Compilers and Formal Languages

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Slides & Progs:	KEATS (also homework is there)

Functional Programming

```
def fib(n) = if n == 0 then 0
             else if n == 1 then 1
             else fib(n - 1) + fib(n - 2);

def fact(n) = if n == 0 then 1 else n * fact(n - 1);

def ack(m, n) = if m == 0 then n + 1
                else if n == 0 then ack(m - 1, 1)
                else ack(m - 1, ack(m, n - 1));

def gcd(a, b) = if b == 0 then a else gcd(b, a % b);
```

Fun-Grammar

$$
\begin{aligned}
\mathit{Exp} &::= \mathit{Var} \mid \mathit{Num} \\
&\mid \mathit{Exp} + \mathit{Exp} \mid \ldots \mid (\mathit{Exp}) \\
&\mid \texttt{if}\ \mathit{BExp}\ \texttt{then}\ \mathit{Exp}\ \texttt{else}\ \mathit{Exp} \\
&\mid \texttt{write}\ \mathit{Exp} \\
&\mid \mathit{Exp} ; \mathit{Exp} \mid \mathit{FunName}\,(\mathit{Exp}, \ldots, \mathit{Exp}) \\
\mathit{BExp} &::= \ldots \\
\mathit{Def} &::= \texttt{def}\ \mathit{FunName}\,(x_1, \ldots, x_n) = \mathit{Exp} \\
\mathit{Prog} &::= \mathit{Def} ; \mathit{Prog} \mid \mathit{Exp} ; \mathit{Prog} \mid \mathit{Exp}
\end{aligned}
$$

Abstract Syntax Trees

```scala
abstract class Exp
abstract class BExp
abstract class Decl

case class Var(s: String) extends Exp
case class Num(i: Int) extends Exp
case class Aop(o: String, a1: Exp, a2: Exp) extends Exp
case class If(a: BExp, e1: Exp, e2: Exp) extends Exp
case class Write(e: Exp) extends Exp
case class Sequ(e1: Exp, e2: Exp) extends Exp
case class Call(name: String, args: List[Exp]) extends Exp

case class Bop(o: String, a1: Exp, a2: Exp) extends BExp

case class Def(name: String,
              args: List[String],
              body: Exp) extends Decl
case class Main(e: Exp) extends Decl
```

Ideas

Use separate JVM methods for each Fun-function.

Compile `exp`s such that the result of the expression is on top of the stack.

- `write(1 + 2)`
- `1 + 2; 3 + 4`

Sequences

Compiling `exp1 ; exp2`:

```
compile(exp1)
pop
compile(exp2)
```

Write

Compiling call to `write(1+2)`:

```
compile(1+2)
dup
invokestatic XXX/XXX/write(I)V
```

needs the helper method

```
.method public static write(I)V
   .limit locals 1
   .limit stack 2
   getstatic java/lang/System/out Ljava/io/PrintStream;
   iload 0
   invokevirtual java/io/PrintStream/println(I)V
   return
.end method
```

Function Definitions

```
.method public static write(I)V
   .limit locals 1
   .limit stack 2
   getstatic java/lang/System/out Ljava/io/PrintStream;
   iload 0
   invokevirtual java/io/PrintStream/println(I)V
   return
.end method
```

We will need methods for definitions like

```
def fname (x1, ... , xn) = ...
```

```
.method public static fname (I...I)I
  .limit locals ??
  .limit stack ??
  ??
.end method
```

Stack Estimation

$$
\begin{array}{lcl}
estimate(n) & \stackrel{\text{def}}{=} & 1 \\
estimate(x) & \stackrel{\text{def}}{=} & 1 \\
estimate(a_1\ aop\ a_2) & \stackrel{\text{def}}{=} & estimate(a_1) + estimate(a_2) \\
estimate(\texttt{if}\ b\ \texttt{then}\ e_1\ \texttt{else}\ e_2) & \stackrel{\text{def}}{=} & estimate(b) + \\
 & & \quad max(estimate(e_1), estimate(e_2)) \\
estimate(\texttt{write}(e)) & \stackrel{\text{def}}{=} & estimate(e) + 1 \\
estimate(e_1; e_2) & \stackrel{\text{def}}{=} & max(estimate(e_1), estimate(e_2)) \\
estimate(f(e_1, ..., e_n)) & \stackrel{\text{def}}{=} & \sum_{i=1..n} estimate(e_i) \\
estimate(a_1\ bop\ a_2) & \stackrel{\text{def}}{=} & estimate(a_1) + estimate(a_2)
\end{array}
$$

Successor Function

```
.method public static suc(I)I
.limit locals 1
.limit stack 2
  iload 0
  ldc 1
  iadd
  ireturn
.end method
```

```
def suc(x) = x + 1;
```

Addition Function

```
.method public static add(II)I
.limit locals 2
.limit stack 5
  iload 0
  ldc 0
  if_icmpne If_else
  iload 1
  goto If_end
If_else:
  iload 0
  ldc 1
  isub
  iload 1
  invokestatic XXX/XXX/add(II)I
  invokestatic XXX/XXX/suc(I)I
If_end:
  ireturn
.end method
```

```
def add(x, y) =
    if x == 0 then y
    else suc(add(x - 1, y));
```

Factorial

```
.method public static facT(II)I
.limit locals 2
.limit stack 6
  iload 0
  ldc 0
  if_icmpne If_else_2
  iload 1
  goto If_end_3
If_else_2:
  iload 0
  ldc 1
  isub
  iload 0
  iload 1
  imul
  invokestatic fact/fact/facT(II)I
If_end_3:
  ireturn
.end method
```

```
def facT(n, acc) =
  if n == 0 then acc
  else facT(n - 1, n * acc);
```

```
.method public static facT(II)I
.limit locals 2
.limit stack 6
facT_Start:
  iload 0
  ldc 0
  if_icmpne If_else_2
  iload 1
  goto If_end_3
If_else_2:
  iload 0
  ldc 1
  isub
  iload 0
  iload 1
  imul
  istore 1
  istore 0
  goto facT_Start
If_end_3:
```

```
def facT(n, acc) =
  if n == 0 then acc
  else facT(n - 1, n * acc);
```

Tail Recursion

A call to `f(args)` is usually compiled as

```
args onto stack
invokestatic  .../f
```

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A call to `f(args)` is usually compiled as

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args onto stack
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A call is in tail position provided:

- `if Bexp then` `Exp` `else` `Exp`
- `Exp ;` `Exp`
- `Exp op Exp`

then a call `f(args)` can be compiled as

```
prepare environment
jump to start of function
```

Tail Recursive Call

```
def compile_expT(a: Exp, env: Mem, name: String): Instrs =
  ...
  case Call(n, args) => if (name == n)
  {
    val stores =
      args.zipWithIndex.map { case (x, y) => i"istore $y" }

    args.map(a => compile_expT(a, env, "")).mkString ++
    stores.reverse.mkString ++
    i"goto ${n}_Start"
  } else {
    val is = "I" * args.length
    args.map(a => compile_expT(a, env, "")).mkString ++
    i"invokestatic XXX/XXX/${n}(${is})I"
  }
```

Dijkstra on Testing

"Program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence."

What is good about compilers: the either seem to work, or go horribly wrong (most of the time).

Proving Programs to be Correct

Theorem: There are infinitely many prime numbers.

Proof ...

similarly

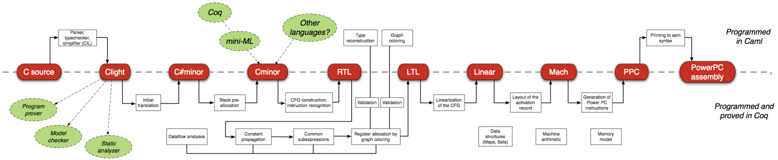
Theorem: The program is doing what it is supposed to be doing.

Long, long proof ...

This can be a gigantic proof. The only hope is to have help from the computer. 'Program' is here to be understood to be quite general (compiler, OS, ...).

Can This Be Done?

- in 2008, verification of a small C-compiler
  - “if my input program has a certain behaviour, then the compiled machine code has the same behaviour”
  - is as good as `gcc -O1`, but much, much less buggy



Fuzzy Testing C-Compilers

- tested GCC, LLVM and others by randomly generating C-programs
- found more than 300 bugs in GCC and also many in LLVM (some of them highest-level critical)
- about CompCert:

> "The striking thing about our CompCert results is that the middle-end bugs we found in all other compilers are absent. As of early 2011, the under-development version of CompCert is the only compiler we have tested for which Csmith cannot find wrong-code errors. This is not for lack of trying: we have devoted about six CPU-years to the task."