Homework 5

1. Define the following regular expressions

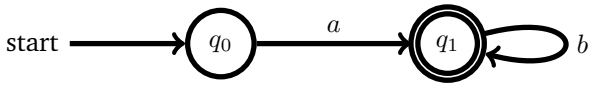
r^+	(one or more matches)
$r^?$	(zero or one match)
$r^{\{n\}}$	(exactly n matches)
$r^{\{m,n\}}$	(at least m and maximal n matches, with the assumption $m \leq n$)

   in terms of the usual regular expressions

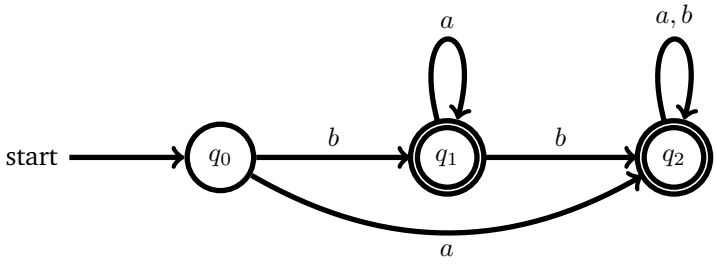
   $$r ::= \varnothing \mid \epsilon \mid c \mid r_1 + r_2 \mid r_1 \cdot r_2 \mid r^*$$

2. Given a deterministic finite automata $A(Q, q_0, F, \delta)$, define which language is recognised by this automaton.

3. Given the following deterministic finite automata over the alphabet $\{a, b\}$, find an automaton that recognises the complement language. (Hint: Recall that for the algorithm from the lectures, the automaton needs to be in completed form, that is have a transition for every letter from the alphabet.)

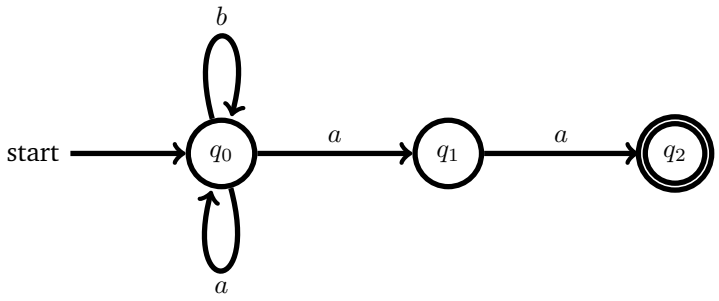
   

4. Given the following deterministic finite automaton

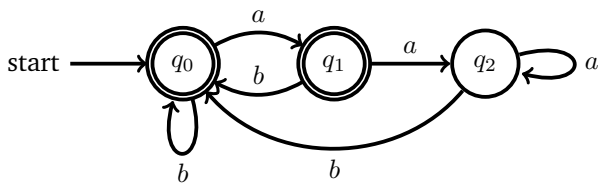
   

   find the corresponding minimal automaton. State clearly which nodes can be merged.

5. Given the following non-deterministic finite automaton over the alphabet $\{a, b\}$, find a deterministic finite automaton that recognises the same language:



6. Given the following finite deterministic automaton over the alphabet $\{a, b\}$:



   Give a regular expression that can recognise the same language as this automaton. (Hint: If you use Brzozwski's method, you can assume Arden's lemma which states that an equation of the form $q = q \cdot r + s$ has the unique solution $q = s \cdot r^*$.)

7. Recall the definitions for Der and der from the lectures. Prove by induction on r the property that

$$L(der\, c\, r) = Der\, c\, (L(r))$$

   holds.