

## Homework 2

Please submit your solutions via email. Please submit only ASCII text or PDFs. Every solution should be preceded by the corresponding question, like:

**Q<sub>n</sub>:**     ...a difficult question from me...  
**A:**        ...an answer from you ...  
**Q<sub>n</sub> + 1**   ...another difficult question...  
**A:**        ...another brilliant answer from you...

**Solutions will only be accepted until 30th December! Please send only one homework per email.**

1. What is the language recognised by the regular expressions  $(0^*)^*$ .
2. Review the first handout about sets of strings and read the second handout. Assuming the alphabet is the set  $\{a, b\}$ , decide which of the following equations are true in general for arbitrary languages  $A, B$  and  $C$ :

$$\begin{aligned}(A \cup B)@C & \stackrel{?}{=} A@C \cup B@C \\ A^* \cup B^* & \stackrel{?}{=} (A \cup B)^* \\ A^*@A^* & \stackrel{?}{=} A^* \\ (A \cap B)@C & \stackrel{?}{=} (A@C) \cap (B@C)\end{aligned}$$

In case an equation is true, give an explanation; otherwise give a counterexample.

3. Given the regular expressions  $r_1 = \mathbf{1}$  and  $r_2 = \mathbf{0}$  and  $r_3 = a$ . How many strings can the regular expressions  $r_1^*$ ,  $r_2^*$  and  $r_3^*$  each match?
4. Give regular expressions for (a) decimal numbers and for (b) binary numbers. (Hint: Observe that the empty string is not a number. Also observe that leading 0s are normally not written.)
5. Decide whether the following two regular expressions are equivalent  $(\mathbf{1} + a)^* \stackrel{?}{=} a^*$  and  $(a \cdot b)^* \cdot a \stackrel{?}{=} a \cdot (b \cdot a)^*$ .
6. Given the regular expression  $r = (a \cdot b + b)^*$ . Compute what the derivative of  $r$  is with respect to  $a, b$  and  $c$ . Is  $r$  nullable?
7. Prove that for all regular expressions  $r$  we have

$$\text{nullable}(r) \quad \text{if and only if} \quad \epsilon \in L(r)$$

Write down clearly in each case what you need to prove and what are the assumptions.

8. Define what is meant by the derivative of a regular expressions with respect to a character. (Hint: The derivative is defined recursively.)
9. Assume the set  $Der c A$  is defined as

$$Der c A \stackrel{\text{def}}{=} \{s \mid c :: s \in A\}$$

What is the relation between  $Der$  and the notion of derivative of regular expressions?

10. Give a regular expression over the alphabet  $\{a, b\}$  recognising all strings that do not contain any substring  $bb$  and end in  $a$ .
11. Do  $(a + b)^* \cdot b^+$  and  $(a^* \cdot b^+) + (b^* \cdot b^+)$  define the same language?
12. Define the function *zeroable* by recursion over regular expressions. This function should satisfy the property

$$\text{zeroable}(r) \text{ if and only if } L(r) = \{\} \quad (*)$$

The function *nullable* for the not-regular expressions can be defined by

$$\text{nullable}(\sim r) \stackrel{\text{def}}{=} \neg(\text{nullable}(r))$$

Unfortunately, a similar definition for *zeroable* does not satisfy the property in (\*):

$$\text{zeroable}(\sim r) \stackrel{\text{def}}{=} \neg(\text{zeroable}(r))$$

Find out why?

13. Give a regular expressions that can recognise all strings from the language  $\{a^n \mid \exists k. n = 3k + 1\}$ .
14. Give a regular expression that can recognise an odd number of *as* or an even number of *bs*.