

Homework 3

Please submit your solutions via email. Please submit only ASCII text or PDFs. Every solution should be preceded by the corresponding question, like:

Qⁿ: ...a difficult question from me...
A: ...an answer from you ...
Qⁿ⁺¹: ...another difficult question...
A: ...another brilliant answer from you...

Solutions will only be accepted until 30th December!

1. What is a regular language? Are there alternative ways to define this notion? If yes, give an explanation why they define the same notion.
2. Why is every finite set of strings a regular language?
3. Assume you have an alphabet consisting of the letters a , b and c only. (1) Find a regular expression that recognises the two strings ab and ac . (2) Find a regular expression that matches all strings *except* these two strings. Note, you can only use regular expressions of the form

$$r ::= \emptyset \mid \epsilon \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

4. Define the function *zeroable* which takes a regular expression as argument and returns a boolean. The function should satisfy the following property:

$$\text{zeroable}(r) \text{ if and only if } L(r) = \emptyset$$

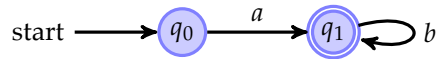
5. Given the alphabet $\{a, b\}$. Draw the automaton that has two states, say q_0 and q_1 . The starting state is q_0 and the final state is q_1 . The transition function is given by

$$\begin{aligned}(q_0, a) &\rightarrow q_0 \\(q_0, b) &\rightarrow q_1 \\(q_1, b) &\rightarrow q_1\end{aligned}$$

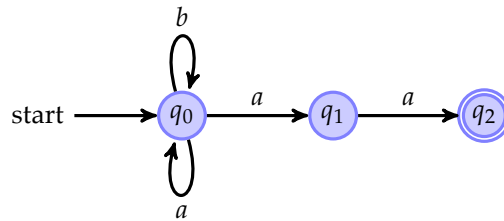
What is the language recognised by this automaton?

6. Give a non-deterministic finite automaton that can recognise the language $L(a \cdot (a + b)^* \cdot c)$.
7. Given a deterministic finite automata $A(Q, q_0, F, \delta)$, define which language is recognised by this automaton. Can you define also the language defined by a non-deterministic automaton?

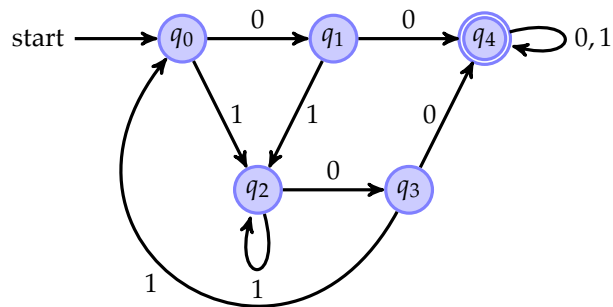
8. Given the following deterministic finite automata over the alphabet $\{a, b\}$, find an automaton that recognises the complement language. (Hint: Recall that for the algorithm from the lectures, the automaton needs to be in completed form, that is have a transition for every letter from the alphabet.)



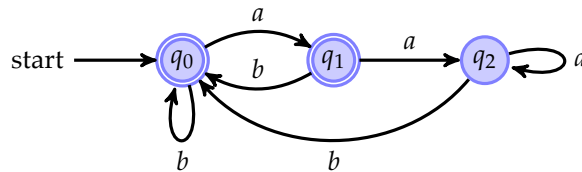
9. Given the following non-deterministic finite automaton over the alphabet $\{a, b\}$, find a deterministic finite automaton that recognises the same language:



10. Given the following deterministic finite automaton over the alphabet $\{0, 1\}$, find the corresponding minimal automaton. In case states can be merged, state clearly which states can be merged.



11. Given the following finite deterministic automaton over the alphabet $\{a, b\}$:



Give a regular expression that can recognise the same language as this automaton. (Hint: If you use Brzozowski's method, you can assume Arden's lemma which states that an equation of the form $q = q \cdot r + s$ has the unique solution $q = s \cdot r^*$.)

12. If a non-deterministic finite automaton (NFA) has n states. How many states does a deterministic automaton (DFA) that can recognise the same language as the NFA maximal need?