

## Homework 2

1. Review the first handout about sets of strings and read the second handout. Assuming the alphabet is  $\{a, b\}$ , decide which of the following equations are true in general for arbitrary languages  $A$ ,  $B$  and  $C$ :

$$(A \cup B)@C = A@C \cup B@C$$

$$A^* \cup B^* = (A \cup B)^*$$

$$A^*@A^* = A^*$$

$$(A \cap B)@C = (A@C) \cap (B@C)$$

In case an equation is true, give an explanation; otherwise give a counterexample.

2. What is the meaning of a regular expression? Give an inductive definition.
3. Given the regular expressions  $r_1 = \epsilon$  and  $r_2 = \emptyset$  and  $r_3 = a$ . How many strings can the regular expressions  $r_1^*$ ,  $r_2^*$  and  $r_3^*$  each match?
4. Give regular expressions for (a) decimal numbers and for (b) binary numbers. (Hint: Observe that the empty string is not a number. Also observe that leading 0s are normally not written.)
5. Decide whether the following two regular expressions are equivalent ( $\epsilon + a$ ) $^* \equiv a^*$  and  $(a \cdot b)^* \cdot a \equiv a \cdot (b \cdot a)^*$ .
6. Given the regular expression  $r = (a \cdot b + b)^*$ . Compute what the derivative of  $r$  is with respect to  $a$  and  $b$ . Is  $r$  nullable?
7. Prove that for all regular expressions  $r$  we have

$$\text{nullable}(r) \quad \text{if and only if} \quad "" \in L(r)$$