

Homework 2

1. Review the first handout about sets of strings and read the second handout. Assuming the alphabet is $\{a, b\}$, decide which of the following equations are true in general for arbitrary languages A , B and C :

$$(A \cup B)@C = A@C \cup B@C$$

$$A^* \cup B^* = (A \cup B)^*$$

$$A^*@A^* = A^*$$

$$(A \cap B)@C = (A@C) \cap (B@C)$$

In case an equation is true, give an explanation; otherwise give a counterexample.

2. What is the meaning of a regular expression? Give an inductive definition.
3. Given the regular expressions $r_1 = \epsilon$ and $r_2 = \emptyset$ and $r_3 = a$. How many strings can the regular expressions r_1^* , r_2^* and r_3^* each match?
4. Give regular expressions for (a) decimal numbers and for (b) binary numbers. (Hint: Observe that the empty string is not a number. Also observe that leading 0s are normally not written.)
5. Decide whether the following two regular expressions are equivalent ($\epsilon + a$)^{*} \equiv ? a^* and $(a \cdot b)^* \cdot a$ \equiv ? $a \cdot (b \cdot a)^*$.
6. Given the regular expression $r = (a \cdot b + b)^*$. Compute what the derivative of r is with respect to a and b . Is r nullable?
7. What is a regular language?
8. Prove that for all regular expressions r we have

$$\text{nullable}(r) \quad \text{if and only if} \quad "" \in L(r)$$