Homework 3

1. The regular expression matchers in Java, Python and Ruby can be very slow with some (basic) regular expressions. What is the main reason for this inefficient computation?

2. What is a regular language? Are there alternative ways to define this notion? If yes, give an explanation why they define the same notion.

3. Why is every finite set of strings a regular language?

4. Assume you have an alphabet consisting of the letters a, b and c only. (1) Find a regular expression that recognises the two strings ab and ac. (2) Find a regular expression that matches all strings *except* these two strings. Note, you can only use regular expressions of the form

$$r ::= \mathbf{0} \mid \mathbf{1} \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

5. Given the alphabet $\{a, b\}$. Draw the automaton that has two states, say Q_0 and Q_1. The starting state is Q_0 and the final state is Q_1. The transition function is given by

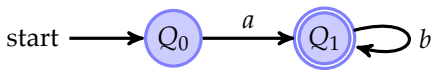
$$\begin{array}{l}(Q_0, a) \to Q_0\\ (Q_0, b) \to Q_1\\ (Q_1, b) \to Q_1\end{array}$$

   What is the language recognised by this automaton?

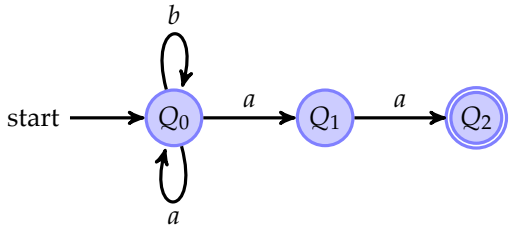
6. Give a non-deterministic finite automaton that can recognise the language $L(a \cdot (a + b)^* \cdot c)$.

7. Given a deterministic finite automaton $A(\Sigma, Q, Q_0, F, \delta)$, define which language is recognised by this automaton. Can you define also the language defined by a non-deterministic automaton?

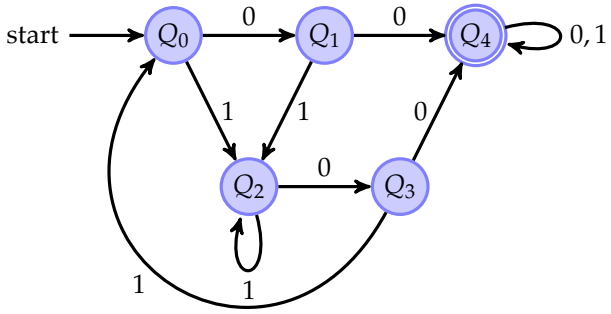
8. Given the following deterministic finite automaton over the alphabet $\{a, b\}$, find an automaton that recognises the complement language. (Hint: Recall that for the algorithm from the lectures, the automaton needs to be in completed form, that is have a transition for every letter from the alphabet.)



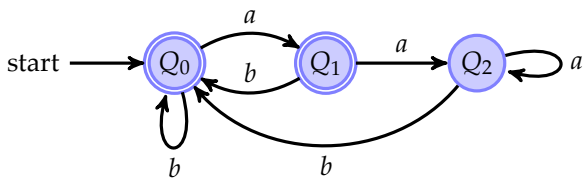
9. Given the following non-deterministic finite automaton over the alphabet $\{a, b\}$, find a deterministic finite automaton that recognises the same language:



10. Given the following deterministic finite automaton over the alphabet $\{0, 1\}$, find the corresponding minimal automaton. In case states can be merged, state clearly which states can be merged.



11. Given the following finite deterministic automaton over the alphabet $\{a, b\}$:



Give a regular expression that can recognise the same language as this automaton. (Hint: If you use Brzozwski's method, you can assume Arden's lemma which states that an equation of the form $q = q \cdot r + s$ has the unique solution $q = s \cdot r^*$.)

12. If a non-deterministic finite automaton (NFA) has n states. How many states does a deterministic automaton (DFA) that can recognise the same language as the NFA maximal need?

13. Rust implements a non-backtracking regular expression matcher based on the classic idea of DFAs. Still, some regular expressions take a surprising amount of time for matching problems. Explain the problem?

14. Prove that for all regular expressions r we have

$$nullable(r) \quad \text{if and only if} \quad [] \in L(r)$$

    Write down clearly in each case what you need to prove and what are the assumptions.

15. **(Optional)** This question is for you to provide regular feedback to me: for example what were the most interesting, least interesting, or confusing parts in this lecture? Any problems with my Scala code? Please feel free to share any other questions or concerns. Also, all my material is ~~crap~~ imperfect. If you have any suggestions for improvement, I am very grateful to hear.

    If *you* want to share anything (code, videos, links), you are encouraged to do so. Just drop me an email or send a message to the Forum.