# Compilers and Formal Languages

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|---|---|
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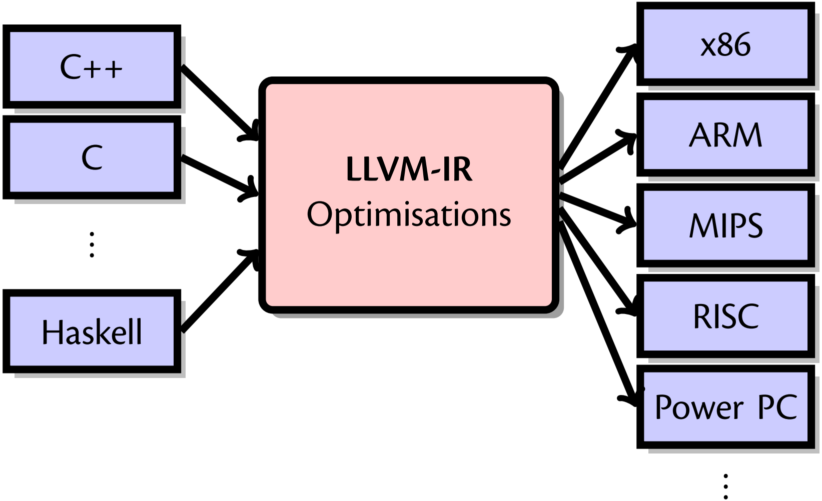
|---|---|
| 1 Introduction, Languages | 6 While-Language |
| 2 Regular Expressions, Derivatives | 7 Compilation, JVM |
| 3 Automata, Regular Languages | 8 Compiling Functional Languages |
| 4 Lexing, Tokenising | 9 Optimisations |
| 5 Grammars, Parsing | 10 LLVM |

# LLVM: Overview



# Static Single-Assignment

$$(1 + a) + (3 + (b * 5))$$

```
let tmp0 = add 1 a in
let tmp1 = mul b 5 in
let tmp2 = add 3 tmp1 in
let tmp3 = add tmp0 tmp2
  in tmp3
```

```
define i32 @fact (i32 %n) {
   %tmp_20 = icmp eq i32  %n, 0
   br i1 %tmp_20, label %if_branch_24, label %else_branch_25
 if_branch_24:
   ret i32 1
 else_branch_25:
   %tmp_22 = sub i32  %n, 1
   %tmp_23 = call i32 @fact (i32 %tmp_22)
   %tmp_21 = mul i32  %n, %tmp_23
   ret i32 %tmp_21
}
```

```
def fact(n) = if n == 0 then 1 else n * fact(n - 1)
```

# LLVM Types

```
boolean  i1
byte     i8
short    i16
char     i16
integer  i32
long     i64
float    float
double   double
*_       pointer to
**_      pointer to a pointer to
[_]      arrays of
```

```
br i1 %var, label %if_br, label %else_br

icmp eq i32  %x, %y     ; for equal
icmp sle i32 %x, %y     ;   signed less or equal
icmp slt i32 %x, %y     ;   signed less than
icmp ult i32 %x, %y     ;   unsigned less than

%var = call i32 @foo(...args...)
```

# Abstract Syntax Trees

```scala
// Fun language (expressions)
abstract class Exp
abstract class BExp

case class Call(name: String, args: List[Exp]) extends Exp
case class If(a: BExp, e1: Exp, e2: Exp) extends Exp
case class Write(e: Exp) extends Exp
case class Var(s: String) extends Exp
case class Num(i: Int) extends Exp
case class Aop(o: String, a1: Exp, a2: Exp) extends Exp
case class Sequence(e1: Exp, e2: Exp) extends Exp
case class Bop(o: String, a1: Exp, a2: Exp) extends BExp
```

# K-(Intermediate)Language

```
abstract class KExp
abstract class KVal

// K-Values
case class KVar(s: String) extends KVal
case class KNum(i: Int) extends KVal
case class Kop(o: String, v1: KVal, v2: KVal) extends KVal
case class KCall(o: String, vrs: List[KVal]) extends KVal
case class KWrite(v: KVal) extends KVal

// K-Expressions
case class KIf(x1: String, e1: KExp, e2: KExp) extends KExp
case class KLet(x: String, v: KVal, e: KExp) extends KExp
case class KReturn(v: KVal) extends KExp
```

# KLet

```
tmp0 = add 1 a
tmp1 = mul b 5
tmp2 = add 3 tmp1
tmp3 = add tmp0 tmp2
```

```
  KLet tmp0 , add 1 a in
   KLet tmp1 , mul b 5 in
    KLet tmp2 , add 3 tmp1 in
     KLet tmp3 , add tmp0 tmp2 in
      ...
```

```
case class KLet(x: String, e1: KVal, e2: KExp)
```

# KLet

```
tmp0 = add 1 a
tmp1 = mul b 5
tmp2 = add 3 tmp1
tmp3 = add tmp0 tmp2
```

```
  let tmp0 = add 1 a in
   let tmp1 = mul b 5 in
    let tmp2 = add 3 tmp1 in
     let tmp3 = add tmp0 tmp2 in
      ...
```

```
case class KLet(x: String, e1: KVal, e2: KExp)
```

# CPS-Translation

```
def CPS(e: Exp)(k: KVal => KExp) : KExp =
  e match { ... }
```

the continuation k can be thought of:

```
let tmp0 = add 1 a in
let tmp1 = mul □ 5 in
let tmp2 = add 3 tmp1 in
let tmp3 = add tmp0 tmp2 in
  KReturn tmp3
```

```scala
def fact(n: Int) : Int = {
  if (n == 0) 1 else n * fact(n - 1)
}


def factC(n: Int, ret: Int => Int) : Int = {
  if (n == 0) ret(1)
  else factC(n - 1, x => ret(n * x))
}

fact(10)
factC(10, identity)
```

```scala
def fibC(n: Int, ret: Int => Int) : Int = {
  if (n == 0 || n == 1) ret(1) else
  fibC(n - 1,
       r1 => fibC(n - 2,
        r2 => ret(r1 + r2)))
}

fibC(10, identity)
```

**Are there more strings in**

$L(a^*)$ **or** $L((a+b)^*)$**?**

# Can you remember this HW?

- (1) How many basic regular expressions are there to match the string *abcd*?
- (2) How many if they cannot include **1** and **0**?
- (3) How many if they are also not allowed to contain stars?
- (4) How many if they are also not allowed to contain $\_ + \_$?

**There are more problems, than there are programs.**

**There are more problems, than there are programs.**

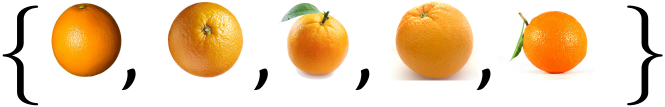
**There must be a problem for which there is no program.**

# Subsets

If $A \subseteq B$ then $A$ has fewer or equal elements than $B$

$A \subseteq B$ and $B \subseteq A$

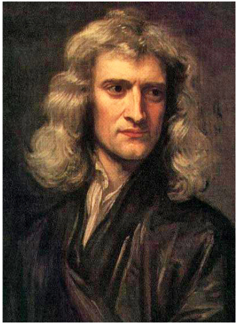
then $A = B$



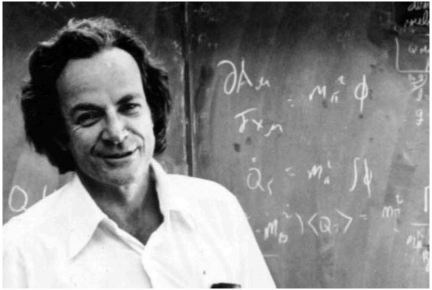
5 elements

3 elements

# Newton vs Feynman



classical physics



quantum physics

# The Goal of the Talk

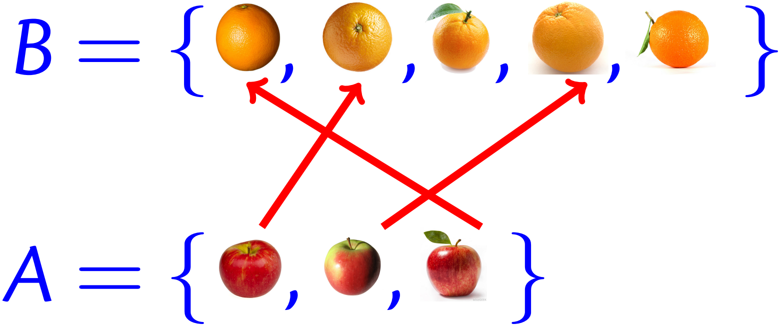
- show you that something very unintuitive happens with very large sets

- convince you that there are more **problems** than **programs**

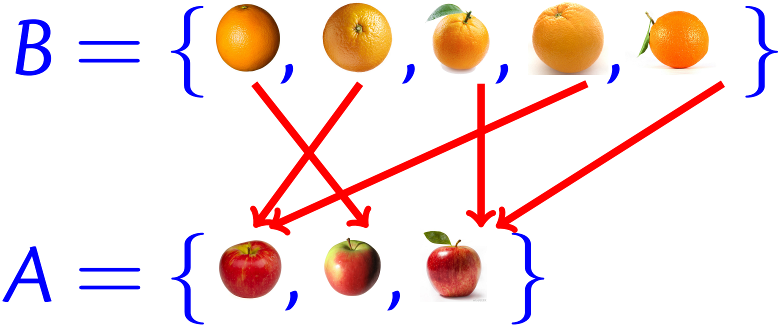
$B = \{$ , , , ,  $\}$

$A = \{$ , ,  $\}$

$$|A| = 5, |B| = 3$$



then $|A| \leq |B|$




for $=$ has to be a **one-to-one** mapping

# Cardinality

$|A| \stackrel{\text{def}}{=}$ "how many elements"

$A \subseteq B \Rightarrow |A| \leq |B|$

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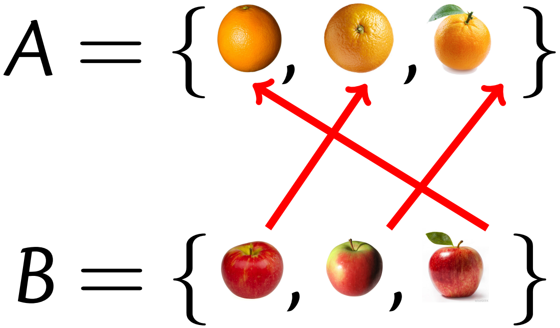
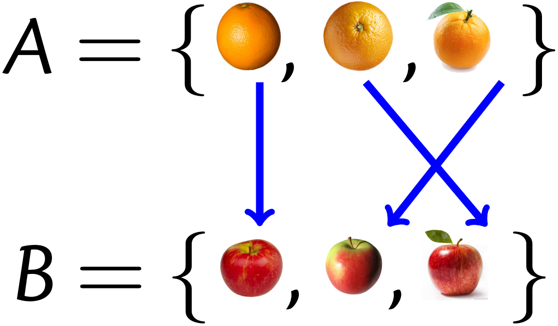
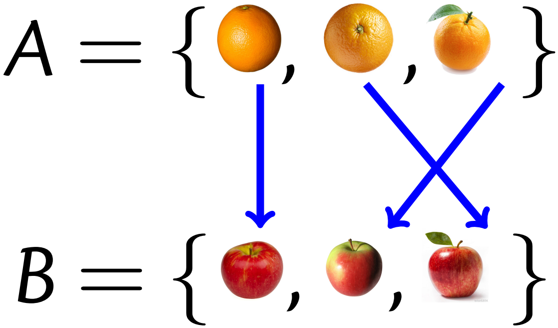
if there is an injective function
$f : A \rightarrow B$ then $|A| \leq |B|$

$$\forall xy.\, f(x) = f(y) \Rightarrow x = y$$


A =
B =


A =
B =



then $|A| = |B|$

# Natural Numbers

$$\mathbb{N} \stackrel{\text{def}}{=} \{0, 1, 2, 3, \ldots\ldots\}$$

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$$\mathbb{N} \stackrel{\text{def}}{=} \{0, 1, 2, 3, \ldots\ldots\}$$

$A$ is countable iff $|A| \leq |\mathbb{N}|$

# First Question

$$|\mathbb{N} - \{0\}| \quad ? \quad |\mathbb{N}|$$

$\geq$ or $\leq$ or $=$ ?

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$$|\mathbb{N} - \{0\}| \quad \color{red}{?} \quad |\mathbb{N}|$$

$\geq$ or $\leq$ or $=$ ?

$x \mapsto x + 1$,

$|\mathbb{N} - \{0\}| = |\mathbb{N}|$

$$|\mathbb{N} - \{0, 1\}| \quad ? \quad |\mathbb{N}|$$

$$|\mathbb{N} - \{0, 1\}| \quad ? \quad |\mathbb{N}|$$

$$|\mathbb{N} - \mathbb{O}| \quad ? \quad |\mathbb{N}|$$

$\mathbb{O} \stackrel{\text{def}}{=}$ odd numbers $\{1, 3, 5......\}$

$$|\mathbb{N} - \{0, 1\}| \quad {\color{red}?} \quad |\mathbb{N}|$$

$$|\mathbb{N} - \mathbb{O}| \quad {\color{red}?} \quad |\mathbb{N}|$$

$\mathbb{O} \stackrel{\text{def}}{=}$ odd numbers $\quad \{1, 3, 5......\}$

$\mathbb{E} \stackrel{\text{def}}{=}$ even numbers $\quad \{0, 2, 4......\}$

$$|\mathbb{N} \cup -\mathbb{N}| \quad ? \quad |\mathbb{N}|$$

$$\mathbb{N} \stackrel{\text{def}}{=} \text{positive numbers} \quad \{0, 1, 2, 3, \ldots\ldots\}$$
$$-\mathbb{N} \stackrel{\text{def}}{=} \text{negative numbers} \quad \{0, -1, -2, -3, \ldots\ldots\}$$

$A$ is countable if there exists an injective $f : A \to \mathbb{N}$

$A$ is uncountable if there does not exist an injective $f : A \to \mathbb{N}$

countable: $|A| \leq |\mathbb{N}|$

uncountable: $|A| > |\mathbb{N}|$

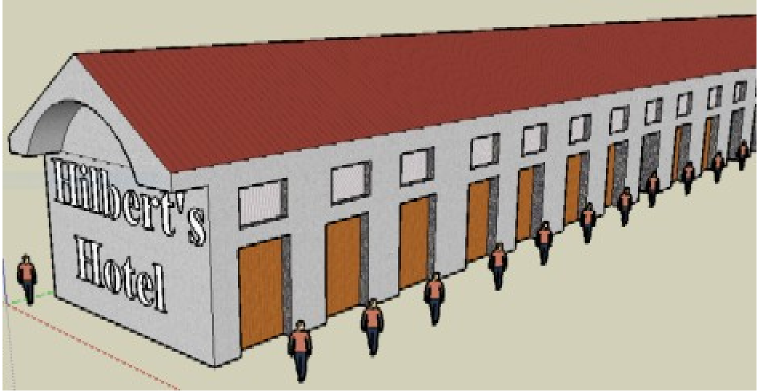
$A$ is countable if there exists an injective $f : A \rightarrow \mathbb{N}$

$A$ is uncountable if there does not exist an injective $f : A \rightarrow \mathbb{N}$

countable: $|A| \leq |\mathbb{N}|$
uncountable: $|A| > |\mathbb{N}|$

Does there exist such an $A$ ?

# Hilbert's Hotel




- ...has as many rooms as there are natural numbers

# Real Numbers between 0 and 1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| 1 | 3 | 3 | 3 | 3 | 3 | 3 | $\ldots$ | $\ldots$ |
| 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 3 | 0 | 1 | 0 | 1 | 0 | $\ldots$ | | |
| 4 | 7 | 8 | 5 | 3 | 9 | $\ldots$ | | |

$\ldots$

# Real Numbers between 0 and 1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | 3 | 3 | 3 | 3 | ... | ... |
| 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 3 | 0 | 1 | 0 | 1 | 0 | ... | | |
| 4 | 7 | 8 | 5 | 3 | 9 | ... | | |

...

# Real Numbers between 0 and 1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | 3 | 3 | 3 | 3 | $\cdots$ | $\cdots$ |
| 2 | 1 | 3 | 3 | 4 | 5 | 6 | 7 | |
| 3 | 0 | 1 | 0 | 1 | 0 | $\cdots$ | | |
| 4 | 7 | 8 | 5 | 3 | 9 | $\cdots$ | | |

$\cdots$

# Real Numbers between 0 and 1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| 1 | 4 | 3 | 3 | 3 | 3 | 3 | $\ldots$ | $\ldots$ |
| 2 | 1 | 3 | 3 | 4 | 5 | 6 | 7 | |
| 3 | 0 | 1 | 1 | 1 | 0 | $\ldots$ | | |
| 4 | 7 | 8 | 5 | 3 | 9 | $\ldots$ | | |
| | $\ldots$ | | | | | | | |

# Real Numbers between 0 and 1

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | 3 | 3 | 3 | 3 | ... |
| 2 | 1 | 3 | 3 | 4 | 5 | 6 | 7 |
| 3 | 0 | 1 | 1 | 1 | 0 | ... | |
| 4 | 7 | 8 | 5 | 4 | 9 | ... | |

...

...

# Real Numbers between 0 and 1

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | 3 | 3 | 3 | 3 | … |
| 2 | 1 | 3 | 3 | 4 | 5 | 6 | 7 |
| 3 | 0 | 1 | 1 | 1 | 0 | … | |
| 4 | 7 | 8 | 5 | 4 | 9 | … | |

…

…

$$|\mathbb{N}| < |R|$$

# The Set of Problems

$\aleph_0$

| | 0 | 1 | 2 | 3 | 4 | 5 | $\ldots$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | $\ldots$ |
| 2 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | $\ldots$ | |
| 4 | 1 | 1 | 0 | 1 | 1 | $\ldots$ | |

$\ldots$

$\ldots$

# The Set of Problems

$\aleph_0$

| | 0 | 1 | 2 | 3 | 4 | 5 | $\ldots$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | $\ldots$ |
| 2 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | $\ldots$ | |
| 4 | 1 | 1 | 0 | 1 | 1 | $\ldots$ | |

$\ldots$

$\ldots$

$$|Progs| = |\mathbb{N}| < |Probs|$$

# Halting Problem

Assume a program $H$ that decides for all programs $A$ and all input data $D$ whether

- $H(A, D) \stackrel{\text{def}}{=} 1$ iff $A(D)$ terminates
- $H(A, D) \stackrel{\text{def}}{=} 0$ otherwise

# Halting Problem (2)

Given such a program $H$ define the following program $C$: for all programs $A$

- $C(A) \stackrel{\text{def}}{=} 0$ iff $H(A, A) = 0$
- $C(A) \stackrel{\text{def}}{=}$ loops otherwise

# Contradiction

$H(C, C)$ is either $0$ or $1$.

- $H(C, C) = 1 \overset{\text{def}\,H}{\Rightarrow} C(C) \downarrow \overset{\text{def}\,C}{\Rightarrow} H(C, C) = 0$
- $H(C, C) = 0 \overset{\text{def}\,H}{\Rightarrow} C(C)$ loops $\overset{\text{def}\,C}{\Rightarrow}$
  $H(C, C) = 1$

Contradiction in both cases. So $H$ cannot exist.

# Take Home Points

- there are sets that are more infinite than others
- even with the most powerful computer we can imagine, there are problems that cannot be solved by any program
- in CS we actually hit quite often such problems (halting problem)

# Big Thank You!

- It is always fun to learn new things in CFL
- I want to add Higher-Order Functions and Algebraic Datatypes to Fun

# Big Thank You!

- Thanks for ALL the EoY feedback:

  *"If all modules were as good as this one I would start recommending KCL over basically every single university instead of suggesting people look somewhere else."*