}}{=} & \{[c]\} \ L(r_{ ext{I}} + r_{ ext{2}}) & \stackrel{ ext{def}}{=} & L(r_{ ext{I}}) \cup L(r_{ ext{2}}) \ L(r_{ ext{I}} \cdot r_{ ext{2}}) & \stackrel{ ext{def}}{=} & \{s_{ ext{I}} @ s_{ ext{2}} \mid s_{ ext{I}} \in L(r_{ ext{I}}) \wedge s_{ ext{2}} \in L(r_{ ext{2}})\} \ L(r^*) & \stackrel{ ext{def}}{=} & \{[]\} \ L(r)^{n+1} & \stackrel{ ext{def}}{=} & L(r) @ L(r)^n \end{aligned}$$

$$L(\mathbf{o}) \stackrel{\mathrm{def}}{=} \{\}$$
 $L(\mathbf{I}) \stackrel{\mathrm{def}}{=} \{[]\}$
 $L(c) \stackrel{\mathrm{def}}{=} \{[c]\}$
 $L(r_1 + r_2) \stackrel{\mathrm{def}}{=} L(r_1) \cup L(r_2)$
 $L(r_1 \cdot r_2) \stackrel{\mathrm{def}}{=} \{s_1 @ s_2 \mid s_1 \in L(r_1) \land s_2 \in L(r_2)\}$
 $L(r^*) \stackrel{\mathrm{def}}{=} \{[]\}$
 $L(r)^{n+1} \stackrel{\mathrm{def}}{=} L(r) @ L(r)^n \quad \text{(append on sets)}$
 $\{s_1 @ s_2 \mid s_1 \in L(r) \land s_2 \in L(r)^n\}$

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 $L(r^*) \stackrel{\mathrm{def}}{=} \bigcup_{o \leq n} L(r)^n$
 $L(r)^o \stackrel{\mathrm{def}}{=} \{[]\}$
 $L(r)^{n+1} \stackrel{\mathrm{def}}{=} L(r) @ L(r)^n \quad \text{(append on sets)}$
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The Meaning of Matching

A regular expression *r* matches a string *s* provided

$$s \in L(r)$$

...and the point of the next lecture is to decide this problem as fast as possible (unlike Python, Ruby, Java)

Written Exam

- Accounts for 80%.
- You will understand the question "Is this relevant for the exam?" is very demotivating for the lecturer!
- Deal: Whatever is in the homework (and is not marked "optional") is relevant for the exam.
- Each lecture has also a handout. There are also handouts about notation and Scala.

Coursework

• Accounts for 20%. Two strands. Choose **one**!

Strand 1

- four programming tasks:
 - matcher (4%, 20.10.)
 - lexer (5%, 03.11.)
 - parser (5%, 24.11.)
 - compiler (6%, 13.12.)

Strand 2

- one task: prove the correctness of a regular expression matcher in the Isabelle theorem prover
- 20%, submission 13.12.
- Solving more than one strand will **not** give you more marks.
- The exam will contain in much, much smaller form elements from both (but will also be in lectures and HW).

Questions?