CSCI 742 - Compiler Construction

Lecture 24

Subtyping

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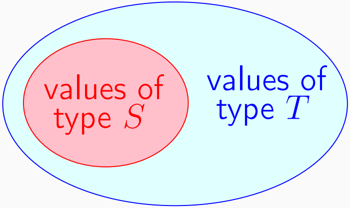
Subtyping

Type: a set of values together with a set of valid operations on those values

Subtype: $S <: T$

A value of type S may be used wherever a value of type T is expected

- values(S) $\subseteq$ values(T)



- All operators valid on T values are valid on S values

Subtyping Example

`Integer <: Object`

```
Object o = new Integer(13);  // ok
Integer i = new Object();    // type error
```

`short <: int`

```
void f(int x) {
  x = 10;
}
short s = 0;
f(s);  // ok
```

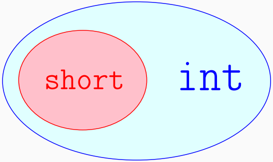
Subtyping Rule

- Rule for subtyping: analogous to set reasoning

$$\frac{\Gamma \vdash e : T_1 \qquad T_1 <: T_2}{\Gamma \vdash e : T_2}$$

In terms of sets

$$\frac{\Gamma \vdash e \in T_1 \qquad T_1 \subseteq T_2}{\Gamma \vdash e \in T_2}$$



Types for Positive and Negative Ints

$$\texttt{int} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$$

$$\texttt{pos} = \{1, 2, \cdots\} \quad \text{(not including zero)}$$

$$\texttt{neg} = \{\cdots, -2, -1\} \quad \text{(not including zero)}$$

types:

$$\texttt{pos} <: \texttt{int}$$

$$\texttt{neg} <: \texttt{int}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{pos}}{\Gamma \vdash x + y : \texttt{pos}}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x \times y : \texttt{neg}}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{pos}}{\Gamma \vdash x / y : \texttt{pos}}$$

sets:

$$\texttt{pos} \subseteq \texttt{int}$$

$$\texttt{neg} \subseteq \texttt{int}$$

$$\frac{\Gamma \vdash x \in \texttt{pos} \qquad \Gamma \vdash y \in \texttt{pos}}{\Gamma \vdash x + y \in \texttt{pos}}$$

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$$\frac{\Gamma \vdash x \in \texttt{pos} \qquad \Gamma \vdash y \in \texttt{pos}}{\Gamma \vdash x / y \in \texttt{pos}}$$

Rules for Positive and Negative Ints

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x + y : ???}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x \times y : ???}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{int}}{\Gamma \vdash x + y : ???}$$

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Rules for Positive and Negative Ints

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x + y : \texttt{int}}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x \times y : \texttt{neg}}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{int}}{\Gamma \vdash x + y : \texttt{int}}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{int}}{\Gamma \vdash x \times y : \texttt{int}}$$

More Rules

$$\frac{\Gamma \vdash x : \texttt{neg} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x \times y : \texttt{pos}}$$

$$\frac{\Gamma \vdash x : \texttt{neg} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x + y : \texttt{neg}}$$

More rules for division:

$$\frac{\Gamma \vdash x : \texttt{neg} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x / y : \texttt{pos}}$$

$$\frac{\Gamma \vdash x : \texttt{pos} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x / y : \texttt{neg}}$$

$$\frac{\Gamma \vdash x : \texttt{int} \qquad \Gamma \vdash y : \texttt{neg}}{\Gamma \vdash x / y : \texttt{int}}$$

Making Rules Useful

- Let x be a variable

$$\frac{\Gamma \vdash x : \texttt{int} \qquad \Gamma \oplus \{(x, \texttt{pos})\} \vdash e_1 : \texttt{void} \qquad \Gamma \vdash e_2 : \texttt{void}}{\Gamma \vdash \left(\texttt{if}\ \ (x > 0)\ e_1\ \texttt{else}\ e_2\right) : \texttt{void}}$$

$$\frac{\Gamma \vdash x : \texttt{int} \qquad \Gamma \vdash e_1 : T \qquad \Gamma \oplus \{(x, \texttt{neg})\} \vdash e_2 : \texttt{void}}{\Gamma \vdash \left(\texttt{if}\ \ (x >= 0)\ e_1\ \texttt{else}\ e_2\right) : \texttt{void}}$$

Exercise

Using type systems prove there is no division by zero in computing `res` (initial environment is $\{(\texttt{x}, \texttt{int}), (\texttt{y}, \texttt{int}), (\texttt{res}, \texttt{int})\}$)

```
if (y > 0) {
  if (x > 0) {
    res = 10 / (x*y);
  }
}
```

Subtyping Example

- Does the last statement type check?

```
pos f(int x) {
  if (x < 0) return -x; else return x+1;}
pos p;
int q;
q = f(p);
```

Initial environment: $\Gamma = \{(\texttt{q}, \texttt{int}), (\texttt{p}, \texttt{pos}), (\texttt{f}, \ \texttt{int} \rightarrow \texttt{pos})\}$

$$\dfrac{\dfrac{(\texttt{q}, \texttt{int}) \in \Gamma}{\Gamma \vdash \texttt{q} : \texttt{int}} \qquad \dfrac{\dfrac{\dfrac{\Gamma \vdash \texttt{p} : \texttt{pos} \qquad \texttt{pos} <: \texttt{int}}{\Gamma \vdash \texttt{p} : \texttt{int}} \qquad \Gamma \vdash \texttt{f} : \texttt{int} \rightarrow \texttt{pos}}{\Gamma \vdash \texttt{f(p)} \ : \ \texttt{pos}} \qquad \texttt{pos} <: \texttt{int}}{\Gamma \vdash \texttt{f(p)} \ : \ \texttt{int}}}{\Gamma \vdash \left(\texttt{q = f(p)}\right) \ : \ \texttt{void}}$$

Subtyping Example

- Does the last statement type check?

```
pos f(pos  x) {
  if (x < 0) return -x; else return x+1;}
int p;
int q;
q = f(p);
```

Type Checking

- Rules for checking code must allow a subtype where a supertype was expected
- Old rule for assignment:

$$\frac{x : T \in \Gamma \qquad \Gamma \vdash e : T}{\Gamma \vdash x = e}$$

- What do we need to change here?

Type Checking

- Rules for checking code must allow a subtype where a supertype was expected
- New rule for assignment:

$$\frac{\Gamma \vdash e : T' \qquad T' <: T \qquad (x, T) \in \Gamma}{\Gamma \vdash x = e} = \frac{\begin{array}{c}\Gamma \vdash e : T' \\ T' <: T\end{array}}{\Gamma \vdash e : T} + \frac{\begin{array}{c}\Gamma \vdash e : T \\ (x, T) \in \Gamma\end{array}}{\Gamma \vdash x = e}$$

Type Checking in Practice

```
class Assignment extends Statement {
  Symbol x;
  Expression e;
  // ...
  Type typeCheck(Environment gamma) {
    Type tp = e.typeCheck(gamma);
    Type t = x.typeCheck(gamma);
    if (tp.subtype(t))
      return t;
    else
      throw new TypeError("type mismatch in assignment");
  }
}
```

$$\frac{\Gamma \vdash e : T' \qquad T' <: T \qquad (x, T) \in \Gamma}{\Gamma \vdash x = e}$$

Parametrized Types

- Suppose we know that $S <: T$ (S is a subtype of T)
- Given a parameterized type constructor TYCON$[T]$, there are three possibilities for the relationship between TYCON$[S]$ and TYCON$[T]$

1. **Invariant:** TYCON$[S]$ and TYCON$[T]$ are unrelated
2. **Covariant:** TYCON$[S] <:$ TYCON$[T]$
3. **Contravariant:** TYCON$[S] :>$ TYCON$[T]$

Subtyping for Products

- Type for a tuple

$$\frac{\Gamma \vdash e_1 : T_1 \qquad \Gamma \vdash e_2 : T_2}{\Gamma \vdash (e_1, e_2) : T_1 \times T_2}$$

$$\frac{\dfrac{\Gamma \vdash e_1 : T_1 \qquad T_1 <: T_1'}{\Gamma \vdash e_1 : T_1'} \qquad \dfrac{\Gamma \vdash e_2 : T_2 \qquad T_2 <: T_2'}{\Gamma \vdash e_2 : T_2'}}{\Gamma \vdash (e_1, e_2) : T_1' \times T_2'}$$

- Covariant subtyping for Product of types

$$\frac{T_1 <: T_1' \qquad T_2 <: T_2'}{T_1 \times T_2 <: T_1' \times T_2'}$$

Analogy with Cartesian Product

$$\frac{T_1 <: T_1' \qquad T_2 <: T_2'}{T_1 \times T_2 <: T_1' \times T_2'}$$

$$\frac{T_1 \subseteq T_1' \qquad T_2 \subseteq T_2'}{T_1 \times T_2 \subseteq T_1' \times T_2'}$$