Automata and Formal Languages (7)

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CFGs

A context-free grammar (CFG) G consists of:

- a finite set of nonterminal symbols (upper case)
- a finite terminal symbols or tokens (lower case)
- a start symbol (which must be a nonterminal)
- a set of rules

$$A \to \text{rhs}_1 | \text{rhs}_2 | \ldots$$

where rhs are sequences involving terminals and nonterminals (can also be empty).

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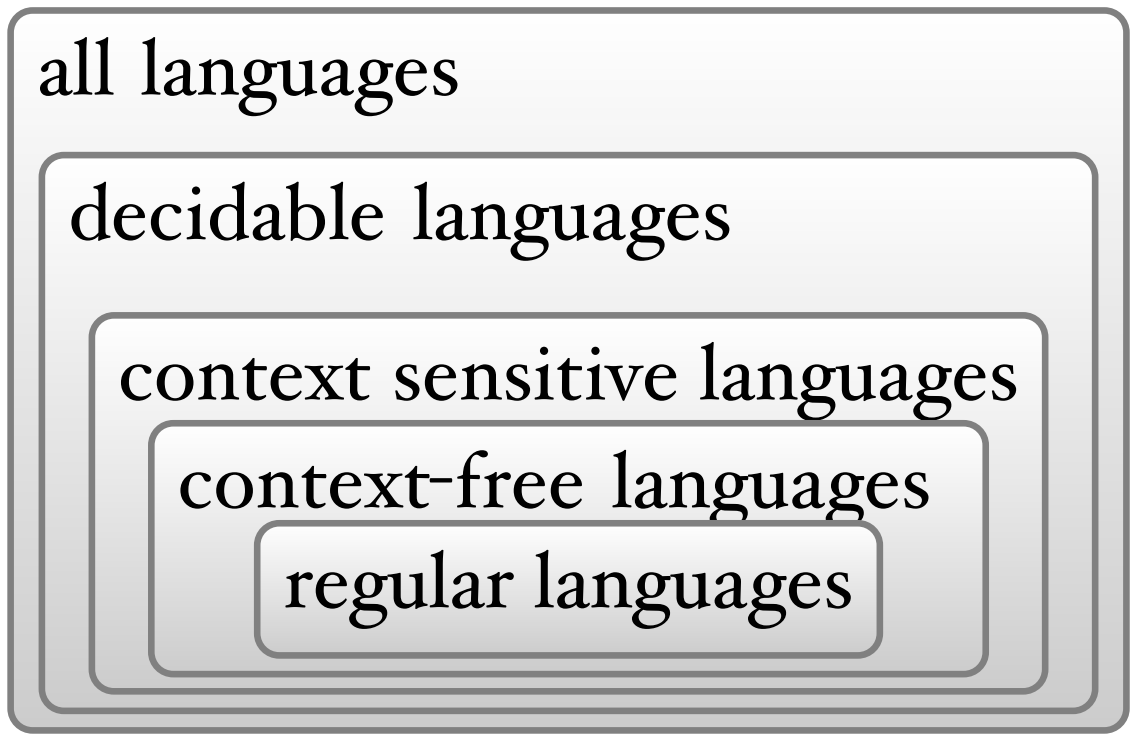
Hierarchie of Languages

Recall that languages are sets of strings.



Numbers

A grammar for numbers:

$$N \;\rightarrow\; N \cdot N \mid 0 \mid 1 \mid \ldots \mid 9$$

Unfortunately left-recursive (and ambiguous).

A problem for recursive descent parsers
(e.g. parser combinators).

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A non-left-recursive grammar for numbers

$$N \;\rightarrow\; 0 \cdot N \mid 1 \cdot N \mid \ldots \mid 0 \mid 1 \mid \ldots \mid 9$$

Chomsky Normal Form

All rules must be of the form

$$A \to a$$

or

$$A \to B \cdot C$$

CYK Algorithm

$$
\begin{array}{lcl}
\boldsymbol{S} & \rightarrow & \boldsymbol{N} \cdot \boldsymbol{P} \\
\boldsymbol{P} & \rightarrow & \boldsymbol{V} \cdot \boldsymbol{N} \\
\boldsymbol{N} & \rightarrow & \boldsymbol{N} \cdot \boldsymbol{N} \\
\boldsymbol{N} & \rightarrow & \text{students} \mid \text{Jeff} \mid \text{geometry} \mid \text{trains} \\
\boldsymbol{V} & \rightarrow & \text{trains}
\end{array}
$$

Jeff trains geometry students

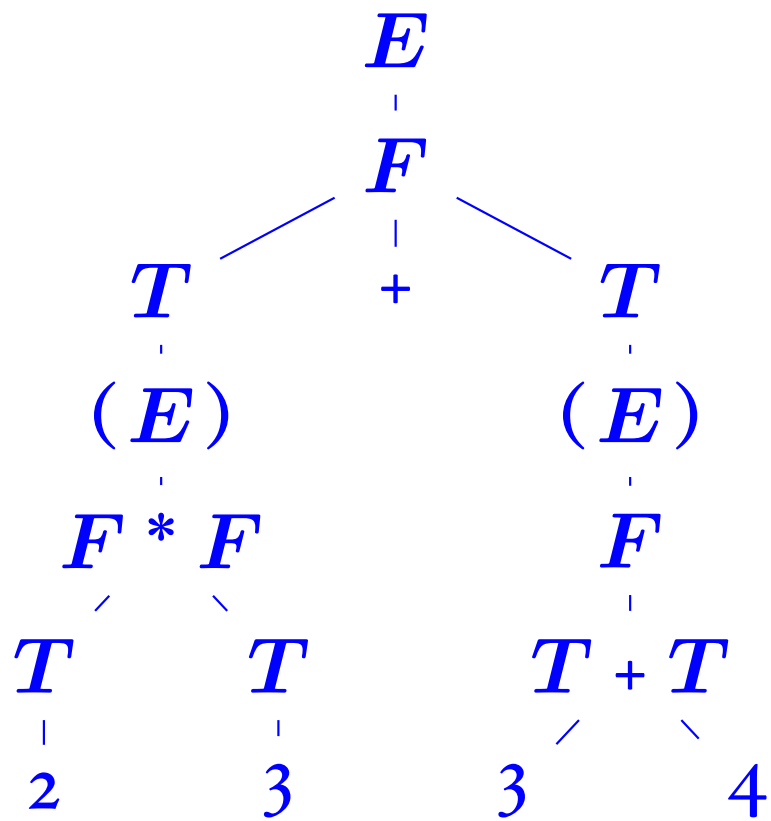
CYK Algorithm

- runtime is $O(n^3)$
- grammars need to be transferred into CNF

Parse Trees

$$
\begin{array}{lcl}
E & \rightarrow & F \mid F \cdot * \cdot F \\
F & \rightarrow & T \mid T \cdot + \cdot T \mid T \cdot - \cdot T \\
T & \rightarrow & num\_token \mid (\cdot E \cdot)
\end{array}
$$

$(2*3)+(3+4)$



Ambiguous Grammars

A CFG is ambiguous if there is a string that has at least parse trees.

$$
\begin{array}{lcl}
E & \rightarrow & num\_token \\
E & \rightarrow & E \cdot + \cdot E \\
E & \rightarrow & E \cdot - \cdot E \\
E & \rightarrow & E \cdot * \cdot E \\
E & \rightarrow & (\cdot E \cdot)
\end{array}
$$

$1 + 2 * 3 + 4$

Dangling Else

Another ambiguous grammar:

$$
\begin{array}{lcl}
E & \rightarrow & \text{if } E \text{ then } E \\
 & | & \text{if } E \text{ then } E \text{ else } E \\
 & | & \text{id}
\end{array}
$$

if a then if x then y else c

A CFG Derivation

1. Begin with a string with only the start symbol S
2. Replace any non-terminal X in the string by the right-hand side of some production $X \to \text{rhs}$
3. Repeat 2 until there are no non-terminals

$$S \to \ldots \to \ldots \to \ldots \to \ldots$$