Compilers and Formal Languages

Email: christian.urban at kcl.ac.uk
Slides & Progs: KEATS (also homework is there)

Functional Programming

```
def fib(n) = if n == 0 then 0
             else if n == 1 then 1
             else fib(n - 1) + fib(n - 2);

def fact(n) = if n == 0 then 1 else n * fact(n - 1);

def ack(m, n) = if m == 0 then n + 1
                else if n == 0 then ack(m - 1, 1)
                else ack(m - 1, ack(m, n - 1));

def gcd(a, b) = if b == 0 then a else gcd(b, a % b);
```

Fun-Grammar

$$
\begin{array}{rcl}
\mathit{Exp} & ::= & \mathit{Var} \;|\; \mathit{Num} \\
& | & \mathit{Exp} + \mathit{Exp} \;|\; \ldots \;|\; (\mathit{Exp}) \\
& | & \texttt{if}\ \mathit{BExp}\ \texttt{then}\ \mathit{Exp}\ \texttt{else}\ \mathit{Exp} \\
& | & \texttt{write}\ \mathit{Exp} \\
& | & \mathit{Exp}\ ;\ \mathit{Exp} \;|\; \mathit{FunName}\,(\mathit{Exp}, \ldots, \mathit{Exp}) \\
\mathit{BExp} & ::= & \ldots \\
\mathit{Def} & ::= & \texttt{def}\ \mathit{FunName}\,(x_1, \ldots, x_n) = \mathit{Exp} \\
\mathit{Prog} & ::= & \mathit{Def}\ ;\ \mathit{Prog} \;|\; \mathit{Exp}\ ;\ \mathit{Prog} \;|\; \mathit{Exp}
\end{array}
$$

Abstract Syntax Trees

```
abstract class Exp
abstract class BExp
abstract class Decl

case class Var(s: String) extends Exp
case class Num(i: Int) extends Exp
case class Aop(o: String, a1: Exp, a2: Exp) extends Exp
case class If(a: BExp, e1: Exp, e2: Exp) extends Exp
case class Write(e: Exp) extends Exp
case class Sequ(e1: Exp, e2: Exp) extends Exp
case class Call(name: String, args: List[Exp]) extends Exp

case class Bop(o: String, a1: Exp, a2: Exp) extends BExp

case class Def(name: String,
               args: List[String],
               body: Exp) extends Decl
case class Main(e: Exp) extends Decl
```

Ideas

Use separate JVM methods for each Fun-function.

Compile exps such that the result of the expression is on top of the stack.

```
write(1 + 2)
1 + 2; 3 + 4
```

Sequences

Compiling `exp1 ; exp2`:

```
compile(exp1)
pop
compile(exp2)
```

Write

Compiling call to `write(1+2)`:

```
compile(1+2)
dup
invokestatic XXX/XXX/write(I)V
```

needs the helper method

```
.method public static write(I)V
   .limit locals 1
   .limit stack 2
   getstatic java/lang/System/out Ljava/io/PrintStream;
   iload 0
   invokevirtual java/io/PrintStream/println(I)V
   return
.end method
```

Function Definitions

```
.method public static write(I)V
   .limit locals 1
   .limit stack 2
   getstatic java/lang/System/out Ljava/io/PrintStream;
   iload 0
   invokevirtual java/io/PrintStream/println(I)V
   return
.end method
```

We will need methods for definitions like

```
def fname (x1, ... , xn) = ...
```

```
.method public static fname (I...I)I
  .limit locals ??
  .limit stack ??
  ??
.end method
```

Stack Estimation

$$
\begin{array}{lcl}
\mathit{estimate}(n) & \stackrel{\text{def}}{=} & 1 \\
\mathit{estimate}(x) & \stackrel{\text{def}}{=} & 1 \\
\mathit{estimate}(a_1 \; \mathit{aop} \; a_2) & \stackrel{\text{def}}{=} & \mathit{estimate}(a_1) + \mathit{estimate}(a_2) \\
\mathit{estimate}(\texttt{if}\; b \;\texttt{then}\; e_1 \;\texttt{else}\; e_2) & \stackrel{\text{def}}{=} & \mathit{estimate}(b) + \\
 & & \quad \mathit{max}(\mathit{estimate}(e_1), \mathit{estimate}(e_2)) \\
\mathit{estimate}(\texttt{write}(e)) & \stackrel{\text{def}}{=} & \mathit{estimate}(e) + 1 \\
\mathit{estimate}(e_1; e_2) & \stackrel{\text{def}}{=} & \mathit{max}(\mathit{estimate}(e_1), \mathit{estimate}(e_2)) \\
\mathit{estimate}(f(e_1, \ldots, e_n)) & \stackrel{\text{def}}{=} & \sum_{i=1..n} \; \mathit{estimate}(e_i) \\
\mathit{estimate}(a_1 \; \mathit{bop} \; a_2) & \stackrel{\text{def}}{=} & \mathit{estimate}(a_1) + \mathit{estimate}(a_2)
\end{array}
$$

Successor Function

```
.method public static suc(I)I
.limit locals 1
.limit stack 2
  iload 0
  ldc 1
  iadd
  ireturn
.end method
```

```
def suc(x) = x + 1;
```

Addition Function

```
.method public static add(II)I
.limit locals 2
.limit stack 5
  iload 0
  ldc 0
  if_icmpne If_else
  iload 1
  goto If_end
If_else:
  iload 0
  ldc 1
  isub
  iload 1
  invokestatic XXX/XXX/add(II)I
  invokestatic XXX/XXX/suc(I)I
If_end:
  ireturn
.end method
```

```
def add(x, y) =
    if x == 0 then y
    else suc(add(x - 1, y));
```

Factorial

```
.method public static facT(II)I
.limit locals 2
.limit stack 6
  iload 0
  ldc 0
  if_icmpne If_else_2
  iload 1
  goto If_end_3
If_else_2:
  iload 0
  ldc 1
  isub
  iload 0
  iload 1
  imul
  invokestatic fact/fact/facT(II)I
If_end_3:
  ireturn
.end method
```

```
def facT(n, acc) =
  if n == 0 then acc
  else facT(n - 1, n * acc);
```

```
.method public static facT(II)I
.limit locals 2
.limit stack 6
facT_Start:
  iload 0
  ldc 0
  if_icmpne If_else_2
  iload 1
  goto If_end_3
If_else_2:
  iload 0
  ldc 1
  isub
  iload 0
  iload 1
  imul
  istore 1
  istore 0
  goto facT_Start
If_end_3:
```

```
def facT(n, acc) =
  if n == 0 then acc
  else facT(n - 1, n * acc);
```

Tail Recursion

A call to `f(args)` is usually compiled as

```
args onto stack
invokestatic  .../f
```

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A call to `f(args)` is usually compiled as

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args onto stack
invokestatic  .../f
```

A call is in tail position provided:

```
if Bexp then [Exp] else [Exp]

Exp ; [Exp]

Exp op Exp
```

then a call `f(args)` can be compiled as

```
prepare environment
jump to start of function
```

Tail Recursive Call

```
def compile_expT(a: Exp, env: Mem, name: String): Instrs =
  ...
  case Call(n, args) => if (name == n)
  {
    val stores =
      args.zipWithIndex.map { case (x, y) => i"istore $y" }

    args.map(a => compile_expT(a, env, "")).mkString ++
    stores.reverse.mkString ++
    i"goto ${n}_Start"
  } else {
    val is = "I" * args.length
    args.map(a => compile_expT(a, env, "")).mkString ++
    i"invokestatic XXX/XXX/${n}(${is})I"
  }
```

Dijkstra on Testing

"Program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence."

What is good about compilers: the either seem to work, or go horribly wrong (most of the time).

Proving Programs to be Correct

Theorem: There are infinitely many prime numbers.

Proof ...

similarly

Theorem: The program is doing what it is supposed to be doing.

Long, long proof ...

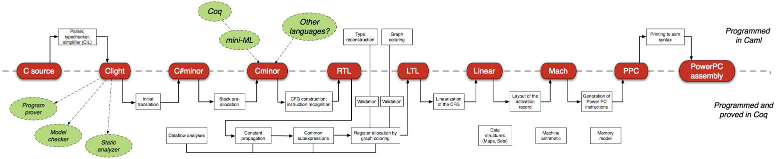
This can be a gigantic proof. The only hope is to have help from the computer. 'Program' is here to be understood to be quite general (compiler, OS, ...).

Can This Be Done?

in 2008, verification of a small C-compiler

> "if my input program has a certain behaviour, then the compiled machine code has the same behaviour"

is as good as `gcc -O1`, but much, much less buggy



Fuzzy Testing C-Compilers

tested GCC, LLVM and others by randomly generating C-programs

found more than 300 bugs in GCC and also many in LLVM (some of them highest-level critical)

about CompCert:

> "The striking thing about our CompCert results is that the middle-end bugs we found in all other compilers are absent. As of early 2011, the under-development version of CompCert is the only compiler we have tested for which Csmith cannot find wrong-code errors. This is not for lack of trying: we have devoted about six CPU-years to the task."