Compilers and Formal Languages (9)

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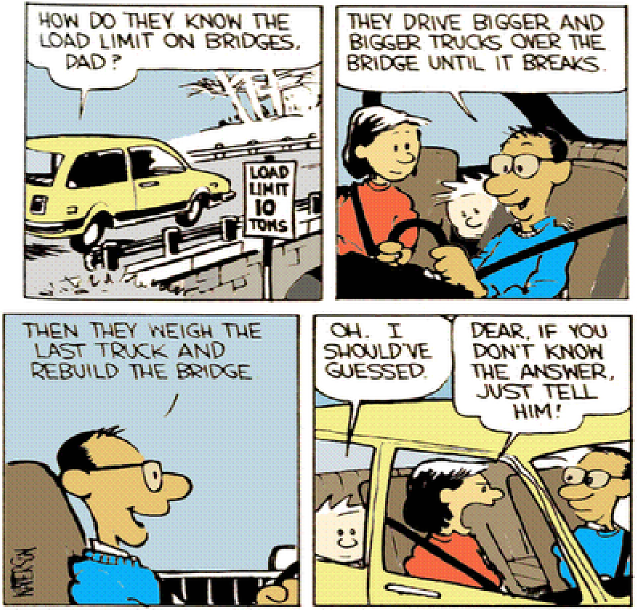
Slides: KEATS (also home work is there)


HOW DO THEY KNOW THE LOAD LIMIT ON BRIDGES, DAD?
LOAD LIMIT 10 TONS
THEY DRIVE BIGGER AND BIGGER TRUCKS OVER THE BRIDGE UNTIL IT BREAKS.
THEN THEY WEIGH THE LAST TRUCK AND REBUILD THE BRIDGE.
OH. I SHOULD'VE GUESSED.
DEAR, IF YOU DON'T KNOW THE ANSWER, JUST TELL HIM!
WATTERSON

Old-Fashioned Eng. vs. CS

bridges:
engineers can "look" at a bridge and have a pretty good intuition about whether it will hold up or not (redundancy; predictive theory)

code:
programmers have very little intuition about their code; often it is too expensive to have redundancy; not "continuous"

Dijkstra on Testing

"Program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence."

Proving Programs to be Correct

Theorem: There are infinitely many prime numbers.

Proof ...

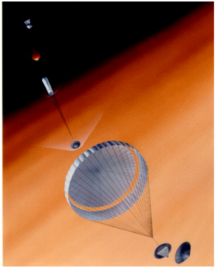
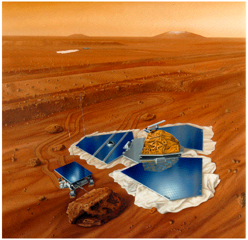
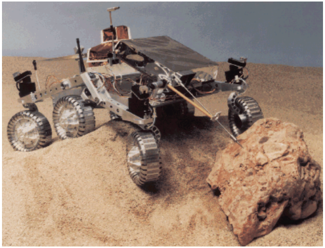
similarly

Theorem: The program is doing what it is supposed to be doing.

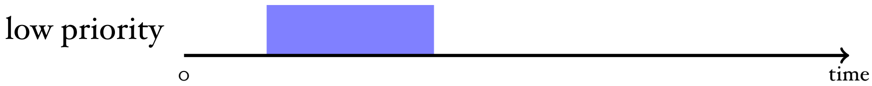
Long, long proof ...

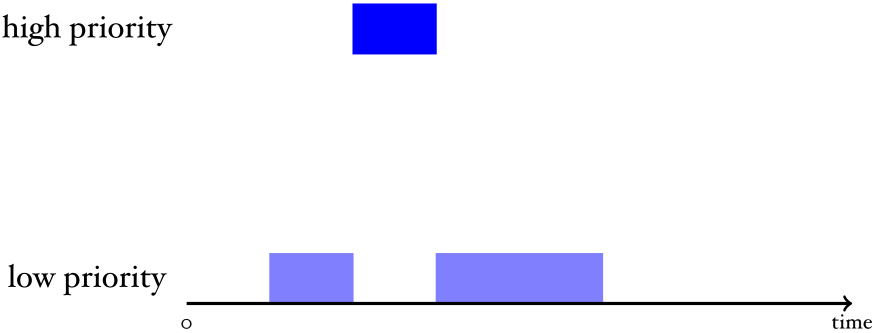
This can be a gigantic proof. The only hope is to have help from the computer. 'Program' is here to be understood to be quite general (protocols, OS, ...).

Mars Pathfinder Mission 1997

- despite NASA's famous testing procedures, the lander crashed frequently on Mars
- a scheduling algorithm was not used in the OS


low priority
0
time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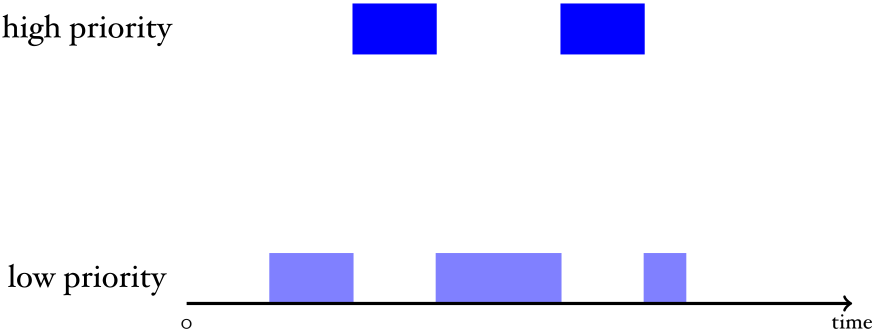
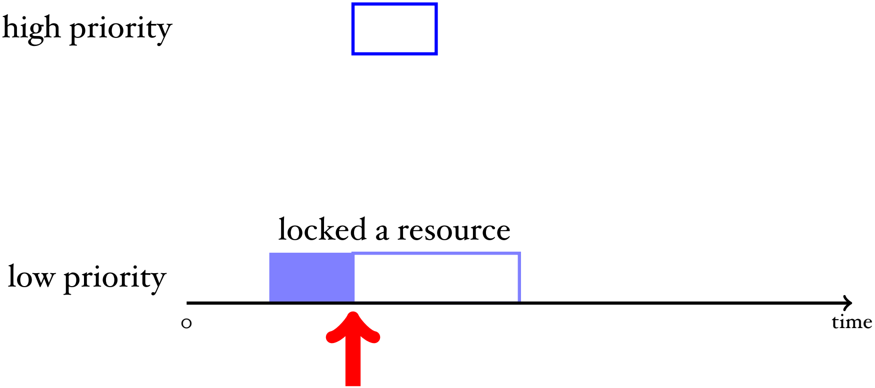


Scheduling: You want to avoid that a high priority process is starved indefinitely.



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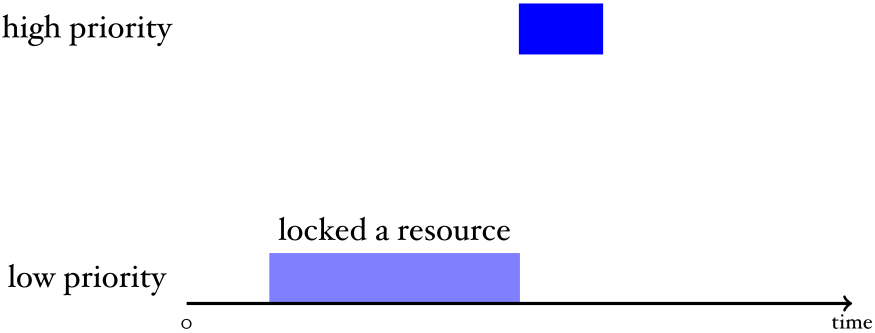

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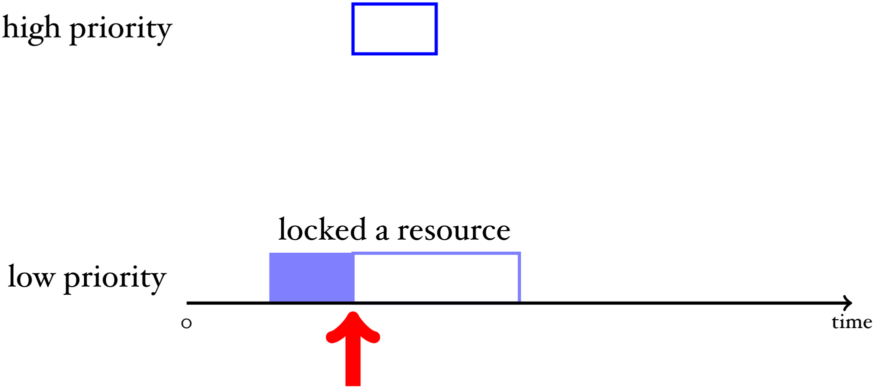

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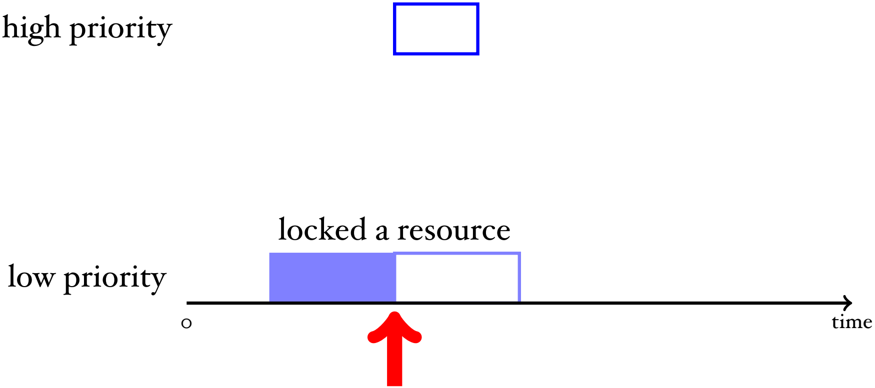

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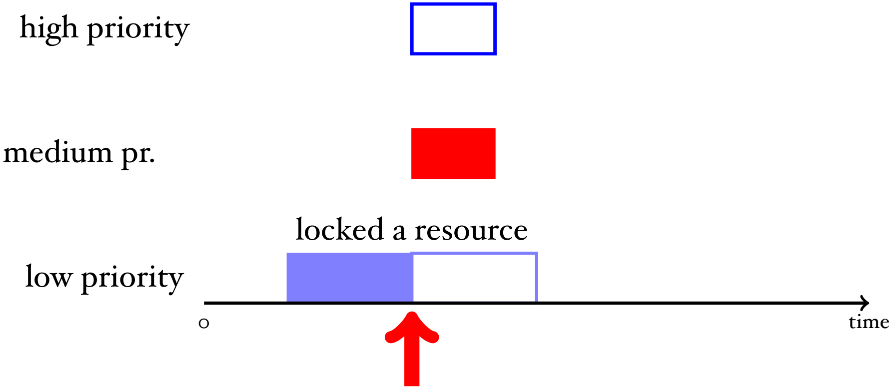

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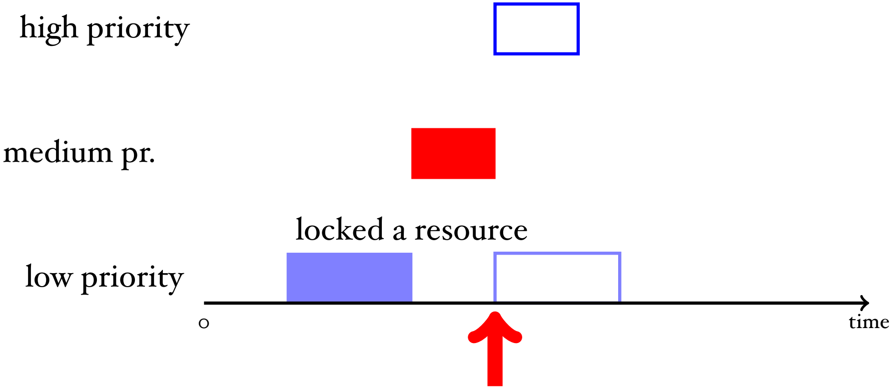

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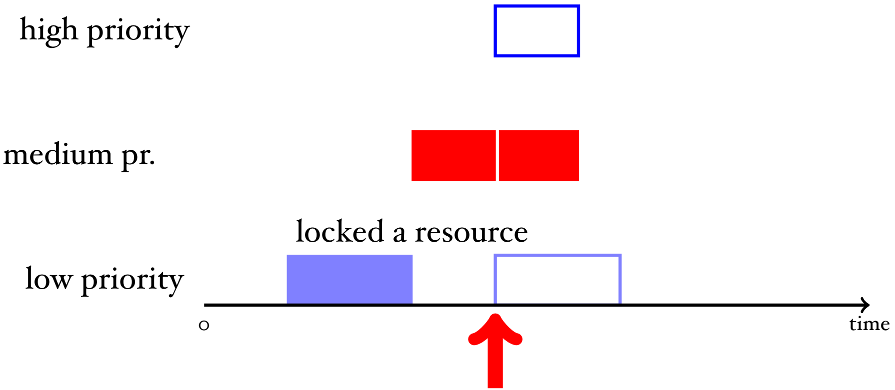

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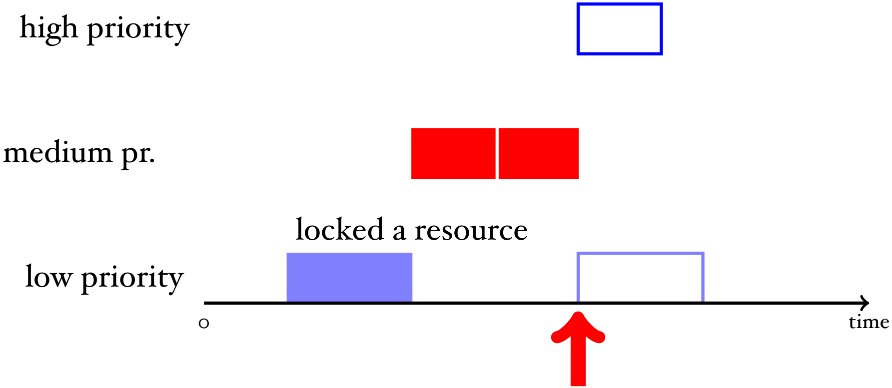

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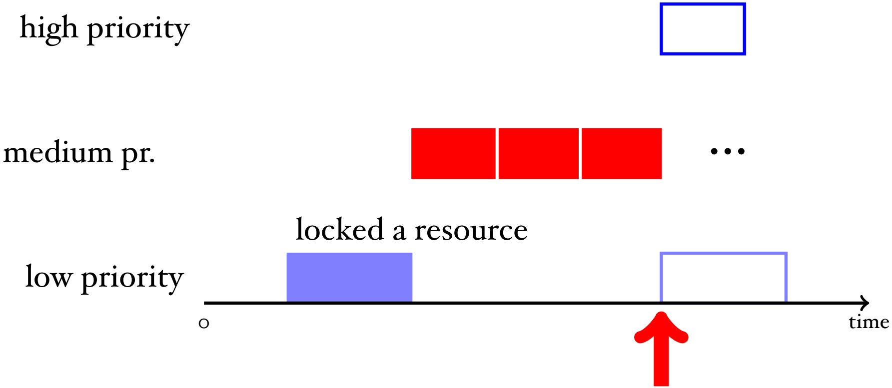

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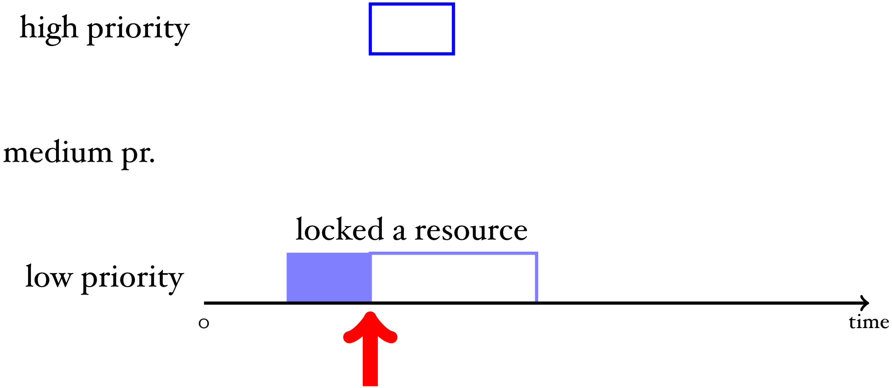

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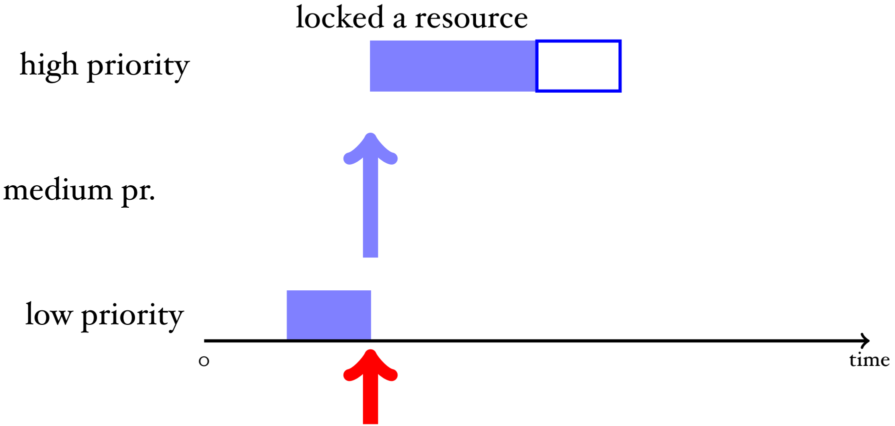

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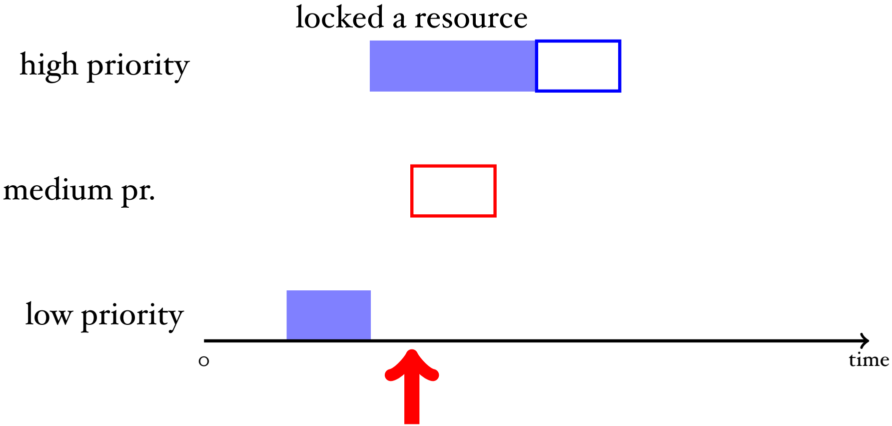
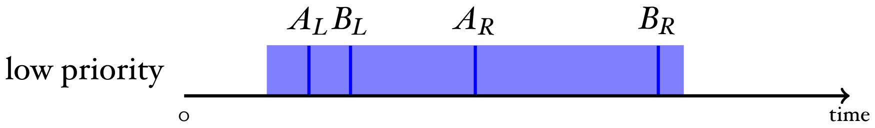


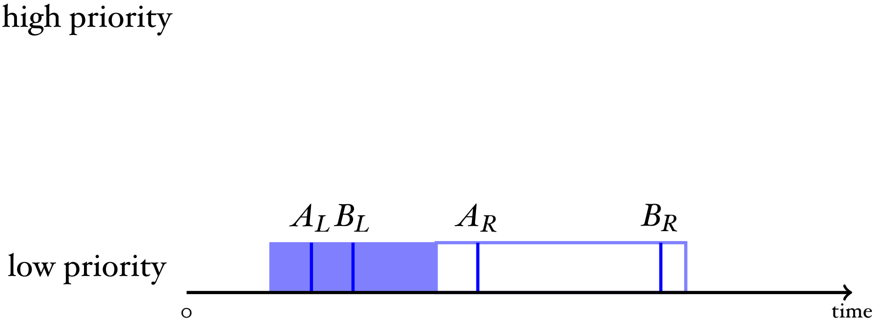
Scheduling: You want to avoid that a high priority process is starved indefinitely.

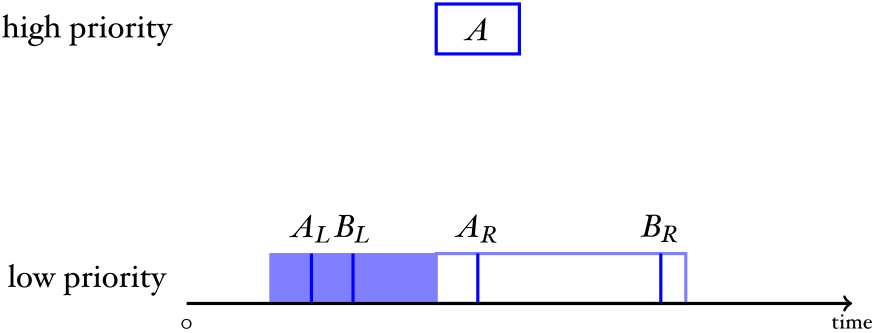
Priority Inheritance Scheduling

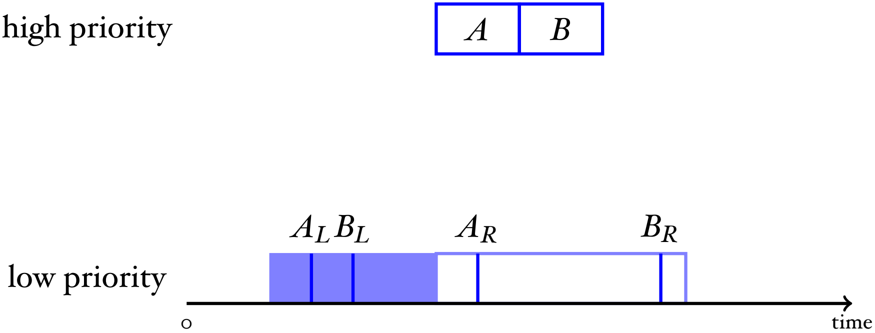
- Let a low priority process L temporarily inherit the high priority of H until L leaves the critical section unlocking the resource.
- Once the resource is unlocked L returns to its original priority level.

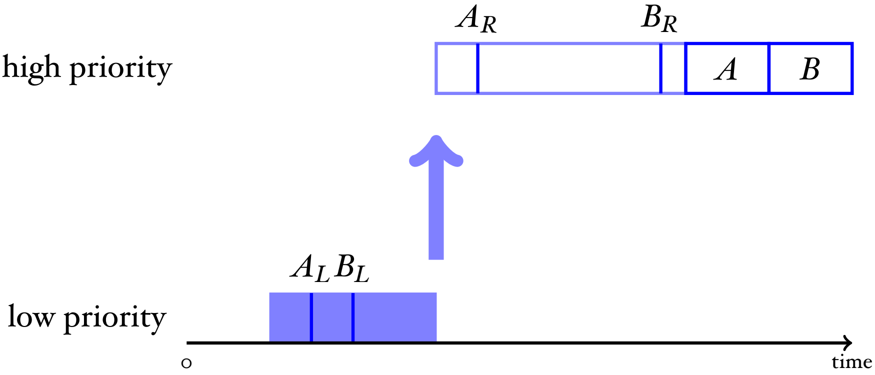


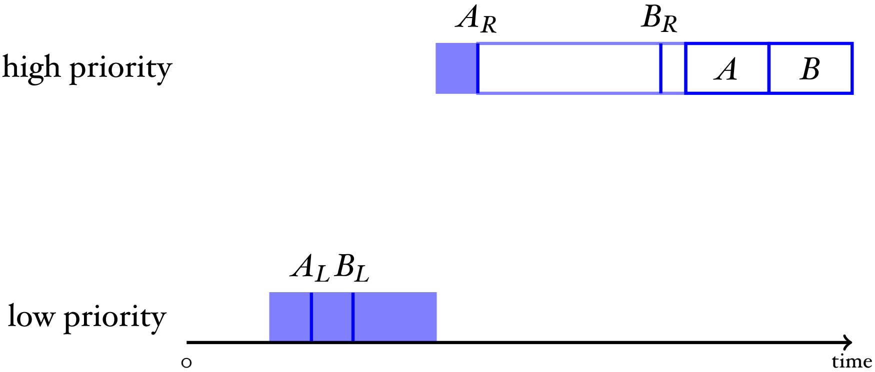
A_L B_L
A_R
B_R
low priority
0
time


high priority
low priority
A_L B_L
A_R
B_R
0
time

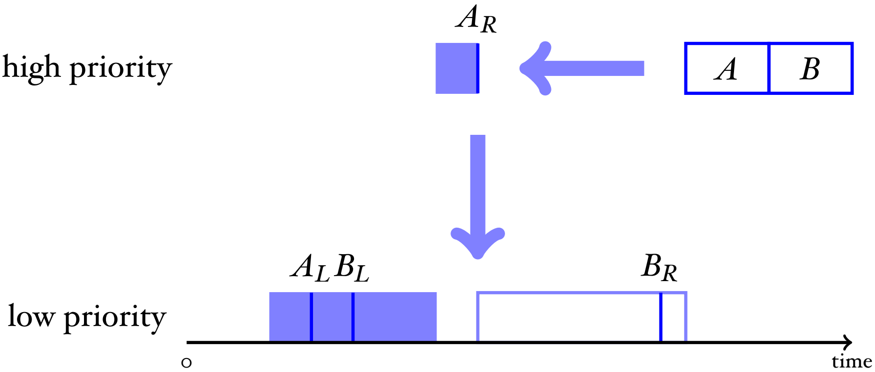

high priority
A
low priority
A_L B_L
A_R
B_R
0
time

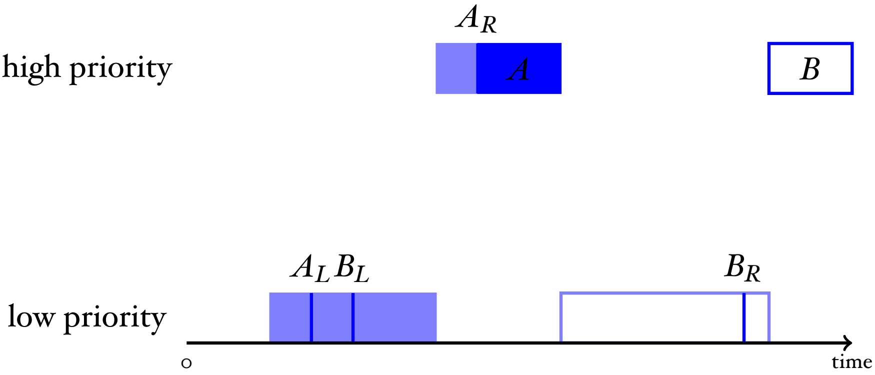

high priority
A
B
low priority
A_L B_L
A_R
B_R
0
time

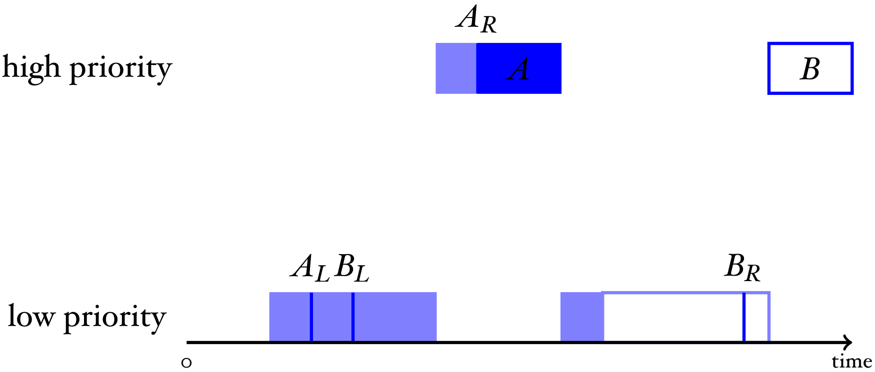

A_R
B_R
high priority
A
B
A_L B_L
low priority
0
time

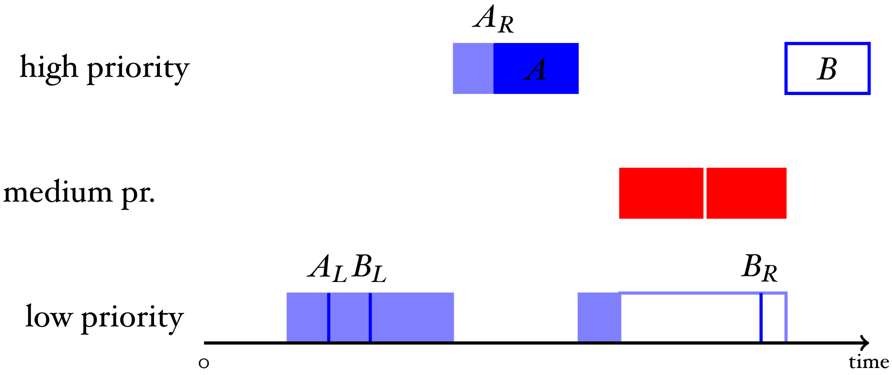


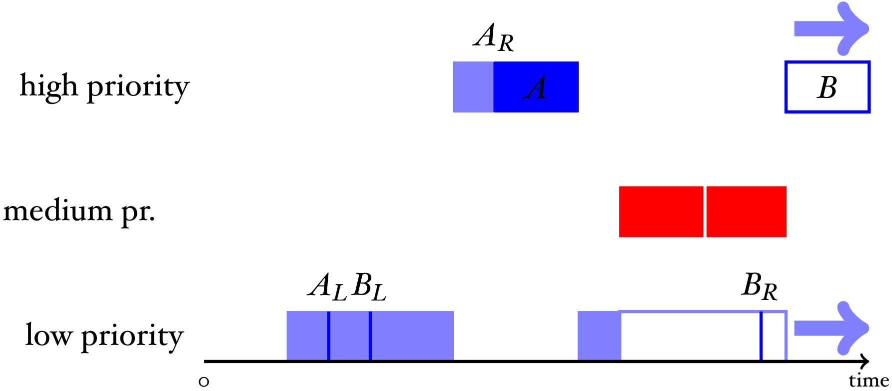
A_R
B_R
high priority
A
B
A_L B_L
low priority
0
time


high priority
A_R
A
B
low priority
A_L B_L
B_R
0
time


A_R
high priority
A
B
A_L B_L
B_R
low priority
0
time


A_R
high priority
A
B
A_L B_L
B_R
low priority
0
time


A_R
high priority
A
B
medium pr.
A_L B_L
B_R
low priority
0
time



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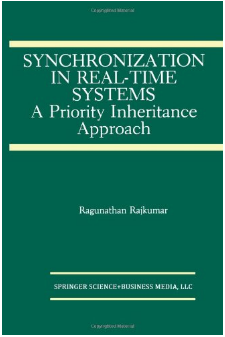
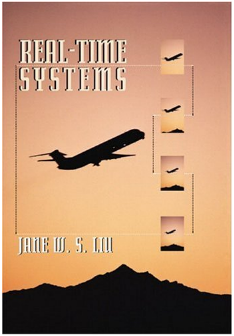
- ...L needs to switch to the highest remaining priority of the threads that it blocks.

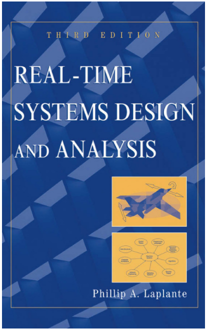
this error is already known since around 1999



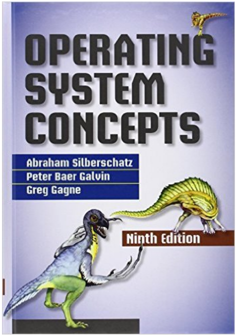
- by Rajkumar, 1991
- *"it resumes the priority it had at the point of entry into the critical section"*



- by Jane Liu, 2000
- *"The job J_l executes at its inherited priority until it releases R; at that time, the priority of J_l returns to its priority at the time when it acquires the resource R."*
- gives pseudo code and totally bogus data structures
- interesting part "*left as an exercise*"



- by Laplante and Ovaska, 2011 ($113.76)
- *"when [the task] exits the critical section that caused the block, it reverts to the priority it had when it entered that section"*



- by Silberschatz, Galvin, and Gagne, 2013 (OS-bible)
- *"Upon releasing the lock, the [low-priority] thread will revert to its original priority."*

Priority Scheduling

- a scheduling algorithm that is widely used in real-time operating systems
- has been "proved" correct by hand in a paper in 1983
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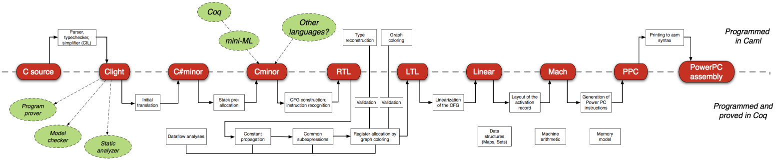
- but this algorithm turned out to be incorrect, despite its “proof”

- we used the corrected algorithm and then **really** proved that it is correct
- we implemented this algorithm in a small OS called PINTOS (used for teaching at Stanford)
- our implementation was much more efficient than their reference implementation

Big Proofs in CS (1)

Formal proofs in CS sound like science fiction?

- in 2008, verification of a C-compiler
  - "if my input program has a certain behaviour, then the compiled machine code has the same behaviour"
  - is as good as `gcc -O1`, but much less buggy



Big Proofs in CS (2)

- in 2010, verification of a micro-kernel operating system (approximately 8700 loc)
  - used in helicopters and mobile phones
  - 200k loc of proof
  - 25 - 30 person years
  - found 160 bugs in the C code (144 by the proof)

> "Real-world operating-system kernel with an end-to-end proof of implementation correctness and security enforcement"

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unhackable kernel (New Scientists, September 2015)

Big Proofs in CS (3)

- verified TLS implementation
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- verified distributed systems (Verdi)
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- Infer static analyser developed by Facebook

How Did This Happen?

Lots of ways!

- better programming languages
  - basic safety guarantees built in
  - powerful mechanisms for abstraction and modularity
- better software development methodology
- stable platforms and frameworks
- better use of specifications

If you want to build software that works or is secure, it is helpful to know what you mean by "works" and by "is secure"!

Goal

Remember the Bridges example?

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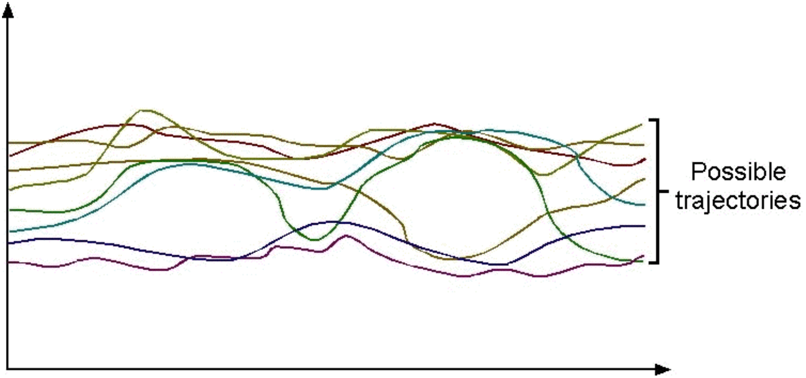
- Can we look at our programs and somehow ensure they are bug free/correct?

- Very hard: Anything interesting about programs is equivalent to the Halting Problem, which is undecidable.

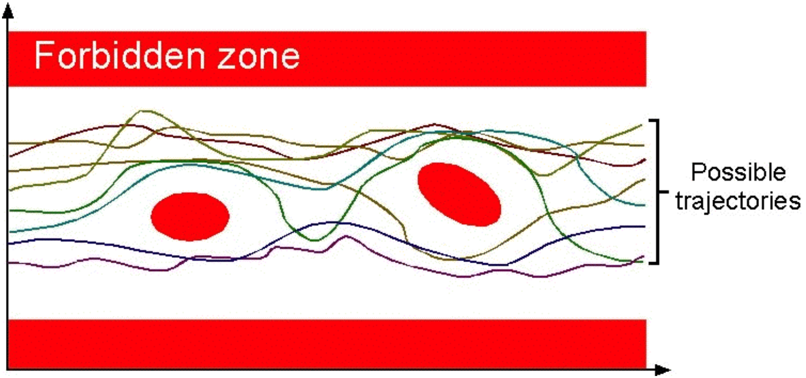
- Solution: We avoid this "minor" obstacle by being as close as possible of deciding the halting problem, without actually deciding the halting problem. $\Rightarrow$ yes, no, don't know (static analysis)

What is Static Analysis?



- depending on some initial input, a program (behaviour) will "develop" over time.

What is Static Analysis?



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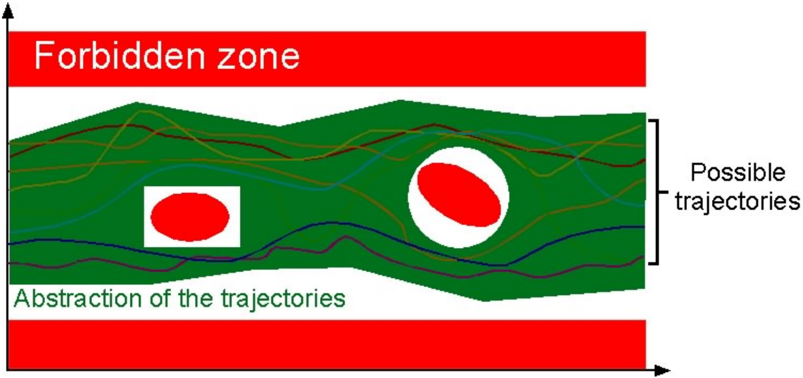

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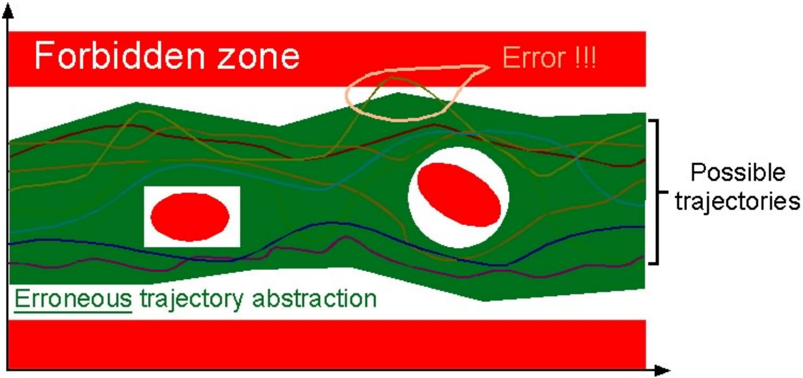

- to be avoided

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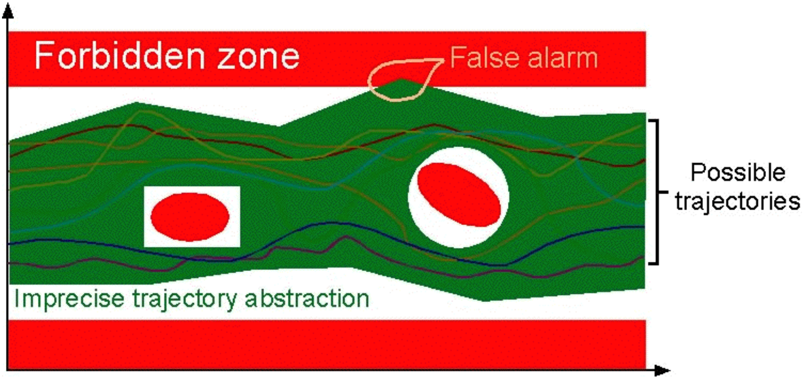
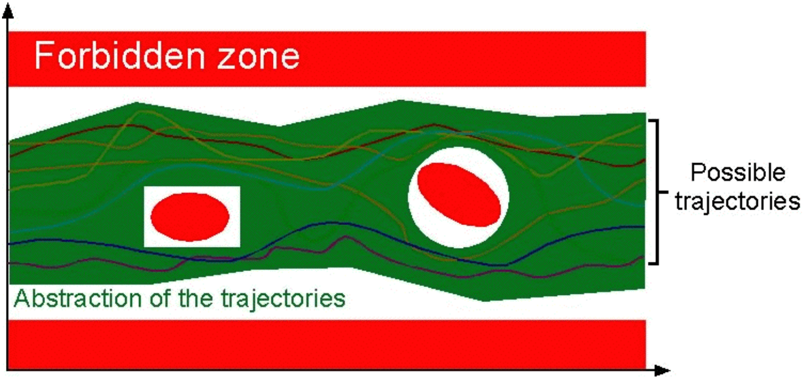


- this needs more work

What is Static Analysis?



Concrete Example: Are Vars Definitely Initialised?

Assuming x is initialised, what about y?

Prog. 1:

```
if x < 1 then y := x else y := x + 1;
y := y + 1
```

Prog. 2:

```
if x < x then y := y + 1 else y := x;
y := y + 1
```

Concrete Example: Are Vars Definitely Initialised?

What should the rules be for deciding when a variable is initialised?

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What should the rules be for deciding when a variable is initialised?

- variable `x` is definitely initialized after `skip` iff `x` is definitely initialized before `skip`.

A is the set of definitely defined variables:

$$\frac{}{A\ \texttt{skip}\ A} \qquad \frac{vars(a) \subseteq A}{A\ \ (\texttt{x := a})\ \ \{x\} \cup A}$$

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$$\frac{A_1\ s_1\ A_2 \quad A_2\ s_2\ A_3}{A_1\ (s_1; s_2)\ A_3}$$

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$$\frac{vars(b) \subseteq A \quad A\ s\ A'}{A\ (\texttt{while}\ b\ \texttt{do}\ s)\ A}$$

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$$\frac{vars(b) \subseteq A \quad A\ s\ A'}{A\ (\texttt{while}\ b\ \texttt{do}\ s)\ A}$$

we start with $A = \{\}$

Concrete Example: Sign-Analysis

$$
\begin{array}{lcl}
\langle \mathit{Exp} \rangle & ::= & \langle \mathit{Exp} \rangle \;\texttt{+}\; \langle \mathit{Exp} \rangle \\
& \mid & \langle \mathit{Exp} \rangle \;\texttt{*}\; \langle \mathit{Exp} \rangle \\
& \mid & \langle \mathit{Exp} \rangle \;\texttt{=}\; \langle \mathit{Exp} \rangle \\
& \mid & \langle \mathit{num} \rangle \\
& \mid & \langle \mathit{var} \rangle \\
\langle \mathit{Stmt} \rangle & ::= & \langle \mathit{label} \rangle \;\texttt{:} \\
& \mid & \langle \mathit{var} \rangle \;\texttt{:=}\; \langle \mathit{Exp} \rangle \\
& \mid & \texttt{jmp?}\; \langle \mathit{Exp} \rangle\; \langle \mathit{label} \rangle \\
& \mid & \texttt{goto}\; \langle \mathit{label} \rangle \\
\langle \mathit{Prog} \rangle & ::= & \langle \mathit{Stmt} \rangle \;\ldots\; \langle \mathit{Stmt} \rangle
\end{array}
$$

```
      a := 1
      n := 5
top:  jmp? n = 0 done
      a := a * n
      n := n + -1
      goto top
done:
```

Concrete Example: Sign-Analysis

$$
\begin{array}{lcl}
\langle \mathit{Exp} \rangle & ::= & \langle \mathit{Exp} \rangle \texttt{ + } \langle \mathit{Exp} \rangle \\
& | & \langle \mathit{Exp} \rangle \texttt{ * } \langle \mathit{Exp} \\
& | & \langle \mathit{Exp} \rangle \texttt{ = } \langle \mathit{Exp} \\
& | & \langle \mathit{num} \rangle \\
& | & \langle \mathit{var} \rangle \\
\langle \mathit{Stmt} \rangle & ::= & \langle \mathit{label} \rangle \texttt{ :} \\
& | & \langle \mathit{var} \rangle \texttt{ := } \langle \mathit{Exp} \\
& | & \texttt{jmp? } \langle \mathit{Exp} \rangle \; \langle \\
& | & \texttt{goto } \langle \mathit{label} \rangle \\
\langle \mathit{Prog} \rangle & ::= & \langle \mathit{Stmt} \rangle \ldots \langle \mathit{Stmt} \rangle
\end{array}
$$

```
      n := 6
      m1 := 0
      m2 := 1
top:  jmp? n = 0 done
      tmp := m2
      m2 := m1 + m2
      m1 := tmp
      n := n + -1
      goto top
done:
```

Concrete Example: Sign-Analysis

$$
\begin{array}{rcl}
\langle \mathit{Exp} \rangle & ::= & \langle \mathit{Exp} \rangle \ \texttt{+} \ \langle \mathit{Exp} \rangle \\
& | & \langle \mathit{Exp} \rangle \ \texttt{*} \ \langle \mathit{Exp} \rangle \\
& | & \langle \mathit{Exp} \rangle \ \texttt{=} \ \langle \mathit{Exp} \rangle \\
& | & \langle \mathit{num} \rangle \\
& | & \langle \mathit{var} \rangle \\
\langle \mathit{Stmt} \rangle & ::= & \langle \mathit{label} \rangle \ \texttt{:} \\
& | & \langle \mathit{var} \rangle \ \texttt{:=} \ \langle \mathit{Exp} \rangle \\
& | & \texttt{jmp?} \ \langle \mathit{Exp} \rangle \ \langle \mathit{label} \rangle \\
& | & \texttt{goto} \ \langle \mathit{label} \rangle \\
\langle \mathit{Prog} \rangle & ::= & \langle \mathit{Stmt} \rangle \ \ldots \ \langle \mathit{Stmt} \rangle
\end{array}
$$

Eval: An Interpreter

$$
\begin{array}{lcl}
[n]_{env} & \stackrel{\text{def}}{=} & n \\
[x]_{env} & \stackrel{\text{def}}{=} & env(x) \\
[e_1 + e_2]_{env} & \stackrel{\text{def}}{=} & [e_1]_{env} + [e_2]_{env} \\
[e_1 * e_2]_{env} & \stackrel{\text{def}}{=} & [e_1]_{env} * [e_2]_{env} \\
[e_1 = e_2]_{env} & \stackrel{\text{def}}{=} & \begin{cases} 1 & \text{if} \quad [e_1]_{env} = [e_2]_{env} \\ 0 & \text{otherwise} \end{cases}
\end{array}
$$

```
def eval_exp(e: Exp, env: Env) : Int = e match {
  case Num(n) => n
  case Var(x) => env(x)
  case Plus(e1, e2) => eval_exp(e1, env) + eval_exp(e2, env)
  case Times(e1, e2) => eval_exp(e1, env) * eval_exp(e2, env)
  case Equ(e1, e2) =>
    if (eval_exp(e1, env) == eval_exp(e2, env)) 1 else 0
}
```

A program

```
      a := 1
      n := 5
top:  jmp? n = 0 done
      a := a * n
      n := n + -1
      goto top
done:
```

The *snippets* of the program:

```
""    a := 1
      n := 5
top:  jmp? n = 0 done
      a := a * n
      n := n + -1
      goto top
done:
```

```
top:  jmp? n = 0 done
      a := a * n
      n := n + -1
      goto top
done:
```

```
done:
```

Eval for Stmts

Let sn be the snippets of a program

$$
\begin{array}{lcl}
[\mathtt{nil}]_{env} & \stackrel{\text{def}}{=} & env \\
[\mathtt{Label}(l:) :: rest]_{env} & \stackrel{\text{def}}{=} & [rest]_{env} \\
[x := a :: rest]_{env} & \stackrel{\text{def}}{=} & [rest]_{(env[x:=[a]_{env}])} \\
[\mathtt{jmp?}\ b\ l :: rest]_{env} & \stackrel{\text{def}}{=} & \begin{cases} [sn(l)]_{env} & \text{if} \quad [b]_{env} = true \\ [rest]_{env} & \text{otherwise} \end{cases} \\
[\mathtt{goto}\ l :: rest]_{env} & \stackrel{\text{def}}{=} & [sn(l)]_{env}
\end{array}
$$

Start with $[sn(\text{""})]_{\varnothing}$

Eval in Code

```
def eval(sn: Snips) : Env = {
  def eval_stmts(sts: Stmts, env: Env) : Env = sts match {
    case Nil => env

    case Label(l)::rest => eval_stmts(rest, env)

    case Assign(x, e)::rest =>
      eval_stmts(rest, env + (x -> eval_exp(e, env)))

    case Jmp(b, l)::rest =>
      if (eval_exp(b, env) == 1) eval_stmts(sn(l), env)
      else eval_stmts(rest, env)

    case Goto(l)::rest => eval_stmts(sn(l), env)
  }

  eval_stmts(sn(""), Map())
}
```

The Idea of Static Analysis

```
      a := 1
      n := 5
top:  jmp? n = 0 done
      a := a * n
      n := n + -1
      goto top
done:
```

$\Rightarrow$

```
      a := '+'
      n := '+'
top:  jmp? n = '0' done
      a := a * n
      n := n + '-'
      goto top
done:
```

Replace all constants and variables by either `+`, `-` or `0`. What we want to find out is what the sign of `a` and `n` is (they should always positive).

Sign Analysis?

e_1	e_2	$e_1 + e_2$
$-$	$-$	$-$
$-$	0	$-$
$-$	$+$	$-, 0, +$
0	x	x
$+$	$-$	$-, 0, +$
$+$	0	$+$
$+$	$+$	$+$

e_1	e_2	$e_1 * e_2$
$-$	$-$	$+$
$-$	0	0
$-$	$+$	$-$
0	x	0
$+$	$-$	$-$
$+$	0	0
$+$	$+$	$+$

$$
\begin{array}{lcl}
[n]_{aenv} & \stackrel{\text{def}}{=} & \begin{cases} \{+\} & \text{if } n > 0 \\ \{-\} & \text{if } n < 0 \\ \{0\} & \text{if } n = 0 \end{cases} \\
[x]_{aenv} & \stackrel{\text{def}}{=} & aenv(x) \\
[e_1 + e_2]_{aenv} & \stackrel{\text{def}}{=} & [e_1]_{aenv} \oplus [e_2]_{aenv} \\
[e_1 * e_2]_{aenv} & \stackrel{\text{def}}{=} & [e_1]_{aenv} \otimes [e_2]_{aenv} \\
[e_1 = e_2]_{aenv} & \stackrel{\text{def}}{=} & \{0, +\}
\end{array}
$$

```
def aeval_exp(e: Exp, aenv: AEnv) : Set[Abst] = e match {
  case Num(0) => Set(Zero)
  case Num(n) if (n < 0) => Set(Neg)
  case Num(n) if (n > 0) => Set(Pos)
  case Var(x) => aenv(x)
  case Plus(e1, e2) =>
    aplus(aeval_exp(e1, aenv), aeval_exp(e2, aenv))
  case Times(e1, e2) =>
    atimes(aeval_exp(e1, aenv), aeval_exp(e2, aenv))
  case Equ(e1, e2) => Set(Zero, Pos)
}
```

AEval for Stmts

Let sn be the snippets of a program

$$
\begin{array}{lcl}
[\texttt{nil}]_{aenv} & \rightarrow & (\texttt{nil}, aenv) \\
[\texttt{Label}(l:) :: rest]_{aenv} & \rightarrow & (rest, aenv) \\
[x \,\texttt{:=}\, e :: rest]_{aenv} & \rightarrow & (rest, aenv[x := [e]_{aenv}]) \\
[\texttt{jmp?}\; e\; l :: rest]_{aenv} & \rightarrow & (sn(l), aenv) \text{ and } (rest, aenv) \\
[\texttt{goto}\; l :: rest]_{aenv} & \rightarrow & (sn(l), aenv)
\end{array}
$$

Start with $[sn(\text{""})]_{\varnothing}$, try to reach all *states* you can find (until a fix point is reached).

Check whether there are only *aenv* in the final states that have your property.

Sign Analysis

- We want to find out whether `a` and `n` are always positive?
- After a few optimisations, we can indeed find this out.
  - equal signs return only `+` or `0`
  - branch into only one direction if you know
  - if x is `+`, then $x +$ `-1` cannot be negative
- What is this good for? Well, you do not need underflow checks (in order to prevent buffer-overflow attacks). In general this technique is used to make sure keys stay secret.

Take Home Points

- While testing is important, it does not show the absence of bugs/vulnerabilities.
- More and more we need (formal) proofs that show a program is bug free.
- Static analysis is more and more employed nowadays in order to automatically hunt for bugs.